Mathematical And
Philosophical Recreations
Vol-3. Part-6.

MATHEMATICAL

AND

PHILOSOPHICAL

RECREATIONS.

PART SIXTH.

Containing the easiest and most curious Problems, as well as the most interesting truths, in Astronomy and Geography, both Mathematical and Physical.

OF all the parts of the mathematics, none are better calculated to excite curiosity than astronomy in its different branches. Nothing indeed can be a stronger proof of the power and dignity of the human mind, than its having been able to raise itself to such abstract knowledge as to discover the causes of the phenomena exhibited by the revolution of the heavenly bodies; the real construction of the universe; the respective distances of the bodies which compose it, &c. At all times therefore this study has been considered as one of the sublimest efforts of genius, and Ovid himself, though a poet, never expresses his thoughts on this subject but with a sort of enthusiasm. Thus, when speaking of the erect posture of man, he says:

Cunctaque cum spectent animalia cætera terram,
Os homini sublime dedit, cælumque tueri
Jussit et erectos in sidera tollere vultus
Metamorph. Lib. 1.

In another place, speaking of astronomers, he says:

Felices animæ! quibus hæc cognoscere primis
Inque domos superas scandere cura fuit.
Credibile est illos pariter vitiisque, jocisque,
Altius humanis exeruisse caput.
Non venus aut vinum sublimia pectora fregit,
Officiumve fori, militiæve labor;
Nec levis ambitio, perfusaque gloria fuco,
Magnarumve fames sollicitavit opum.
Admovere oculis distantia sidera nostris,
Ætheraque ingenio supposuere suo.

If astronomy at that period excited admiration, what ought it not to do at present, when the knowledge of this science is far more extensive and certain than that of the ancients; who as we may say were acquainted only with the rudiments of it! How great would have been the enthusiasm of the poet, how sublime his expressions, had he foreseen only a part of the discoveries which the sagacity of the moderns has enabled them to make with the assistance of the telescope!—The moons which surround Jupiter and Saturn; the singular ring that accompanies the latter; the rotation of the sun and planets around their axes; the various motions of the earth; its immense distance from the sun; the still more incredible distance of the fixed stars; the regular course of the comets; the discovery of new planets and comets; and in the last place, the arrangement of all the celestial bodies, and their laws of motion, now as fully demonstrated as the truths of geometry. With much more reason would we have called those who have ascended to these astronomical truths, and who have placed them beyond all doubt, privileged beings, and of an order superior to human nature.

CHAPTER I.

Elementary Problems of Astronomy and Geography.

PROBLEM I.

To find the Meridian Line of any Place.

THE determination of the meridian line, is certainly the basis of every operation, both in astronomy and geography; for which reason we shall make it the first problem relating to this subject.—There are several methods of determining this line, which we shall here describe.

1. On any horizontal plane, fixed obliquely, and in a firm manner, a spike or sharp pointed piece of iron, with the point uppermost, as AB, pl. 1 fig. 1. Then provide a double square, that is, two squares joined together so as to form an angle, and by its means find, on the horizontal plane, the point C, corresponding in a perpendicular direction with the summit of the style. From this point describe several concentric circles, and mark, in the forenoon, where the summit of the shadow touches them. Do the same thing in the afternoon; and the two points D and E being thus determined in the same circle, divide into two equal parts the arc intercepted between them. If a straight line be then drawn through the centre, and this point of bisection, it will be the meridian line required.

By taking two points in one of the other circles, and repeating the same operation; if the two lines coincide, it will be a proof, or at least afford a strong presumption, that the operation has been accurately performed: if they do not coincide, some error must have arisen; and therefore it will be necessary to recommence the operation with more care.

Two observations, the least distant from noon, ought in general to be preferred; both because the sun is then more brilliant and the shadow better defined, and because the change in the sun's declination is less; for this operation

supposes that the sun neither recedes from nor approaches to the equator, at least in a sensible manner, during the interval between the two observations.

In short, provided these two observations have been made between 9 o'clock in the morning and 3 in the afternoon, even if the sun be near the equator, the meridian found by this method will be sufficiently exact, in the latitude of from 45 to 60 degrees; for we have found that, in the latitude of Paris, and making the most unfavourable suppositions, the quantity which such a meridian may err, will not be above 20″. If it be required with perfect exactness, nothing is necessary but to make choice of a time when the sun is either in one of the tropics, particularly that of Cancer, or very near it, so that in the interval between the two operations his declination may not have sensibly changed.

We are well aware that, for the nice purposes of astronomy, something more precise will be necessary; but the object of this work is merely to give the simplest and most curious operations in this science. The following however is a second method of finding the meridian by means of the pole star.

II. To determine the meridian line in this manner, it will be necessary to wait till the pole star, which we here suppose to be known, has reached the meridian. But this will be the case when that star and the first in the tail of the Great Bear, or the one nearest the square of that constellation, are together in the same line perpendicular to the horizon; for about the year 1700 these two stars passed over the meridian exactly at the same time; so that when the star in the Great Bear was below the pole, the polar star was above it; but though this is not precisely the case at present, these stars, as we shall here show, may be still employed for several years, and without any sensible error.

Having suspended a plumb line in a motionless state,

wait till the pole star, and that in the Great Bear above described, are together concealed by the thread; and at that moment suspend a second plumb line, in such a manner that it shall hide the former and the two stars. These two threads will then comprehend between them a plane which will be that of the meridian; and if the two points on the ground, corresponding to the extremities of the two plumb lines, be joined by a straight line, you will have the direction of the meridian.

The hour at which the pole-star, or any other star, passes the meridian on any given day, may be easily found by a calculation, for which precepts are given in the Nautical Almanac, in White's Ephemeris, and most books on practical Astronomy; but, to save trouble, we shall here present the reader with a table containing the precise time at which the pole-star passes the meridian, both above and below the pole, on the first day of every month.

Months.	Above the pole.			Below the pole.		
January	6h	6m	Ev.	6h	8m	Mor.
February	3	55		3	57	
March	2	6		2	8	
April	0	12		0	14	
May	10	14	Mor.	10	12	Ev.
June	8	11		8	9	
July	6	7		6	5	
August	4	2		4	0	
September	2	7		2	5	
October	0	19		0	17	
November	10	27	Ev.	10	29	Mor.
December	8	24		8	26	

This table indeed is calculated only for the year 1802; but the pole-star changes its place so little, that the difference cannot amount to more than 3 or 4 minutes in half a century.

Attention however must be paid to the day of the

month; for, from the beginning of any month to the end there is a difference of nearly two hours. The daily anticipation being $3^m\ 56^s$ per day*, $3^m\ 56^s$ must be multiplied by the number of days of the month which have elapsed, and the product must be subtracted from the time of the star's passing the meridian on the first of the month, as given in the table: the remainder will be the time of its passage on the proposed day.

Thus, if it were required to trace out a meridian by the pole-star on the 15th of March, multiply $3^m\ 56^s$ by 14, which will give 55^m; and if 55^m be subtracted from $2^h\ 8^m$, the remainder $1^h\ 13^m$ will be the hour in the morning when the pole star passes the meridian, below the pole, on the 15th of March.

On account of the great length of the days in some months, such as June, July, and part of August, neither of these passages is visible; as they take place in the day, or during the twilight. This inconvenience however may be remedied in the following manner. Find the hour at which the pole-star will pass the meridian above the pole on the proposed day, and then examine whether, by counting 6 hours more, that hour will fall in the night-time: should this be the case, wait for that moment, and then proceed according to the rules above given. By these means you will obtain the position of the azimuth circle passing through the zenith and the pole-star, when it has attained to its greatest distance towards the west; for if it passes the meridian at a certain hour, it is evident that 6 hours after it will be at its greatest distance from it. But

* If the earth had only a diurnal, without an annual motion, any given meridian would revolve from the sun to the sun again in the same time as from any star to the same star again; because the sun would never change his place in regard to the stars. But as the earth advances almost a degree eastward in its orbit, in the time that it turns eastward round its axis, whatever star passes over the meridian on any day with the sun, will pass over the same meridian on the next day when the sun is almost a degree short of it; that is 3 minutes 56 seconds sooner.

it will be found by calculation that the angle which this azimuth forms with the meridian for the latitude of London, 51° 31′, is 3° 11′; therefore if a line be drawn in such a manner as to form with the line found, an angle of 3° 11′ towards the east, you will have a true meridian line.

If the 6 hours counted after the star's passage of the meridian above the pole, do not fall in the night, nothing is necessary but to count 6 hours less: the hour thus found will certainly be one of those of the night, and will show the time when the pole-star will be at its greatest distance from the meridian towards the east; in this case the angle of 3° 11′ must be laid off towards the west.

It will perhaps be found troublesome to make an angle of 3° 11′; but it may be done in the following manner. In the line from which you are desirous of laying off an angle of 3° 11′, assume any point A, pl. 1 fig. 2; and from that point, towards the north, take the length of 1000 lines, or 6 feet 11 in. 4 lin. from the point B, where this length terminates, raise a perpendicular towards the west, if the proposed angle is intended to be laid off on that side, or towards the east if intended to be laid off on the other. On this perpendicular set off 55½ lines; and let this length terminate at the point C: if AC be then drawn, it will form with AB the required angle of 5° 11′; and this angle will be much more exact than if any other method had been employed.

REMARK.—As several physical methods of finding a meridian line are given in the preceding editions of this work, it is necessary that we should here mention them; were it only that the reader may be able to appreciate how far they are likely to answer the purpose.

To find the meridian without a compass or magnetic needle, some have proposed the following method, which would answer, they say, in the bowels of the earth. Take a common small sewing needle exceedingly well polished, and lay it gently on the surface of some water in a state

of perfect rest in any vessel: this needle, they tell us, will place itself in the direction of the meridian.

This experiment, in some respects, is true: if the needle is long and delicate, it will remain at the surface of the water, where it will form for itself a small cavity; the air which adheres to it will preserve it for some time from coming into contact with the water; and if this should not be the case, the same effect might be produced by greasing it with a little tallow: it will then easily maintain itself on the water, and will move till it approaches the direction of the meridian. This we have often confirmed by experiment.

But it is false that the direction it assumes is the exact meridian of the place, for it is only the magnetic meridian, because every long slender piece of iron, when delicately suspended, is a magnetic needle. The magnetic meridian however is only the direction of the current of the magnetic fluid; and this direction, as is well known, forms in almost every part of the earth an angle of greater or less extent, with the astronomical or true meridian. At London, for example, at present (1813), it is 24° 16′½. Besides, unless the north and south points were known, it would be impossible in this manner to distinguish them from each other.

Kircher proposes a method by which he says, that the south and the north may be easily known. If the trunk of a very straight tree, growing in the middle of a plane, at a distance from any eminence or other shelter, that could defend it from the wind or the sun, be cut horizontally, several curved lines closer on the one side than the other, will be observed on the section. The side where the curved lines are closest will be the north; because the cold coming from that quarter contracts, while the heat coming from the other dilates the juices, and other matter of which the strata of the tree are formed.

There is some truth and reason in the principle on which this method is founded; but, besides that all trees do not

exhibit this phenomenon, it is not true that the north wind is every where the coldest: it is often, according to the position of the place, the north-west or the north-east: in this case, one of these points would be mistaken for the north.

PROBLEM II.

To find the Latitude of any place.

The latitude of any place on the earth, is its distance from the equator; and is measured by an arc of the celestial meridian, intercepted between the zenith of the place and the equator; for this arc is similar to that comprehended on the earth between the place and the terrestrial equator. This is equal to the elevation of the pole, which is the arc of the meridian intercepted between the pole and the horizon. To those therefore who live under the equator, the poles are in the horizon; and if there were inhabitants at either pole, the equator would be in their horizon.—The latitude of any place on the earth may be easily found by various methods.

1st. By the meridian altitude of the sun on any given day. For if the sun's declination for that day, when the sun is in any of the northern signs, and the given place in the northern hemisphere, be subtracted from the altitude, the remainder will be the elevation of the equator, the complement of which is the elevation of the pole, or the latitude. If the sun be in any of the southern signs, it may be readily seen that, to find the elevation of the equator, the declination must be added.

2d. If the meridian altitude of one of the circumpolar stars, which do not set, be taken twice in the course of the same night, namely once when directly above the pole, and again when exactly below it; and if from each of these altitudes the refraction be subtracted; the mean between these two altitudes will be that of the pole, or the latitude. Or, take any two altitudes of such a star at the interval of

$11^h\ 58^m$ of time, correcting them by subtracting the refractions as before; then the mean between them will be the height of the pole, or the latitude of the place.

3d. Look, in some catalogue of the fixed stars, for the distance of any star from the equator, that is, its declination; then take its meridian altitude, and by adding or subtracting the declination, you will have the elevation of the equator, the complement of which, as before said, is the latitude.

PROBLEM II.

To find the Longitude of any place on the earth.

The longitude of any place, or the second element of its geographical position, is the distance of its meridian from a certain meridian, which by common consent is considered as the first. This first meridian is commonly supposed to be that passing through the island of Ferro, the most eastern of the Canaries. But the meridian of the observatory of Paris is for the most part used by the French, and that of the Royal Observatory of Greenwich by the English.

Formerly the longitude was reckoned, from west to east, throughout the whole circumference of the equator; but at present it is almost the general practice to reckon both ways from the first meridian, or the meridian accounted as such; that is to say east and west, so that the longitude according to this method can never exceed 180 degrees: and in the tables it is marked whether it be east or west. We shall now proceed to show in what manner the longitude is determined.

If two terrestrial meridians, distant from each other 15°, for example, be supposed to be continued to the heavens; it is evident that they will intercept, in the equator and all its parallels, arcs of the same number of degrees. It may be readily seen also that the sun will arrive first at the more eastern meridian, and that he will

then have to pass over 15° in the equator, or the parallel which he describes that day during his diurnal rotation, before he arrives at the more western meridian. But to pass over 15° the sun requires one hour, since he employs 24 hours to pass over 360°; hence it follows, that when it is noon at the more eastern place, it will be only 11 o'clock in the morning at the more western. If the distance of the meridians of the two places be greater or less, the difference of the hours will be greater or less, in the proportion of one hour for 15°; and consequently of 4 minutes for a degree, 4 seconds for a minute, and so on.

Thus it is seen, that to determine the longitude of a place, nothing is necessary but to know what hour it is there, when it is a certain hour in another place situated under the first meridian, or the distance of which from the first meridian is known; for if this difference of time be changed into degrees and parts of a degree, allowing 15° for one hour of time, one degree for 4 minutes, and so on, then the longitude of the proposed place will be obtained.

To find this difference of hours, the usual method is to employ the observation of some celestial phenomenon, that happens exactly at the same moment to every place on the earth, such for example as eclipses of the moon. Two observers stationed at two places, the difference of the longitude of which is required, observe, by means of a well regulated clock, the moments when the shadow successively reaches several remarkable spots on the moon's disk; they then compare their observations, and by the difference of the time which they reckoned when the shadow reached the same spot, they determine, as above explained, the difference of the longitude of the two places.

Let us suppose, by way of example, that an observer at London found, by observation, that the shadow reached the spot called Tycho at 1h 45m 50s in the morning;

and that another stationed at a place A made a similar observation at 24m 30s after midnight: the difference of this time is 1h 21m 20s, which reduced to degrees and minutes of the equator, gives 20° 20′. This is the difference of longitude; and as it was later at London when the phenomenon was observed, than at the place A, it thence follows that the place A is situated 20° 20′ farther west than London.

As eclipses of the moon are very rare, and as it is difficult to observe with precision when the shadow comes into contact with the moon's disk, so as to determine the commencement of the eclipse, and also the exact period when the shadow reaches any particular spot, the modern astronomers make use of the immersions, that is the eclipses, of Jupiter's satellites, and particularly those of the first, which, as it moves very fast, experiences frequent eclipses that end in a few seconds. The case is the same with the emersion or return of light to the satellite, which takes place almost instantaneously. For the sake of illustration we shall suppose that an observer, stationed at the place A, observes an immersion of the first satellite to have happened on a certain day at 4h 55m, in the morning; and another stationed at a place B at 3h 25m. The difference being 1h 30m, it gives 22° 30′ for the difference of longitude. We may therefore conclude that the place A is farther to the east than B, since the inhabitants at the former reckoned an hour more at the time of the phenomenon.

Remark.—These observations of the satellites, which since the discovery of those of Jupiter, have been often repeated in every part of the globe, have in some measure made an entire reformation in geography; for the position in longitude of almost all places was determined merely by itinerary distances very incorrectly measured; so that in general the longitudes were counted much greater than they really were. Towards the end of the seventeenth

century there were more than 25° to be cut off from the extent in longitude assigned to the old continent from the western ocean to the eastern coast of Asia.

This method, so evident and demonstrative, was however criticised by the celebrated Isaac Vossius, who preferred the itinerary results of travellers, or the estimated distances of navigators; but by this he only proved that, though he possessed a great deal of erudition badly digested, he had a weak judgment, and was totally unacquainted with the elements of astronomy.

A knowledge of the latitude and longitude of the different places of the earth, is of so much importance to astronomers, geographers, &c, that we think it our duty to give a table of those of the principal places of the earth. This table, which is very extensive, contains the position of the most considerable towns both in England and in France, as well as of the greater part of the capitals and remarkable places in every quarter of the globe; the whole founded on the latest astronomical observations, and the best combinations of distances and positions.

The reader must observe, that the longitude is reckoned from the meridian of London, both east and west. When east it is denoted by the letter E, and when west by the letter W. In regard to the latitude it is distinguished, in the same manner, by the letters N and S, which denote north and south.

A TABLE

CONTAINING THE LONGITUDES AND LATITUDES OF THE CHIEF TOWNS AND MOST REMARKABLE PLACES OF THE EARTH.

Names of places.	Countries.	Latitude, or el of the pole.	Longitude, or dif of merids.
Abbeville .	France .	50° 7′N	1° 55′E
Aberdeen .	Scotland .	57 6 N	1 44 W
Abo .	Finland .	60 27 N	22 15 E
Acapulco .	America	17 30 N	106 23 W
Achen .	Sumatra .	5 22 N	9› 40 E
Adrianople .	Turkey .	41 40 N	26 31 E
Agra .	India .	26 43 N	76 49 E
Aleppo .	Syria .	35 45 N	37 25 E
Alxandretta .	Syria .	36 35 N	36 20 E
Alexandria .	Egypt .	31 11 N	30 17 E
Algiers .	Algiers .	36 49 N	2 18 E
Alicant .	Spain .	38 31 N	0 7 W
Altona .	Germany .	53 38 N	9 55 E
Altorf .	Germany .	49 17 N	11 11 E
Amiens .	France .	49 53 N	2 23 E
Amboyna I. .	India .	4 25 N	127 25 E
Amsterdam .	Holland .	52 23 N	4 52 E
Anabona I. .	Ethiopia .	2 36 S	5 35 E
Ancona .	Italy .	43 38 N	13 31 E
Andrews St .	Scotland .	56 18 N	2 37 W
Angers .	France .	47 28 N	0 31 W
Angoulême .	France .	45 39 N	0 14 E
Anapolis Royal .	Nova Scotia .	44 52 N	64 0 W
Antego I. .	Caribbee .	16 57 N	62 4 W
Antibes .	France .	43 35 N	7 14 E
Antiochetta .	Syria .	36 8 N	36 17 E
Antwerp .	Flanders .	51 13 N	4 24 E
Archangel .	Russia .	64 34 N	38 59 E
Arcot .	India .	12 51 N	79 33 E
Arles .	France .	43 40 N	4 43 E
Arras .	France .	50 18 N	2 50 E
Ascension I. .	Brazil .	7 56 S	14 16 W

Names of places.		Countries.	Lat. or el. of the pole.	Lon. or dif. of merids.
Astracan	.	Siberia	46° 21′ N	48° 8′ E
Athens	.	Turkey	38 5 N	23 52 E
Auch	.	France	43 39 N	0 40 E
Augustine St.	.	Florida	30 10 N	81 29 W
Augsburg	.	Germany	48 24 N	10 26 E
Avignon	.	France	43 57 N	4 54 E
Avranches	.	France	48 41 N	1 18 W
Aurillac	.	France	44 55 N	2 32 E
Auxerre	.	France	47 48 N	3 39 E
Awatcha	.	Kamtschatka	53 1 N	158 30 E
Azoph	.	Crimea	47 10 N	40 55 E
Bagdad	.	Mesopotamia	33 20 N	44 26 E
Bahama I.	.	America	26 45 N	78 35 W
Baldivia	.	Chili	39 38 S	73 20 W
Bale	.	Swisserland	45 55 N	7 40 E
Bangalore	.	India	13 0 N	77 42 E
Bantry Bay	.	Ireland	51 45 N	10 46 W
Barcelona	.	Spain	41 26 N	2 18 E
Bassora	.	Arabia	29 45 N	47 40 E
Batavia	.	Java I.	6 12 S	106 45 E
Bayeux	.	France	49 16 N	0 38 W
Bayonne	.	France	43 30 N	1 30 W
Beechy Head	.	England	50 44 N	0 25 E
Belfast	.	Ireland	54 43 N	5 52 W
Bencoolen	.	Sumatra I.	3 49 S	102 5 E
Belgrade	.	Turkey	45 3 N	21 27 E
Bender	.	Turkey	46 50 N	29 41 E
Bengal	.	India	22 0 N	92 45 E
Bergen	.	Norway	60 10 N	6 14 E
Berlin	.	Germany	52 33 N	13 26 E
Bermuda	.	Bahama I.	32 35 N	63 23 W
Berne	.	Swisserland	46 58 N	7 31 E
Berwick	.	England	55 45 N	1 50 W
Besançon	.	France	47 13 N	6 8 E
Bezieres	.	France	43 20 N	3 18 E
Bilboa	.	Spain	43 26 N	3 18 W
Blois	.	France	47 35 N	1 24 E

Names of places.	Countries.	Lat. or el. of the pole.	Lon. or dif. of merids.
Bologna .	Italy .	44° 29′ N	11° 26′ E
Bolkereskoy .	Kamtschatka	52 54 N	156 25 E
Bombay .	India .	18 57 N	72 43 E
Borneo .	Borneo I. .	5 0 N	112 15 E
Boston .	England .	53 10 N	0 25 E
Boston .	America .	42 25 N	70 32 W
Botany Bay .	N. Holland .	34 6 S	151 20 E
Boulogne .	France .	50 44 N	1 40 E
Bourdeaux .	France .	44 50 N	0 30 W
Bourg-en-Bresse	France .	40 12 N	5 19 E
Bourges .	France .	47 4 N	2 28 E
Bremen .	Germany .	53 30 N	9 0 E
Breslaw .	Silesia .	51 3 N	17 13 E
Brest .	France .	48 23 N	4 26 W
Bridge Town .	Barbadoes I. .	13 5 N	59 36 W
Bristol .	England .	51 27 N	2 35 W
Bruges .	Flanders .	51 11 N	3 12 E
Brussels .	Flanders .	50 51 N	4 27 E
Buchan-ness .	Scotland .	57 29 N	1 23 W
Bucharest .	Wallachia .	44 27 N	26 13 E
Buda .	Turkey .	47 28 N	19 51 E
Buenos Ayres .	Brasil .	34 35 S	58 26 W
Cadiz .	Spain .	36 31 N	6 7 W
Caen .	France .	49 11 N	0 17 W
Caffa .	Crimea .	44 45 N	35 55 E
Cagliari .	Sardinia I. .	39 25 N	9 38 E
Cairo .	Egypt .	30 2 N	31 26 E
Calais .	France .	50 57 N	1 56 E
Calcutta .	India .	22 35 N	88 34 E
Calicut .	India .	11 15 N	75 39 E
Callao .	Peru .	12 2 S	76 53 W
Camboida .	India .	10 35 N	104 45 E
Cambray .	France .	50 10 N	3 19 E
Cambridge .	England .	52 13 N	0 9 E
Canaria I. .	Canaries .	28 1 N	15 0 W
Candia .	Ceylon .	7 54 N	81 53 E
Canterbury .	England .	51 17 N	1 22 E

Names of places.	Countries.	Lat. or el. of the pole.	Lon or dif. of merids.
Cape Comorin .	India .	7° 55′ N	78° 7′ E
Cape Finisterre .	Spain .	42 52 N	9 12 W
Cape François .	St. Domingo I.	19 57 N	71 22 W
Cape Town .	Caffraria .	33 55 S	18 23 E
Cape Kamtschatka	Russia .	51 3 N	160 12 E
Cape Ortegal .	Bay of Biscay .	43 47 N	7 34 W
Cape St. Lucas .	California .	23 28 N	109 20 W
Cape Verd .	Negroland .	14 45 N	17 28 W
Caracas .	South America	10 6 N	66 45 W
Carcassone .	France .	43 12 N	2 25 E
Carlescrona .	Sweden .	56 20 N	15 31 E
Carlisle .	England .	54 47 N	2 35 W
Carthagena .	Spain .	37 37 N	1 3 W
Carthagena .	South America	10 27 N	75 22 W
Casan .	Russia .	55 45 N	48 40 E
Cassel .	Germany .	51 19 N	9 21 E
Castres .	France .	43 57 N	2 20 E
Cayannebourg .	Finland .	64 13 N	41 9 E
Cayenne I. .	South America .	4 50 N	52 10 W
Cay St. Louis .	St. Domingo I. .	18 19 N	73 1 W
Cephalonia I. .	Turkey .	38 20 N	20 11 E
Cette .	France .	43 20 N	0 21 W
Ceuta .	Barbary .	35 49 N	5 25 W
Cezene .	Italy .	44 8 N	12 17 E
Chalons-sur-Marne	France .	48 57 N	0 23 E
Chalons-sur-Saône	France .	46 47 N	4 56 E
Chandernagor	Bengal .	22 51 N	88 34 E
Charlestown .	Carolina .	33 22 N	79 50 W
Chartres .	France .	48 26 N	1 34 E
Cherbourg .	France .	49 28 N	1 33 W
Chester .	England .	53 10 N	2 25 W
Christiana .	Norway .	59 25 N	10 30 E
Christianstadt	Sweden .	62 47 N	22 50 E
Civita Vecchia	Italy .	42 5 N	11 41 E
Clagenfurth .	Carinthia .	47 20 N	14 57 E
Clermont-Ferrand	France .	45 46 N	3 10 E
Cochin .	India .	9 50 N	76 5 E

Names of places.	Countries.	Lat. or el. of the pole.	Lon. or dif. of merids
Colchester	England	52°00′ N	0°58′ E
Collioure	France	42 31 N	3 10 E
Cologne	Germany	50 55 N	7 10 E
Compiegne	France	49 25 N	2 55 E
Conception la	Chili	36 43 S	73 13 W
Congo R.	Congo	5 45 S	11 53 E
Constance	Swisserland	47 42 N	8 58 E
Constantinople	Turkey	41 00 N	28 53 E
Copenhagen	Denmark	55 41 N	12 40 E
Cordova	Spain	[illegible] 42 N	3 47 W
Corfu	Turkey	39 50 N	19 48 E
Corinth	Turkey	37 30 N	23 00 E
Corke	Ireland	51 54 N	8 30 W
Corsica { N. part	Italy	42 53 N	9 40 E
Corsica { S. part	Italy	41 22 N	9 26 E
Coutance	France	49 3 N	1 22 W
Cowes	Isle of Wight	50 46 N	1 15 W
Cracow	Poland	50 10 N	19 55 E
Cremsmunster	Germany	48 3 N	14 8 E
Cruz St. I.	Antilles	17 53 N	64 55 W
Cuddalore	India	11 41 N	79 51 E
Curassoa	West Indies	11 56 N	68 20 W
Cusco	Peru	12 25 S	73 35 W
Dabul	India	18 24 N	73 33 E
Danzic	Poland	54 22 N	18 39 E
Dartmouth	England	50 27 N	3 36 W
Deseada I.	Caribbees	16 36 N	61 10 W
Dieppe	France	49 55 N	0 9 E
Dijon	France	47 19 N	3 7 E
Dilingen	Germany	48 30 N	10 19 E
Dol	France	48 33 N	1 41 W
Dole	France	45 5 N	5 34 E
Domingo St.	Antilles	18 25 N	69 30 W
Dordrecht	Netherlands	52 00 N	4 26 E
Dover	England	51 7 N	1 24 E
Dresden	Saxony	51 6 N	13 31 E
Drontheim	Norway	63 26 N	11 8 E

Names of places.	Countries.	Lat. or el. of the pole.	Lon. or dif. of merids.
Dublin	Ireland	53°21′N	6° 10′W
Dunbar	Scotland	55 58 N	2 22 W
Dundee	Scotland	56 26 N	2 48 W
Dungeness	England	50 55 N	1 3 E
Dunkirk	France	51 2 N	2 27 E
Durazzo	Turkey	41 58 N	25 00 E
Edinburgh	Scotland	55 58 N	3 7 W
Elba I.	Italy	42 52 N	10 38 E
Elbing	Poland	54 12 N	20 35 E
Elsinburg	Sweden	56 00 N	13 35 E
Elsinore	Denmark	56 00 N	13 23 E
Embden	Germany	53 5 N	7 26 E
Enchuysen	Holland	52 43 N	5 6 E
Ephesus	Natolia	38 00 N	27 53 E
Erfurth	Germany	51 6 N	10 20 E
Erivan	Armenia	40 30 N	41 25 E
Erzerum	Armenia	39 57 N	48 41 E
Eustatia	Caribbee	17 30 N	63 4 W
Faenza	Italy	44 17 N	11 55 E
Falmouth	England	50 8 N	4 58 W
Fernambouc	Brasil	8 13 S	35 5 W
Ferrara	Italy	44 50 N	11 40 E
Ferro I.	Canaries	27 48 N	17 40 W
Finisterre C.	France	42 52 N	0 12 W
Fladstrand	Denmark	57 27 N	10 37 E
Florence	Italy	43 46 N	11 7 E
Flushing	Holland	51 33 N	3 20 W
Forbisher's Straits	Greenland	62 5 N	47 18 W
Formosa I. N. p	China	25 25 N	121 25 E
Formosa I. SpE	China	22 00 N	120 40 E
Frankfort on the Mayn	Germany	50 6 N	8 40 E
Frankfort on the Oder	Germany	52 26 N	14 38 E
Fredeickstadt	Norway	59 00 N	11 10 E
Frejus	France	43 26 N	6 50 E
Gallipoli	Turkey	40 36 N	27 2 E

Names of places.	Countries.	Lat. or el. of the pole.	Lon. or dif of merids.
Gambia R.	Negroland	13°00′ N	14°58′ W
Geneva	Swisserland	46 12 N	6 5 E
Genoa	Italy	44 25 N	8 41 E
Ghent	Netherlands	51 4 N	3 47 E
Gibraltar	Spain	36 5 N	5 17 W
Glasgow	Scotland	55 52 N	4 10 W
Gloucester	England	51 50 N	2 16 W
Gluckstadt	Holstein	53 48 N	9 31 E
Goa	India	15 31 N	73 50 E
Gombroon	Persia	27 40 N	55 20 E
Good Hope Cape	Africa	34 29 S	18 28 E
Gottenburg	Sweden	57 42 N	11 44 E
Gottingen	Germany	51 32 N	9 58 E
Granville	France	48 50 N	1 32 W
Gratz	Styria	47 4 N	15 29 E
Greenwich	England	51 29 N	0 5 E
Grenoble	France	45 11 N	5 39 W
Grypswald	Pomerania	54 4 N	13 43 E
Guadaloupe I.	Caribbee	16 00 N	61 55 W
Guiaquil	Peru	2 10 S	81 5 W
Guernsey I.	England	49 30 N	2 47 W
Hague	Holland	52 4 N	4 22 E
Halifax	Nova Scotia	44 46 N	63 20 W
Halle	Saxony	51 31 N	11 46 E
Hamburgh	Germany	53 34 N	9 55 E
Harlem	Holland	52 24 N	4 10 E
Harwich	England	52 11 N	1 18 E
Hastings	England	50 52 N	0 46 E
Havannah	Cuba I.	23 12 N	82 13 W
Havre de Grace	France	49 30 N	0 11 E
Helena St. I.	Africa	15 55 S	5 44 W
Holy Head	Wales	53 23 N	4 40 W
Horn Cape	South America	55 59 S	67 21 W
Hull	England	53 50 N	0 28 W
Hydrabad	India	17 12 N	78 56 E
Jacoutsk	Russ. Tartary	62 20 N	129 46 E
Jafnapatan C.	Ceylon I.	9 47 N	80 55 E

Names of places.	Countries.	Lat. or el. of the pole.	Lon. or dif. of meridians.
Jago St. .	Cape Verd I. .	15° 7′ N	23°30′ W
Jamaica { W. end	West Indies .	18 45 N	78 00 W
Jamaica { E. end	West Indies .	18 00 N	76 40 W
Jassey .	Moldavia .	47 9 N	27 35 E
Java Head .	Java I. .	6 49 S	105 6 E
Jeddo .	Japan .	36 00 N	139 40 E
Jena .	Germany .	51 2 N	11 23 E
Jersey I. .	England .	49 7 N	2 26 W
Jerusalem .	Palestine .	31 55 N	35 25 E
Jeniseik .	Russ. Tartary .	58 27 N	91 25 E
Ingolstadt .	Germany .	48 46 N	11 28 E
Inspruc .	Tyrol .	47 18 N	12 00 E
Inverness .	Scotland .	57 33 N	4 2 W
Jsanna I. .	Zanguebar .	12 5 S	45 45 E
Joppa .	Syria .	32 45 N	36 00 E
Ipswich .	England .	52 14 N	1 00 E
Ismail .	Turkey .	45 21 N	28 55 E
Ispahan .	Persia .	32 25 N	52 55 E
Juan Fernandez I.	Chili .	33 45 S	78 37 W
Judda .	Arabia .	21 29 N	29 27 E
Ivica I. .	Spain .	38 54 N	1 15 E
Kamtschatka lower	Russia .	56 11 N	159 25 E
Kamtschatka upper	Russia .	54 48 N	162 10 E
Kilda St. I. .	Scotland .	57 44 N	8 18 W
Kinsale .	Ireland .	51 41 N	8 23 W
Kongkitao Cape	Corea .	37 30 N	116 27 E
Konigsberg .	Prussia .	54 42 N	21 23 E
Lancaster .	England .	54 42 N	4 36 W
Landau .	France .	49 11 N	8 13 E
Lands End .	England .	50 6 N	5 20 W
Landscrona .	Sweden .	55 52 N	12 55 E
Langres .	France .	47 50 N	5 26 E
Lausanne .	Swisserland .	46 31 N	6 50 E
Leeds .	England .	53 48 N	1 33 W
Leghorn .	Italy .	43 33 N	10 25 E
Leipsic .	Germany .	51 19 N	12 25 E
Leestoff .	England .	52 38 N	1 54 E

Names of places.	Countries.	Lat. or el. of the pole.	Lon. or dif. of merids.
Lepanto	Turkey	38°20′N	22° 3′E
Leyden	Holland	52 10 N	4 33 E
Liverpool	England	53 22 N	3 10 W
Liege	Germany	50 36 N	5 40 E
Lima	Peru	12 1 S	76 44 W
Limeric	Ireland	52 22 N	10 00 W
Lisbon	Portugal	38 42 N	9 4 W
Lizard	England	49 57 N	5 10 W
London	England	51 31 N	0 0
Londonderry	Ireland	55 1 N	7 31 W
Loretto	Italy	43 27 N	13 38 E
Louisburg	Cape Briton	45 54 N	59 50 W
Louvain	Netherlands	50 50 N	4 55 E
Lubec	Germany	54 00 N	11 40 E
Lucia St. I.	Caribbee	13 25 N	60 46 W
Lucca	Italy	43 50 N	10 35 E
Lunden	Sweden	55 42 N	13 20 E
Luxembourg	Netherlands	49 37 N	6 17 E
Lynn	England	52 46 N	0 30 E
Macao	China	22 12 N	113 46 E
Macassar	Celebes I.	5 9 S	119 50 E
Madras	India	13 5 N	80 34 E
Madrid	Spain	40 25 N	03 21 W
Madura	India	9 54 N	78 18 E
Mahon Port	Minorca	39 51 N	3 53 E
Majorca I.	Spain	39 35 N	2 35 E
Malacca	India	2 12 N	102 10 E
Malta I.	Italy	35 54 N	14 34 E
Manchester	England	53 24 N	2 20 W
Manilla	Luconia I.	14 36 N	120 58 E
Mantua	Italy	45 2 N	10 15 E
Marseilles	France	43 18 N	5 27 E
Martinico I.	West Indies	14 36 N	61 4 W
Masulipatam	India	16 28 N	81 40 E
Mauritius I.	Africa	20 10 S	57 33 E
Meaco	Japan	35 35 N	133 20 E
Meaux	France	48 58 N	2 55 E

Names of places.	Countries.	Lat. or el. of the pole.	Lon. or dif. of merids.
Mecca .	Arabia .	21° 40′ N	41° 00′ E
Mechlin .	Netherlands .	51 2 N	4 34 E
Medina .	Arabia .	24 58 N	39 53 E
Memel .	Courland .	55 48 N	22 23 E
Messina .	Sicily .	38 21 N	16 21 E
Metz .	France .	49 7 N	7 16 E
Mexico .	Mexico .	19 54 N	100 1 W
Milan .	Italy .	45 28 N	9 15 E
Mocha .	Arabia .	13 45 N	44 4 E
Modena .	Italy .	44 34 N	11 18 E
Montpelier .	France .	48 36 N	3 57 E
Montreal .	Canada .	45 52 N	73 11 W
Mosambique .	Zangue .	15 00 S	41 40 E
Moscow .	Russia .	55 45 N	37 51 E
Munich .	Germany .	48 10 N	11 35 E
Munster .	Germany .	52 00 N	7 40 E
Namur .	Netherlands .	50 25 N	4 50 E
Nangasaki .	Japan .	32 32 N	128 50 E
Nankin .	China .	32 7 N	118 35 E
Nantes .	France .	47 13 N	1 29 W
Naples .	Italy .	40 51 N	14 19 E
Narbonne .	France .	43 11 N	3 5 E
Narva .	Livonia .	59 23 N	29 27 E
Naze .	Norway .	57 50 N	7 32 E
Negapatnam .	India .	10 46 N	80 2 E
Nevis I. .	Caribbee .	17 11 N	62 52 W
Newcastle .	England .	55 3 N	1 28 W
Nice .	Italy .	43 42 N	7 22 E
Nieuport .	Flanders .	51 8 N	2 50 E
Nombre de Dios	South America	9 43 N	78 35 W
Nootka Sound .	America .	49 30 N	126 36 W
Noyon .	France .	49 34 N	1 44 E
Nuremberg .	Germany .	49 27 N	11 12 E
Ochozk .	Tartary .	59 20 N	143 18 W
Oczakow .	Turkey .	45 12 N	34 40 E
Olinda .	Brazil .	8 13 S	35 00 W
Olmutz .	Moravia .	49 43 N	17 37 E

Names of places.	Countries.	Lat. or el. of the pole.	Lon. or dif. of merids.
Oneglia .	Italy .	43° 57′ N	7° 52′ E
Oporto .	Portugal .	41 10 N	8 22 W
Oran .	Barbary .	35 45 N	0 00
Orenburg .	Astracan .	51 46 N	55 14 E
Orkney isles .	Scotland .	{ 59 24 N	3 23 W
		{ 58 44 N	2 11 W
Orleans New .	Louisiana .	30 00 N	89 54 W
Orleans .	France .	47 54 N	1 59 E
Ormus I. .	Persia .	27 30 N	55 17 E
Orotava .	Canaries .	28 23 N	16 19 W
Ostend .	Flanders .	51 14 N	3 00 E
Ozaca .	Japan .	35 10 N	134 5 E
Padua .	Italy .	45 22 N	11 59 E
Paita .	Peru .	5 20 S	80 35 W
Palermo .	Sicily .	38 10 N	13 43 E
Palikate .	India .	13 40 N	80 50 E
Pampeluna .	Spain .	42 44 N	1 35 W
Panama .	Mexico .	8 45 N	80 16 W
Panorma .	Turkey .	40 5 N	21 40 E
Para .	South America	1 30 S	47 5 W
Paris .	France .	48 50 N	2 25 E
Parma .	Italy .	44 45 N	10 00 E
Passau .	Germany .	48 30 N	13 5 E
Patmos I. .	Natolia .	37 22 N	26 48 E
Pavia .	Italy .	45 40 N	9 16 E
Pegu .	India .	17 00 N	96 58 E
Pekin .	China .	39 55 N	116 29 E
Perpignan .	France .	42 42 N	2 59 E
Petersburg .	Russia .	59 56 N	30 24 E
Philadelphia .	America .	39 57 N	75 8 W
Pico I. .	Azores .	38 29 N	28 19 W
Pisa .	Italy .	43 43 N	10 17 E
Plymouth .	England .	50 22 N	4 10 W
Pondicherry .	India .	11 42 N	79 58 E
Port Mahon .	Minorca I. .	39 51 N	3 53 E
Porto Bello .	New Spain .	9 33 N	79 45 W
Porto Praya .	C. Verde .	14 54 N	23 24 W

Names of places.		Countries.		Lat. or el. of the pole.	Lon. or dif. of merids.
Port Royal	.	Jamaica	.	17° 59′ N	76° 40′ W
Port Royal	.	Martinico	.	14 36 N	61 4 W
Port Royal	.	Acadia	.	45 2 N	65 00 W
Portsmouth	.	England	.	50 48 N	1 1 W
Prague	.	Bohemia	.	50 4 N	14 50 E
Presburg	.	Hungary	.	48 8 N	17 33 E
Quebec	.	Canada	.	46 49 N	71 10 W
Quiloa	.	Zanguebar	.	9 30 S	39 9 E
Quimper	.	France	.	47 58 N	4 2 W
Quinam	.	Cochin China		12 52 N	109 10 E
Quito	.	Peru	.	0 13 S	77 50 W
Ragusa	.	Dalmatia	.	42 45 N	20 00 E
Rajapoor	.	India	.	17 19 N	73 50 E
Ramsgate	.	England	.	51 20 N	1 22 E
Ratisbon	.	Germany	.	49 2 N	12 1 E
Ravenna	.	Italy	.	44 26 N	12 21 E
Rennes	.	France	.	48 6 N	1 37 W
Reims	.	France	.	49 14 N	4 8 E
Revel	.	Livonia	.	59 26 N	24 21 E
Riga	.	Livonia	.	56 56 N	23 44 E
Rimini	.	Italy	.	44 3 N	4 8 E
Rio Janeiro	.	Brazil	.	22 54 S	42 40 W
Rochelle	.	France	.	46 10 N	1 5 W
Rochester	.	England	.	51 20 N	0 30 E
Rome	.	Italy	.	41 54 N	12 34 E
Rostock	.	Germany	.	54 10 N	12 50 E
Rotterdam	.	Holland	.	51 56 N	4 33 E
Rouen	.	France	.	49 27 N	1 10 E
Rye	.	England	.	51 3 N	0 45 E
Saffia	.	Barbary	.	32 30 N	8 50 W
Saint-Flour	.	France	.	45 2 N	3 11 E
Saint-Malo	.	France	.	48 39 N	1 57 W
Saint-Omer	.	France	.	50 44 N	2 30 E
Salerno	.	Italy	.	40 39 N	14 48 E
Sallee	.	Barbary	.	33 58 N	6 20 W
Salonicha	.	Turkey	.	40 41 N	23 13 E
Sarragossa	.	Spain	.	41 40 N	0 39 W

Names of places.	Countries.	Lat. or el. of the pole.	Lon. or dif. of merid.
Scanderoon .	Syria .	36° 35′ N	36° 25′ E
Schamaki .	Persia .	40 30 N	37 5 E
Scilly Isles .	England .	50 00 N	6 45 W
Selinginsk .	Russ. Tartary	51 6 N	106 42 E
Senegal R. .	Negroland .	15 53 N	16 26 W
Senlis .	France .	49 13 N	2 39 E
Sens .	France .	48 12 N	3 22 E
Seringapatam .	India .	12 32 N	76 52 E
Seville .	Spain .	37 21 N	6 4 W
Sheerness .	England	51 25 N	0 50 E
Siam .	India	14 18 N	100 55 E
Sienna .	Italy	43 20 N	11 26 E
Sierra Leone .	Guinea .	8 30 N	12 7 W
Shields .	England .	55 2 N	1 20 W
Shetland I. .	Scotland .	60 47 N 59 51 N	0 10 W 1 31 W
Skalolt .	Iceland .	64 10 N	17 25 W
Smyrna .	Natolia .	38 28 N	27 25 E
Socatora I. .	Africa .	12 15 N	52 55 E
Soissons .	France .	49 21 N	3 24 E
Southampton .	England .	50 55 N	1 00 W
Spoletto .	Italy .	41 57 N	12 50 E
Spurn .	England .	53 35 N	0 30 E
Start Point .	England .	50 14 N	3 39 W
Stettin .	Pomerania .	53 36 N	15 25 E
Stockholm .	Sweden .	59 22 N	18 12 E
Stockton .	England .	54 33 N	1 15 W
Straelsund .	Germany .	54 23 N	14 10 E
Strasburgh .	France .	48 34 N	7 51 E
Stromness .	Orkneys .	58 56 N	3 26 W
Stuttgard .	Germany .	48 40 N	9 7 E
Sukadana .	Borneo I. .	1 00 S	110 40 E
Sunderland .	England .	54 55 N	1 00 W
Surat .	India .	21 10 N	72 28 E
Surinam .	South America	6 30 N	55 30 W
Swansey .	Wales .	51 40 N	4 25 W
Syracuse .	Sicily .	37 4 N	15 31 E

Names of places.		Countries.		Lat. or el. of the pole.	Lon. or dif. of merids.
Tangier	.	Barbary	.	35° 55′ N	5° 45′ W
Tarento	.	Italy	.	40 43 N	17 31 E
Tauris	.	Persia	.	38 5 N	46 55 E
Teflis	.	Georgia	.	42 55 N	46 25 E
Tellichery	.	India	.	11 42 N	75 30 E
Temeswar	.	Hungary	.	44 42 N	22 00 E
Teneriff Peak	.	Canaries	.	28 13 N	16 24 W
Tetuan	.	Barbary	.	35 27 N	4 50 W
Tinmouth	.	England	.	55 3 N	1 17 W
Thessalonica	.	Greece	.	48 36 N	23 13 E
Tobago I.	.	Caribbee	.	11 15 N	60 27 W
Tobolski	.	Siberia	.	58 12 N	68 20 E
Toledo	.	Spain	.	39 50 N	2 15 W
Tonquin	.	India	.	20 50 N	105 55 E
Tonsberg	.	Norway	.	58 50 N	10 5 E
Torbay	.	England	.	50 31 N	3 36 W
Tornea	.	Sweden	.	65 51 N	24 16 E
Toulon	.	France	.	43 7 N	0 2 E
Toulouse	.	France	.	43 36 N	1 31 E
Tours	.	France	.	47 23 N	0 40 E
Trente	.	Italy	.	45 43 N	10 45 E
Trieste	.	Carniola	.	45 51 N	14 3 E
Trinquemalee	.	Ceylon I.	.	8 50 N	83 24 E
Tripoli	.	Syria	.	34 53 N	30 7 E
Tripoli	.	Barbary	.	32 54 N	13 10 E
Truxilla	.	Peru	.	8 00 S	78 35 W
Tunis	.	Barbary	.	36 47 N	10 16 E
Turin	.	Italy	.	45 5 N	7 45 E
Tyrnau	.	Hungary	.	48 23 N	17 39 E
Valencia	.	Spain	.	39 30 N	0 40 W
Valladolid	.	Spain	.	41 42 N	5 34 W
Valpariso	.	Chili	.	33 3 N	72 14 W
Vannes	.	France	.	47 39 N	2 41 W
Venice	.	Italy	.	45 27 N	12 9 E
Vera Cruz	.	New Spain	.	19 12 N	97 25 W
Verona	.	Italy	.	45 26 N	11 24 E
Versailles	.	France	.	48 48 N	2 12 E

Names of places.		Countries.		Lat. or el. of the pole.	Lon. or dif. of merids.
Vienna	.	Germany	.	48° 13′ N	16° 28′ E
Vigo	.	Spain	.	42 14 N	8 23 W
Vilna	.	Poland	.	54 41 N	25 46 E
Viterbo	.	Italy	.	42 25 N	12 12 E
Upsal	.	Sweden	.	59 52 N	17 47 E
Uraniburg	.	Denmark	.	55 54 N	12 57 E
Urbino	.	Italy	.	43 43 N	12 43 E
Wardhus'	.	Lapland	.	70 23 N	51 12 E
Warsaw	.	Poland	.	52 14 N	21 5 E
Waterford	.	Ireland		52 7 N	7 42 W
Wells	.	England	.	53 7 N	1 00 E
Wexford	.	Ireland	.	52 13 N	6 56 W
Weymouth	.	England	.	52 40 N	2 34 W
Whitby	.	England	.	54 30 N	0 50 W
Whitehaven	.	England	.	54 25 N	3 15 W
Wicklow	.	Ireland	.	52 50 N	6 30 W
Wittenberg	.	Saxony	.	51 43 N	12 38 E
Wurtzburg	.	Franconia	.	49 46 N	10 19 E
Wybourg	.	Finland	.	60 55 N	30 20 E
Yamboa	.	Arabia	.	24 25 N	38 54 E
Yarmouth	.	England	.	52 55 N	1 40 E
Yellow River	.	China	.	34 6 N	120 10 E
Ylo	.	Peru	.	17 36 S	71 8 W
York New	.	America	.	40 43 N	74 4 W
Youghal	.	Ireland	.	51 46 N	8 6 W
Zacatula	.	Mexico	.	17 10 N	105 00 W
Zagrab	.	Croatia	.	46 6 N	10 19 E
Zante I.	.	Italy	.	37 50 N	21 30 E
Zara	.	Dalmatia	.	44 15 N	16 55 E
Zurich	.	Swisserland	.	47 22 N	9 21 E

PROBLEM IV.

To find what o'clock it is at any place of the earth, when it is a certain hour at another.

As the earth makes one revolution on its axis in the course of a common day, or of 24 hours, every point of the equator will describe the whole circle of 360 degrees in that period; and therefore if 360 be divided by 24, the quotient 15 will be the number of degrees that correspond to one hour of time. Hence it is evident that two places which are 15 degrees of longitude distant from each other, will differ one hour in their computation of time, one of them making it earlier or later according as it is situated to the east or west of the other. To determine this problem therefore, find by the preceding table the difference of longitude of the two places, which may be done by subtracting the longitude of the one from that of the other if they are both east or both west of London, or by adding them if the one is east and the other west, and then change the sum or difference into time: this time added to or subtracted from the hour at one of the given places, will give for result the hour at the other. If London be one of the places proposed, the difference of longitude will be found in the last column to the right in the preceding table.

To change the difference of longitude into time, multiply by 24, and divide by 360, or multiply by 4, and divide by 60; or only divide by 15; or find the hours and minutes corresponding to the given degrees and minutes in the subjoined table, which will greatly facilitate operations of this kind.

Now let it be proposed to find what o'clock it is at Cayenne, when it is noon at London. The difference of longitude, or of meridians, between London and Cayenne,

is 52° 10′; which converted into time, gives 3 hours 28 minutes 40 seconds; and as Cayenne lies to the west of London, if 3h 28m 40s be subtracted from 12 hours, the remainder will be 8 hours 31 minutes 20 seconds: hence it appears that when it is noon at London, it is only 8h 31m 20s in the morning at Cayenne; consequently when it is noon at Cayenne, it is 3h 28m 40s in the afternoon at London.

When it is noon at London, required the hour at Pekin? The difference of meridians between London and Pekin is 116° 29′, which is equal in time to 7 hours 45 minutes 56 seconds. But as Pekin lies to the east of London, these 7h 45m 56s must be added to 12 hours; and hence it is evident that when it is noon at London, it is 7h 45m 56s in the evening at Pekin. On the other hand, to find what o'clock it is at London when it is noon at Pekin, these 7h 45m 56s must be subtracted from 12 hours, and the result will be 4h 14m 4s in the morning.

When the two given places are both to the west of London, to find their difference of meridians, the longitude of the one must be subtracted from that of the other. If Madrid and Mexico, for instance, be proposed; as the longitude of the first is 3° 21′, and that of the second 100° 1′, if the former be subtracted from the latter, the remainder 96° 40′ will be their difference of longitude; which changed into time, gives 6 hours 26 minutes 40 seconds. Hence, when it is noon at Madrid, it is 5h 33m 20s in the morning at Mexico.

If one of the proposed places lies to the east and the other to the west of London, the longitude of the one must be added to that of the other, in order to have their difference of longitude; and the sum must then be converted into time, and added or subtracted as before.

By way of example we shall take Constantinople and Mexico, the former of which lies to the east of London. The longitude of Constantinople is 28° 53′, and that of Mexico 100° 1′, which added give for difference of longitude 128° 54′ = in time to 8h 35m 36s. When it is noon therefore at Constantinople, it is only 3h 24m 24s in the morning at Mexico; and when it is noon at the latter, it is 8h 35m 36s in the evening at Constantinople.

A table for changing degrees and minutes into hours, minutes, and seconds, or the contrary.

D / M	H M / M S	D / M	H M / M S	D / M	H M / M S	D / M	H M / M S
1	0 4	46	3 4	91	6 4	136	9 4
2	0 8	47	3 8	92	6 8	137	9 8
3	0 12	48	3 12	93	6 12	138	9 12
4	0 16	49	3 16	94	6 16	139	9 16
5	0 20	50	3 20	95	6 20	140	9 20
6	0 24	51	3 24	96	6 24	141	9 24
7	0 28	52	3 28	97	6 28	142	9 28
8	0 32	53	3 32	98	6 32	143	9 32
9	0 36	54	3 36	99	6 36	144	9 36
10	0 40	55	3 40	100	6 40	145	9 40
11	0 44	56	3 44	101	6 44	146	9 44
12	0 48	57	3 48	102	6 48	147	9 48
13	0 52	58	3 52	103	6 52	148	9 52
14	0 56	59	3 56	104	6 56	149	9 56
15	1 0	60	4 0	105	7 0	150	10 0
16	1 4	61	4 4	106	7 4	151	10 4
17	1 8	62	4 8	107	7 8	152	10 8
18	1 12	63	4 12	108	7 12	153	10 12
19	1 16	64	4 16	109	7 16	154	10 16
20	1 20	65	4 20	110	7 20	155	10 20
21	1 24	66	4 24	111	7 24	156	10 24
22	1 28	67	4 28	112	7 28	157	10 28
23	1 32	68	4 32	113	7 32	158	10 32
24	1 36	69	4 36	114	7 36	159	10 36
25	1 40	70	4 40	115	7 40	160	10 40
26	1 44	71	4 44	116	7 44	161	10 44
27	1 48	72	4 48	117	7 48	162	10 48
28	1 52	73	4 52	118	7 52	163	10 52
29	1 56	74	4 56	119	7 56	164	10 56
30	2 0	75	5 0	120	8 0	165	11 0
31	2 4	76	5 4	121	8 4	166	11 4
32	2 8	77	5 8	122	8 8	167	11 8
33	2 12	78	5 12	123	8 12	168	11 12
34	2 16	79	5 16	124	8 16	169	11 16
35	2 20	80	5 20	125	8 20	170	11 [illegible]
36	2 24	81	5 24	126	8 24	171	11 24
37	2 28	82	5 28	127	8 28	172	11 28
38	2 32	83	5 32	128	8 32	173	11 32
39	2 36	84	5 36	129	8 36	174	11 36
40	2 40	85	5 40	130	8 40	175	11 40
41	2 44	86	5 44	131	8 44	176	11 44
42	2 48	87	5 48	132	8 48	177	11 48
43	2 52	88	5 52	133	8 52	178	11 52
44	2 56	89	5 56	134	8 56	179	11 56
45	3 0	90	6 0	135	9 0	180	12 0

In the above tables the narrow columns contain degrees or minutes, and the broad ones hours and minutes, or minutes and seconds. Thus, if 4 in the first narrow column represent degrees, the 16 opposite to it in the broad column will be minutes; and if 4 represent minutes, the 16 will be seconds. If it be required to change 4° 20′ into time, opposite to 4 will be found 16, which in this case is minutes, and opposite to 20′ stands 1 minute 20 seconds, which added to 16 minutes, gives 17 minutes 20 seconds, the time answering to 4° 20′.

PROBLEM V.

How two men may be born on the same day, die at the same moment, and yet the one may have lived a day, or even two days more than the other.

It is well known to all navigators, that if a ship sails round the world, going from east to west, those on board when they return will count a day less than the inhabitants of the country. The cause of this is, that the vessel, following the course of the sun, has the days longer, and in the whole number of the days reckoned, during the voyage, there is necessarily one revolution of the sun less.

On the other hand, if the ship proceeds round the world from west to east, as it goes to meet the sun, the days are shorter, and during the whole circumnavigation, the people on board necessarily count one revolution of the sun more.

Let us now suppose that there are two twins, one of whom embarks on board a vessel which sails round the world from east to west, and that the other has remained at home. When the ship returns, the inhabitants will reckon Thursday, while those on board the vessel will reckon only Wednesday; and the twin who embarked will have a day less in his life. Consequently if they should die the same day, one of them would count a day older than the other, though they were born at the same hour.

But let us next suppose that, while the one circumnavigates the globe from east to west, the other goes round it from west to east, and that on the same day they return to port, where the inhabitants reckon Thursday for example: in this case, the former will count Wednesday, and the latter Friday, so that there will be two days difference in their ages.

In fact, it is evident that the one is as old as the other; the only difference is, that in the course of their voyage the one has had the days longer and the other shorter.

If the latter returned on a Wednesday and the former on a Friday, the former would count the day of his arrival Thursday: next day would be Thursday to the inhabitants, and the day after would be a Thursday to those who arrived in the second vessel; which, notwithstanding the popular proverb, would give three Thursdays in one week.

PROBLEM VI.

To find the length of the day in any proposed latitude, when the sun is in any given degree of the ecliptic.

Let the circle ABCX, pl. I fig. 3, represent a meridian, and AC the horizon. Assume the arc CE, equal to the elevation of the pole of the proposed place, for example London, which is 51° 31′; and having drawn DE, draw DF perpendicular to it, or make the arc AF equal to the complement of CE, and draw FD: it is here evident that DE will represent the circle of 6 hours, and DF the equator.

After this is done, find by the Ephemeris the sun's declination, when in the proposed degree of the ecliptic, or determine it by an operation which we shall show how to perform hereafter. We shall suppose that the declination is north: assume the arc FM, towards the arctic pole, equal to the declination, and through the point M draw MN parallel to FD, meeting the line DE in O, and the horizon AC in N. Then from the point O, as a centre, with the radius

OM, describe an arc of a circle MT, comprehended between the point M and NT parallel to DE. Having measured the number of the degrees comprehended in this arc, which may be easily done by means of a protractor, and having changed them into time, at the rate of 1 hour to 15 degrees, &c, the double of the result will be the length of the day.

Thus, if the length of the day at London, at the time when the sun has attained to the greatest northern declination, be required; as the greatest declination is 23° 28′, make FB equal to 23° 28′, and the arc BI will be found to be 124° 17′, which corresponds to 8h 17′, and this doubled gives 16h 34′, as the length of the day.

If you have no table of the sun's declination for each degree of the ecliptic, this deficiency may be supplied in the following manner. Find the number of degrees which the sun is distant from the nearest solstice, whether he has not yet reached it, or has passed it. We shall suppose that he is in the 23d degree of Taurus. The nearest solstice is that of Cancer, from which the sun, according to this supposition, is distant 37°. Draw the line BD representing a quarter of the ecliptic; and having assumed, from the point B, the arcs BK and B*k*, each equal to 37°, draw K*k*, intersecting BD, in L: if MN be then drawn through the point L, it will give the position of the parallel required.

All these things may be found much more correctly by trigonometrical calculation; but on that head we must refer the reader to works on astronomy

PROBLEM VII.

The longest Day in any Place being given, to find the Latitude.

This problem is the converse of the preceding, and may be solved without much difficulty; for the longest day, in all places of the northern hemisphere, always happens when the sun has just entered the sign Cancer. Let FD

(pl. 1 fig. 4) then represent the celestial equator, or rather its diameter, and BL that of the tropic of Cancer. On the latter describe a circle BKL; and having assumed the arc BK equal to the number of degrees corresponding to half the length of the given day at the rate of 15° for one hour, draw KM perpendicular to BL; if the diameter NMO be then drawn through the point M, the angle PCO will be the elevation of the pole or latitude of the place.

It would thence be easy to deduce a trigonometrical solution, and to determine the latitude by calculation; but, consistent with our plan, we must here confine ourselves to this graphic construction.

PROBLEM VIII.

The latitude of a place being given, to find the climate in which it is situated.

In astronomy, the name climate is given to an interval, on the surface of the earth, comprehended between two parallels under which the difference of the longest days is half an hour: thus the days in summer, under the parallel, whether north or south, distant from the equator 8° 25′, being 12^h 30^m, this interval, or the zone comprehended between the equator and that parallel, is called the first climate.

The limits of the different climates may therefore be easily determined, by finding in what latitudes the days are $12\frac{1}{2}$ hours, 13, $13\frac{1}{2}$, 14, &c. The following is a tal of all these climates.

Climates.	Most southern paral. of lat.		Most northern paral. of lat.
I	0° 0′	.	8° 25′
II	8 25	.	16 25
III	16 25	.	23 50
IV	23 50	.	30 20
V	30 20		36 28
VI	36 28		41 22

Climates.	Most southern paral. of lat.		Most northern paral. of lat.	
VII	41	22	45	29
VIII	45	29	49	21
IX	49	21	51	28
X	51	28	54	27
XI	54	27	56	37
XII	56	37	58	29
XIII	58	29	59	58
XIV	59	58	61	18
XV	61	18	62	25
XVI	62	25	63	22
XVII	63	22	64	6
XVIII	64	6	64	49
XIX	64	49	65	21
XX	65	21	65	47
XXI	65	47	66	6
XXII	66	6	66	20
XXIII	66	20	66	28
XXIV	66	28	66	31

As the longest day at the polar circle is 24 hours, and at the pole 6 months, there are supposed to be six climates between that circle and the pole.

Climates.	Most southern paral. of lat.		Most northern paral. of lat.	
XXV	66°	31′	67°	30′
XXVI	67	30	69	30
XXVII	69	30	73	20
XXVIII	73	20	78	20
XXIX	78	20	84	00
XXX	84	00	90	00

Now if it be asked in what climate London is, it may be easily replied that it is in the tenth; its latitude being 51° 31′, and its longest day $16^h\ 34^m$.

REMARK.—The idea of climates belongs to the ancient astronomy; but the modern pays no attention to this di-

vision, which in a great measure is destitute of correctness, in consequence of the refraction; for if the refraction be taken into account, as it ought to be, whatever Ozanam may say, it will be found that, under the polar circle the longest day, instead of 24, will be several times 24 hours; for as the horizontal refraction elevates the centre of the sun 32′ at least, the centre of that luminary ought consequently never to set between the 9th of June and the 3d or 4th of July; and the upper limb from the 6th of June to the 6th of July; this makes a complete month, during which the sun would never be out of sight.

PROBLEM IX.

To measure a degree of a great circle of the earth, and even the earth itself.

The rotundity of the earth, that is to say its being a globe, or of a form approaching very near to one, is proved by a number of astronomical phenomena; but we think it needless to enumerate these proofs, which must be known by those who are in the least acquainted with the principles of philosophy and the mathematics.

We shall here then suppose that the earth is perfectly spherical, as it apparently is; and shall begin our reasoning on that hypothesis.

What is called a degree of the meridian on the earth, is nothing else than the distance between two observers, the distance between whose zeniths is equal to a degree, or the geometrical distance between two places lying under the same meridian, the latitudes of which, or their elevation of the pole, differ a degree. Hence, if a person proceeds along a meridian of the earth, measuring the way he travels, he will have passed over a degree when he finds a degree of difference between the latitude of the place which he left, and that at which he has arrived; or when any star near the zenith of his first station has approached or receded a degree.

Nothing then is necessary but to make choice of two places, situated under the same meridian, the distance and latitudes of which are exactly known; for if the less latitude be taken from the greater, the remainder will be the arc of the meridian comprehended between the two places; and thus it will be known that a certain number of degrees and minutes correspond to a certain number of toises, or yards or feet, &c: all then that remains to be done, is to make use of the following proportion: as the given number of degrees and minutes, is to the given number of toises, yards or feet, so is one degree to a fourth term, which will be the toises, yards or feet corresponding to a degree.

But as the stations chosen may not be exactly under the same meridian, but nearly so, as Paris and Amiens, the meridional distance between their two parallels must be measured geometrically; and when this distance, as well as the difference of latitude of the two places is known, the number of toises, yards or feet corresponding to a degree, may be found by a proportion similar to the preceding.

This was the method employed by Picard, to determine the length of a terrestrial degree of the meridian in the neighbourhood of Paris. By a series of trigonometrical operations, he measured the distance between the pavilion of Malvoisine, to the south of Paris, as far as the steeple of Amiens, reducing it to the meridian, and found it to be 78907 toises. He found also by astronomical observations, that the cathedral of Amiens was 1° 22′ 58″ farther north than the pavilion of Malvoisine. By making this proportion then: as 1° 22′ 58″ are to one degree, so are 78907 toises to 57057, he concluded that a degree was equal to 57057 toises.

Picard's measurement having been since rectified in some points, it has been found that this degree is equal to 57070 toises.

COROLLARIES.

I. Thus, if we suppose the [illegible] spherical, its circumference will be 20545200 Frenc[illegible]es = 24881·8 English miles.

II. Its diameter will easily be found by making use of the following proportion: as the circumference of the circle is to its diameter, or as 314159 is to 100000, so is the above number to a fourth term, which is 6530196 toises = the diameter of the earth = 7920·12 English miles.

III. If we suppose its surface to be as smooth as that of the sea during a calm, its superficial content will be found to be 134164182859200 square toises = 197063856 English square miles. The rule for obtaining this result is: Multiply half the circumference by half the diameter, and then quadruple the product; or still shorter, multiply the circumference by the diameter.

IV. To find the solidity: multiply the superficial content, above found, by a third of the radius, which will give 146019735041736067200 cubic toises = 260124289920 English cubic miles.

Remark.—The operation performed by Picard between Paris and Amiens, was afterwards continued throughout the whole extent of the kingdom, both north and south; that is to say, from Dunkirk, where the elevation of the pole is 51° 2′ 27″, to Collioure, the latitude of which is 42° 31′ 16″: the distance therefore between the parallels of these two places is 8° 31′ 11″. But it was found at the same time by measurement, that the distance between these parallels was 486058 toises, which gives for a mean degree in the whole extent of France 57051 toises; and by corrections made afterwards, this number was reduced to 57038.

During this operation care was taken to determine the distance of the first meridian, which in France is that of the observatory of Paris, from the principal places between

which it passes. As it may perhaps afford gratification to some of our readers, we shall here present them with a table, the first column of which contains the names of these places, and the second the number of toises they are distant from the meridian, whether to the east or west of it. The place where the meridian was met by a perpendicular drawn to it, from the steeple of the cathedral of Bourges, was marked by a pillar.

Table of the places in France nearest to the meridian of the observatory of Paris.

Names of the Places.	Toises.
Fort de Revers	1206 E.
Dunkirk	1414 E.
Saint Omer	3011 E.
Dourlens	W
Villers Boccage	580 W.
Amiens	1252 W.
Sourdon	2541 E.
Saint Denis	E.
Montmartre	0
Paris	0
Lay	0
Juvisy	1350 E.
Orleans	16396 W.
Bourges	2358 E.
Saint Sauvier	345 W.
Mauriac	382 W.
Rhodez	9528 E.
Alby	8316 W.
Castres	3911 W.
Carcassone	246 E.
Perpignan	23461 E.
The summit of the Canigou	4664 E.

The meridian of France continued, then enters Spain, leaving Gironne on the east, at the distance of about ¼ of

a degree; passes two or three thousand toises to the east of Barcelona, traverses very nearly the island of Majorca, to the east of that city, and then enters Africa, about 7 minutes of a degree west of Algiers. But we shall not follow its course farther through unknown nations and countries: we shall only observe that it issues from Africa in the kingdom of Ardra. The astronomers of France have, since the above, repeated the measurement of the said arc through the country, with no great difference; from which they have deduced the length of the meridional quadrant, which has been assumed as the standard of the new universal measures. Also several degrees of the meridian through England are now measuring by Lt. Col. Mudge, of the Royal Artillery, under the auspices of the Master General and Board of Ordnance.

PROBLEM X.

Of the real figure of the earth.

We have already said that the rotundity of the earth is proved by various astronomical and physical phenomena; but these phenomena do not prove that it is a perfect sphere. Accurate methods for measuring it were no sooner employed, than doubts began to be entertained respecting its perfect sphericity. In fact, it is now demonstrated that our habitation is flattened or depressed towards the poles, and elevated about the equator; that is to say, the section of it through its axis, instead of being a circle, is a figure approaching very near to an ellipse, the less axis of which is the axis of the earth, or the distance from the one pole to the other, and the greater the diameter of the equator. Newton and Huygens first established this truth, on physical reasoning deduced from the centrifugal force

duce... -tation of the earth; and it has since been confirmed

During th... -al observations.

distance of the first ... hich Newton and Huygens reasoned;

observatory of Paris, from ... the earth originally spherical.

and motionless, it would be a globe, the greater part of the surface of which would be covered with water. But it is at present demonstrated, that the earth has a rotary motion around its axis, and every one knows that the effect of circular motion is to make the revolving bodies recede from the centre of motion: thus the waters under the equator will lose a part of their gravity, and therefore they must rise to a greater height, to regain by that elevation the force necessary to counterbalance the lateral columns, extended to other points of the earth, where the centrifugal force, which counterbalances their gravity, is less, and acts in a less direct manner. The waters of the ocean then must rise under the equator as soon as the earth, supposed to be at first motionless, assumes a rotary motion round its axis: the parts near the equator will rise a little less, and those in the neighbourhood of the poles will sink down; for the polar column, as it experiences no centrifugal force, will be the heaviest of all. This reasoning cannot be weakened, but by supposing that the nucleus of the earth is of an elongated form; or by supposing a singular contexture in its interior parts, expressly adapted for producing that effect; but this is altogether improbable.

The philosophers however on the continent persisted a long time in refusing to admit this truth. Their principal arguments against it were founded on the measurement of the degrees of the meridian made in France; by which it appeared that a degree was less in the northern part of the kingdom than in the southern, and hence they concluded that the figure of the earth was a spheroid elongated at the poles. If the earth, said they, were perfectly spherical, by advancing uniformly under the same meridian, the elevation of the pole would be uniformly changed. Thus, in advancing from Paris, for example, towards the north 57070 toises, the elevation of the pole would vary a degree; and to make the elevation of the pole increase

another degree, it would be necessary to advance towards the north 57070 toises more; and so on throughout the whole circumference of a meridian.

If, in proportion as we proceed northwards, it is found necessary to travel farther than the above number of toises before the latitude is changed one degree, there is reason to conclude that the earth is not spherical, but that it is less curved or more flattened towards the north, and that the curvature decreases the nearer we approach the pole, which is the property of an ellipsis having its poles at the extremities of its less axis. In the contrary case, it would be a proof that the curvature of the earth decreased towards the equator; which is the property of a body formed by the revolution of an ellipsis around its greater axis.

But it was believed in France at first, that the degrees of the meridian were found to increase the more they approached the south. The degree measured in the neighbourhood of Collioure, the austral boundary of the meridian, appeared to be equal to 57192 toises, while that in the neighbourhood of Dunkirk, which was the most northern, seemed to be only 56954. There was reason therefore to conclude that the earth was an elongated spheroid, or formed by the revolution of an ellipsis around its greater axis.

The partisans of the Newtonian philosophy, at that time too little known in France, replied, that these observations proved nothing, because the above difference, being so inconsiderable, could be ascribed only to the errors unavoidable in such operations. As 19 toises correspond to about a second, the 238 toises of difference would amount only to about 12 seconds; an error which might have arisen from various causes: they even asserted that this difference might be on the opposite side.

To decide the contest, it was then proposed to measure two degrees as far distant from each other as possible, one

under the equator, and the other as near the pole as the cold of the polar regions would admit. For this purpose, Maupertuis, Camus, and Clairaut, were dispatched by the king in the year 1735, to measure a degree of the meridian at the bottom of the Gulph of Bothnia, under the arctic polar circle; and Bouguer, Godin, and Condamine, were sent to the neighbourhood of the equator, where they measured, not only a degree of the meridian, but almost three. It resulted from these operations, performed with the utmost care and attention, that a degree near the polar circle was equal to 57422 toises, and that a degree near the equator contained 56750, which gives a difference of 672 toises, and therefore too considerable to be ascribed to the errors unavoidable in the necessary observations. Since that time it has never been contested that the earth is flattened towards the poles, as Newton and Huygens asserted. We shall here add that the measurements formerly made in France having been repeated, it was found that the degree goes on increasing from south to north as ought to be the case, if the earth be an oblate spheroid.

This truth has been since confirmed by other measurements of the meridian, made in different parts of the earth. The Abbé de la Caille having measured a degree at the Cape of Good Hope, that is under the latitude of about 33° south, found it to be 57037 toises: and in 1755, Fathers Marré and Boscovich, two Jesuits, having measured a degree in Italy, in latitude 43°, found it to be 56979: it is therefore certain that the degrees of the terrestrial meridian go on increasing from the equator towards the poles, and that the earth has the form of an oblate spheroid.

Other operations of the same kind for measuring a degree of the terrestrial meridian have been since undertaken at different times, as by the Abbé Liesganig in Germany, near Vienna; by Father Beccaria in Lombardy; and by

Messrs. Mason and Dixon, members of the Royal Society of London, in North America; and again more lately by Mechain and De Lambre in France. They all confirm the diminution of the terrestrial degrees as they approach the equator, though with inequalities difficult to be reconciled with a regular figure. But it may here be asked, why should the earth have a figure perfectly regular?

It is, indeed, impossible to determine with accuracy the proportion between the axis of the earth and its diameter at the equator: it has been proved that the former is shorter; but to find their exact ratio would require observations which can be made only at the pole. However the most probable ratio is that of 177 to 178.

Consequently, if this ratio be admitted, the axis of the earth from the one pole to the other, will be 6525376 toises, and the diameter of the equator 6562242.

In the last place, the difference between the distance of any point of the equator on a level with the sea, to the centre of the earth, and the distance of the pole from the same centre, will be 18433 toises, or about 22 English miles.

Since Montucla wrote the above, however, the French astronomers Mechain and De Lambre, in 1799, completed their measurement of the meridian, from Dunkirk in France, to near Barcelona in Spain, an extent of almost 10 degrees; from which it has been more accurately deduced, that the flattening of the earth at the poles is only the 334th part, the ratio of the axes being that of 334 to 333; that the polar axis is 7899$\frac{3}{4}$ English miles, the equatorial diameter 7923$\frac{4}{5}$ miles, their half difference only 11$\frac{5}{8}$ miles, which is the height of the equator more than that at the pole, from the centre; the mean diameter 7911$\frac{4}{9}$ miles, the mean circumference 24873$\frac{5}{9}$ miles, the greatest or equatorial circumference 24892$\frac{2}{9}$ miles, the least or meridional circle 24855 miles, and the difference of the two 37$\frac{2}{9}$ miles.

COROLLARIES.

I. From what has been said, several curious truths may be deduced. The first is, that all bodies, except those placed under the equator and the poles, do not tend to the centre of the earth; for a circle is the only figure in which all the lines perpendicular to its circumference tend to the same point. In other figures, the curves of which are continually varying, as is the case with the meridians of the earth, the lines perpendicular to the circumference all pass through different points of the axis.

II. The elevation of the waters under the equator, and their depression under the poles, being the effect of the earth's rotation round its axis, it may be readily conceived that if this rotary motion should be accelerated, the elevation of the waters under the equator would increase; and as the solid part of the earth has assumed, since its creation, a consistence which will not suffer it to give way to such an elevation, the rising of the waters might become so great, that all the countries lying under the equator would be inundated; and in that case the polar seas, if not very deep, would be converted into dry land.

On the other hand, if the diurnal motion of the earth should be annihilated, or become slower, the waters accumulated, and now sustained under the equator, by the centrifugal force, would fall back towards the poles, and overwhelm all the northern parts of the earth: new islands and new continents would be formed in the torrid zone by the sinking down of the waters, which would leave new tracts of land uncovered.

Remark.—We cannot help here remarking one advantage which France, and all countries near the mean latitude of about 45 degrees, would in this case enjoy. If such a catastrophe should take place, these countries would be sheltered from the inundation, because the spheroid, which is the real figure of the earth at present, and the globe or less oblate spheroid into which it would be

changed, would have their intersection about the 45th degree; consequently the sea would not be altered in that latitude.

PROBLEM XI.

To determine the length of a degree on any given parallel of latitude.

As the difference between the greater and less diameter of the earth does not amount to the 300th part, in this and the following problems we shall consider it as absolutely spherical; especially as the solution of these problems, if we supposed the earth to be a spheroid, would be attended with difficulties inconsistent with the plan of this work.

Let it be proposed then to determine how many miles or yards are equal to a degree on the parallel passing through London; that is to say under the latitude of 51 degrees 31 minutes. This problem may be solved either geometrically or by calculation, according to the following methods

1st. Draw any straight line AB, pl. 1 fig. 5, and divide it into 23 equal parts, because a degree of the equator contains 69·14 miles or about 23 leagues. Then from the point A as a centre, with the distance AB, describe the arc BC, equal to 51° 31′; and from the point C draw CD perpendicular to AB: the part AD will indicate the number of leagues contained in a degree on the parallel of 51° 31′.

2d. This however may be found much more correctly by trigonometrical calculation; for which purpose nothing is necessary but to make use of the following proportion:

As Radius	1000000
is to the cosine of lat. 51° 31′ . .	622287
So are the miles in a degree of the equator	$69\frac{1}{2}$
to a fourth term which will be .	43·0267

This last term 43·0267, or 43 nearly, is the number of miles contained in a degree on the parallel of 51° 31′.

The above example is worked by means of the natural sines and the common rule of three, but the same thing may be done by logarithms in the following manner:

As Radius		10 0(00000
is to the cosine of the latitude 51° 31′ .		9·7939907
So are the miles in a degree viz .	69 14	1·8397294
to a fourth term		1·6337201

which in the table of logarithms will be found answering to 43 025 miles, as before nearly. A degree therefore on the parallel of London contains nearly 43 miles, or about 75643 yards.

The demonstration of this rule is easy, if it be recollected that the circumferences of two circles, or degrees of these circles, are to each other in the ratio of their radii. But the radius of the parallel of London is the cosine of the latitude; whereas the radius of the earth, or of the equator, is the real radius or sine of 90°, and hence the above rule.

3d If the circumference of the earth at the given parallel be required, nothing is necessary but to multiply the degree found as above by 360 thus as a degree on the parallel of London is equal to 43 miles, if this number be multiplied by 360, we shall have 15480 miles, for the whole circumference of the circle of that parallel.

The following table, which shows the number of miles contained in a degree on every parallel, from the equator to the pole, is computed on the supposition that the length of the degrees of the equator are equal to those of the meridian, at the medium latitude of 45°, which length is nearly 69 $\frac{1}{15}$ English miles.

Deg. of Lat	English miles.	Deg. of Lat.	English miles.	Deg. of Lat.	English miles.	Deg. of Lat	English miles.
0	69 07	23	63·51	46	47·93	69	24·73
1	69·06	24	63·03	47	47·06	70	23·60
2	69·03	25	62·53	48	46·16	71	22·47
3	68·97	26	62·02	49	45·26	72	21·32
4	68·90	27	61·48	50	44·35	73	20·17
5	68·81	28	60·93	51	43·42	74	19·02
6	68·62	29	60·35	52	42·48	75	17·86
7	68·48	30	59·75	53	41·53	76	16·70
8	68·31	31	59·13	54	40·56	77	15·52
9	68·15	32	58 51	55	39 58	78	14·35
10	67·95	33	57·87	56	38·58	79	13·17
11	67·73	34	57·20	57	37·58	80	11·98
12	67·48	35	56·51	58	36·57	81	10·79
13	67·21	36	55·81	59	35·54	82	9·59
14	66·95	37	55·10	60	34·50	83	8·41
15	66 65	38	54·37	61	33·45	84	7·21
16	66·31	39	53·62	62	32·40	85	6·00
17	65·98	40	52·85	63	31·33	86	4·81
18	65·62	41	52·07	64	30 24	87	3·61
19	65 24	42	51·27	65	29·15	88	2·41
20	64·84	43	50 46	66	28·06	89	1·21
21	64·42	44	49·63	67	26·96	90	0·00
22	63·97	45	48·78	68	25 85		

PROBLEM XII.

Given the latitude and longitude of any two places on the earth, to find the distance between them.

We must here observe, that the distance of any two places on the surface of the earth, ought to be the arc of the great circle intercepted between them. The distance therefore of any two places, lying under the same parallel, is not the arc of that parallel intercepted between them, but an arc of a great circle having the same extremities as that arc; for on the surface of a sphere, *it* is the shortest way from one point to another, as a straight line is upon a surface.

This being premised, it may be readily seen that this problem is susceptible of several cases; for the two places proposed may lie under the same meridian, that is to say, have the same longitude, but different latitudes; or they may have the same latitude, that is lie under the equator or under the same parallel; or in the last place their longitudes and latitudes may be both different: there is also a sub-division into two cases, viz. one where the two places are in the same hemisphere, and another where one is in the northern and the other in the southern hemisphere. But we shall confine ourselves to the solution of the only case which is attended with any difficulty.

For it is evident that if the two places are under the same meridian, the arc which measures their distance is their difference of latitude, provided they are in the same hemisphere, or the sum of these latitudes if they are in different hemispheres. Nothing then is necessary but to reduce this arc into leagues, miles or yards, and the result will be the distance of the two places in similar parts.

If the places lie under the equator, the amplitude of the arc which separates them may be determined with equal ease; and can then be reduced into leagues, miles, &c.

Let us suppose then, which is the only case attended with difficulty, that the places differ both in longitude and latitude, as London and Constantinople, the former of which is 28° 53′ farther west than the latter, and 10° 31′ farther north. If we conceive a great circle passing through these two cities, the arc comprehended between them will be found by the following construction.

From A as a centre (pl. 1 fig. 6 n° 1), with any opening of the compasses taken at pleasure, describe the semicircle BCDE, representing the meridian of London. Take the arc BF equal to 51° 31′, which is the latitude of London, in order to find its place in F, and draw the radius AF.

In the same semi-circle, if the arcs BC and ED be taken each equal to 41°, the latitude of Constantinople, the line

CD will be the parallel of Constantinople, the place of which must be found in the following manner.

On CD as a diameter, describe the semi-circle CGD; and in the circumference of it take the arc CG equal to the difference of longitude between London and Constantinople, that is 28° 53′; then from the point G draw GH, perpendicular to CD, to have in H the projection of the place of Constantinople; and from the point H draw HI, perpendicular to AF and terminated at I by the arc BCDE: if the arc FI be measured, it will give the distance required in degrees and minutes. In this case it is about 22 degrees*.

If one of the places be on the other side of the equator, as the city of Fernambouc in Brasil is in regard to London, being in 7° 30′ of south latitude, the arc BC must be assumed on the other side of the diameter BE (fig. 6 n° 2), equal to the latitude of the second place given, which is here 7° 30′; and as the difference of longitude between London and Fernambouc is 35° 5′, it will be necessary to make the arc CG = 35° 5′. By these means the arc FI will be found to be equal to about 66°†, which reduced into miles of 69·07 to a degree, gives 4558 miles, for the distance between London and the above city of Brasil.

REMARK.—When the distance between the two places is not very considerable, as is the case with Lyons and Geneva, the latter being only 36′ farther north than the former, and more to the east by 6 minutes of time, which is equal to 1° 30′ under the equator, the calculation may be greatly shortened.

For this purpose, take the mean latitude of the two places, which in this instance is 46° 4′, and find by the preceding problem the extent of a degree on the parallel passing through that latitude, which will be = 47·922

* Calculation by spherical trigonometry gives 22° 23′.

† Trigonometrical calculation gives 66° 15′.

miles. The difference of longitude between these places is 1° 30′, which on that parallel, allowing 47·922 miles to a degree, gives 71·88 miles, and the miles corresponding to the difference of latitude are 41·44.

If we therefore suppose a right angled triangle, one of the sides of which adjacent to the right angle is 41·44 miles, and the other 71 88, by squaring these two numbers, adding them together, and extracting the square root of the sum, we shall have the hypothenuse equal to 82·97 miles; which will be the distance, in a straight line, between Lyons and Geneva.

As this is the proper place for making known the measures employed by different nations, in measuring itinerary distances, it will doubtless be gratifying to our readers to find here a table of them, especially as it is difficult to collect them: for the same reason we have added some of the itinerary measures of the ancients, the whole expressed in English feet.

TABLE OF ITINERARY MEASURES,

Ancient and Modern.

	Feet.
ANCIENT GREECE.	
The olympic stadium	604
A smaller stadium	482
The least stadium	322
EGYPT.	
The schœnus	19421
PERSIA.	
The parasang or farsang	14499
ROMAN EMPIRE.	
The mile (*milliare*)	4833
JUDEA.	
The rast or stadium	486
The berath or mile	3640

	Feet.
ANCIENT GAUL.	
The league (*leug*)	7249
GERMANY.	
The rast or league	14498
The mile 12½ to a degree . . .	28995
The same 15 to a degree . . .	24292
ARABIA.	
The mile	6929
FRANCE.	
The mile of 1000 French toises .	6392
The small league of 30 to a degree .	12159
The mean league of 25 to a degree .	14594
The great or marine league of 20 to a degree	18238
SWEDEN.	
The mile	35050
DENMARK.	
The mile	25123
ENGLAND.	
The mile	5280
SCOTLAND.	
The mile	7332
IRELAND.	
The mile	6724
SPAIN.	
The league (*legale of* 5000 *vares*) .	13724
The common league 17½ to a degree .	20846
ITALY.	
The Roman mile .	4909
The Lombard mile .	5425
The Venetian mile .	6341

	Feet
POLAND.	
The league	18223
RUSSIA.	
The ancient werst	4193
The modern werst	3497
TURKEY.	
The agash	16211
INDIA.	
The little coss	8579
The great coss	9857
The gau of the Malabar coast . .	38356
The nari or nali of the same . . .	5753
CHINA.	
The present li	1885
The pu, equal to 10 lis . . .	18857

These evaluations are extracted from a work by Danville, entitled *Traité des Mesures itinéraires anciennes et modernes,* Paris, 1768, 8vo, in which this subject is treated with great erudition and sagacity; so that, amidst the uncertainty which prevails in regard to the precise relation between these measures and ours, the evaluations given by Danville may be considered as the most probable, and the best founded. We have deviated therefore in many points from those given by Christiani, in his book *Delle Misure d'ogni genere antiche è moderne.* This work is valuable in some respects; but the subject is far from being examined there in so profound a manner as it has been by Danville.

PROBLEM XIII.

To represent the terrestrial globe in plano.

A map, which represents the whole superficies of the terrestrial globe on a flat surface, is called a planisphere, or general map of the world.

A map of this kind is generally represented in two hemispheres; because the artificial globe, which represents the globe of the earth, cannot be all seen at one view: hence, when delineated in plano, it is necessary to divide it into two halves, each of which is called a hemisphere. It may be thus represented in three ways.

The first is to represent it as divided by the plane of the meridian into two hemispheres, one eastern the other western. This method is that generally used for a map of the world, because it exhibits the old continent in the one hemisphere, and the whole of the new in the other.

The second is to represent it as divided by the equator into two hemispheres, the one northern and the other southern. This representation is in some cases attended with advantage, because the disposition of the most northern and most southern countries are better seen. Some maps of this kind have been published, in which the tracks pursued by our modern navigators, and all the discoveries made by them in the South Seas, are accurately delineated.

The third method is to exhibit the globe of the earth as divided by the horizon into two hemispheres; the upper and lower, according to the position of each.

Under certain circumstances this form has its advantages also. The disposition of the different parts of the earth, in regard to the proposed place, are better seen, and a great many geographical problems can be solved by it with much greater facility.

Father Chrysologue of Gy, in Franche-Comté, published some years ago two hemispheres of this kind, the centre of one of which was occupied by Paris; and he added an explanation of the different uses to which they might be applied.

Two methods may be employed in these representations. According to one of them, the globe is supposed to be seen by the eye placed without it; and such as it would appear at an infinite distance.

According to the other, each hemisphere is supposed to be viewed on the concave side; as if the eye were placed at the end of the central diameter, or at the pole of the opposite hemisphere; and it is conceived to be projected on the plane of its base. Hence arise the different properties of these representations, which we shall here describe.

I. When the globe is represented as seen on the convex side, and divided into two hemispheres by the plane of the first meridian, the eye is supposed to be at an infinite distance, opposite to the point where the equator is intersected by the 90th meridian. All the meridians are then represented by ellipses, the first excepted, which is represented by a circle, and the 90th which becomes a straight line: the parallels of latitude also are represented by straight lines. This representation is attended with one great fault, viz, that the parts near the first meridian are very much contracted, on account of the obliquity under which they present themselves.

When the hemispheres are represented by the second method, that is, as seen on the concave side, and projected on the plane of the meridian, the contrary is the case. It is supposed, in regard to the eastern hemisphere, that the eye is placed at the extremity of the diameter which passes through the place where the equator and the 90th meridian intersect each other. In this case there is more equality between the distances of the meridians; and even the parts of the earth represented in the middle of the map lie somewhat closer than those towards the edges. Besides, all the meridians and parallels are represented by arcs of a circle, which is very convenient in constructing the map. It is attended however with this inconvenience, that the parts of the earth have an appearance different from what they have when seen from without. Asia for example is seen on the left, and Europe on the right; but this may be easily remedied by a counter-impression.

II. If a projection of the earth on the plane of the

equator be required, the eye according to the first method may be supposed at an infinite distance in the axis produced: the pole will then occupy the centre of the map; the parallels will be concentric circles, and the meridians straight lines. But it is attended with this inconvenience, that the parts of the earth near the equator will be very much contracted.

For this reason it will be better to have recourse to the second method, which supposes the northern hemisphere to be seen by an eye placed at the south pole, and *vice versa*: as there is here an inversion of the relative position of the places, it may be remedied in like manner by a counter-impression.

III. If the eye be supposed in the zenith of any determinate place, as of London for example, and at an infinite distance, we shall have on the plane of the horizon a representation of the terrestrial hemisphere, the pole of which is occupied by London, and which is of the third kind. But this representation will still be attended with the inconvenience of the places near the horizon being too much crowded.

This defect however may be remedied by employing the second method, or by supposing the above hemisphere to be seen through the horizon by an eye placed in the pole of the lower hemisphere: the different meridians will then be represented by arcs of a circle, as will also the parallels: the circles representing the distance from the proposed place, to all other places of the earth, will be straight lines. The inversion of position may be remedied as in the preceding cases.

The numerous uses to which this particular kind of projection can be applied, may be seen in a work published by Father Chrysologue in 1774, and which was intended as an explanation of his double map of the world, already mentioned.

Various other projections of the globe might be con-

ceived; and by supposing the eye in some other point than the pole of the hemisphere, more equality might be preserved between the parts lying near to the centre and the edges of the projection; but this would be attended with other inconveniences, viz, that the circles on the surface of the sphere or globe, would not be represented by circles or straight lines, which would render a description of them difficult. It is therefore better to adhere to the projection where the eye is supposed to be in the pole of the hemisphere opposite to that intended to be represented; whether the terrestrial globe, as in common maps, is to be projected on the plane of the first meridian, or whether it be required to project it on the plane of the equator, or on that of the horizon of any determinate place.

PROBLEM XIV.

The latitude and longitude of two places, London and Cayenne for example, being given; to find with what point of the horizon the line drawn from the one to the other corresponds; or what angle the azimuth circle drawn from the former of these places through the other makes with the meridian.

The solution of this problem is attended with very little difficulty, if spherical trigonometry be employed, as it is reduced to the following: the two sides of a spherical triangle and the included angle being given, to find one of the other two angles. But for want of trigonometrical tables, which I had lost with all my baggage in consequence of shipwreck, I found myself obliged on a certain occasion to solve this problem by a simple geometrical construction, which I shall here describe. I cannot however help mentioning the singular circumstance which conducted me to it.

Being at the island of Socotora, near Madagascar, on board a vessel belonging to the East India company, which had touched there, I formed an acquaintance with a de-

vout Mussulam, one of the richest and most respectable inhabitants of the island. As he soon learned, by the astronomical observations which he saw me make, that I was an astronomer, he requested me to determine in his chamber the exact direction of Mecca, that he might turn himself towards that venerable place when he repeated his prayers. I at first hesitated on account of the object; but the good Iahia (that was his name) begged with so much earnestness, that I was not able to refuse. Having neither charts nor globes, and knowing only the latitude and longitude of the two places, I had recourse to a graphic construction on a pretty large scale. I determined the angle of position, which Mecca formed with the above island; and traced out, on the floor of his oratory, the line in the direction in which he ought to look, in order to be turned towards Mecca. Words can hardly express how much the good Iahia was gratified by my compliance with his wishes; and I have no doubt, if still alive, that he offers up grateful prayers to his prophet for my conversion. But let us return to our problem, in which we shall take, by way of example, London and Cayenne.

To resolve it by a geometrical construction, describe a circle to represent the horizon of London, which we shall suppose to be in the centre P: the larger this circle is, the more correct will the operation be. Draw the two diameters AB and CD, cutting each other at right angles; and having assumed DN, equal to the distance of London from the pole, draw the radius NP, and PE perpendicular to it, which will represent a radius of the equator: make the arc EK equal to the distance of the second place from the equator, which in regard to Cayenne is 4° 56′; draw also KF and KG, perpendicular to the radii PB and PN; and from the point G draw GO perpendicular to the diameter AB, and continue it on both sides: if from O as a centre, with the radius GK, a semi-circle RHQ be then described on the line ROQ, the points R and Q will neces-

sarily fall within the circle; because PG being greater than PO, we shall have, on the other hand, GK or OK less than OS.

Having described the semi-circle RHQ, assume the arc HI equal to the difference of the longitudes of the two places, that is towards the side C, which we here suppose to represent the west, and towards the south if the second place lies to the west of London and farther south, which is the case in the proposed example; for Cayenne is situated to the west of London, and lies much nearer the equator. Hence it may be readily seen what ought to be done, if the second place lay farther north, or to the east, &c. The arc HI then having been taken equal to 52° 11′, draw IL perpendicular to the diameter RQ; and draw HI till it meet, in M, that diameter continued; if MF be then drawn, which will cut LI in T, the point T will represent the projection of Cayenne on the horizon of London; and consequently, by drawing the line PT, the angle TPA will be that formed by the azimuth of London passing through Cayenne.

It will be found, by this operation, that the line of position of Cayenne, in regard to London, makes with the meridian an angle of 61° 48′, consequently Cayenne bears from London south west by west ½ west nearly.

It must however be allowed that this problem can be solved mechanically, by means of a globe, with much more ease and convenience; for nothing more is necessary than to rectify the globe for the latitude of London; to screw fast the quadrant of altitude to that point, and then to turn it till the edge of it corresponds with Cayenne: if the number of degrees intercepted between it and the meridian be then counted on the horizon, you will have the angle it forms with the meridian. But as a globe may not always be at hand, nor tables of sines and tangents to solve it trigonometrically, this want may be supplied by the graphic construction above described.

THEOREM.

The heavenly bodies are never seen in the place where they really are: thus, for example, the whole face of the sun is seen above the horizon after he is actually set.

Though this has the appearance of a paradox, it is a truth acknowledged by all astronomers, and which philosophers explain in the following manner.

The earth is surrounded by a stratum of a fluid much denser than that which fills the expanse of the celestial regions. A small portion of the terrestrial globe enveloped by this stratum, commonly called the atmosphere, is seen represented fig. 8 pl. 2. If the sun then be in S, a central ray SE, when it reaches the atmosphere, instead of continuing its course in a straight line, is refracted towards the perpendicular, and assumes the direction EF. A spectator at F, must consequently see the sun in the line FE; and as we always judge the object to be in the direct continuation of the ray by which the eye is affected, the spectator at F sees the centre of the sun at *s*, a little nearer the zenith than he really is; and this deviation is greater, the nearer the body is to the horizon, because the ray then falls with more obliquity on the surface of the atmospheric fluid.

Astronomers have found that when the body is on the horizon, this refraction is about 33 minutes; therefore when the upper limb of the sun is in the horizontal line, so that if there were no atmosphere he would seem only beginning to peep over the horizon, he appears to be elevated 33 minutes; and as the apparent diameter of the sun is less than 33 minutes, his lower limb will appear to touch the horizon. Thus the sun is risen in appearance, though he is not really so, and even when he is entirely below the horizon. Hence follow several curious consequences, which deserve to be remarked.

I. More than one half of the celestial sphere is always

seen; though in every treatise on the globes it is supposed that we see only the half; for besides the upper hemisphere, we see also a band round the horizon of about 33 minutes in breadth, which belongs to the lower hemisphere.

II. The days are every where longer, and the nights shorter, than they ought to be according to the latitude of the place; for the apparent rising of the sun precedes the real rising, and the apparent setting follows the real setting; therefore, though the quantity of day and night ought to be equally balanced at the end of the year, the former exceeds the latter in a considerable degree.

III. The effect of refraction, above described, serves also to account for another astronomical paradox, which is as follows.

The moon may be seen eclipsed even totally and centrally, when the sun is above the horizon.

A total and central eclipse of the moon cannot take place but when the sun and moon are directly opposite to each other. We here suppose that the reader is acquainted with the causes of these phenomena, an explanation of which may be found in every elementary work on astronomy. When the centre of the moon therefore, at the time of a total eclipse, is in the rational horizon, the centre of the sun ought to be in the opposite point; but by the effect of refraction these points are raised 33 minutes above the horizon. The apparent semi-diameter of the sun and moon being only about 15 minutes; the lower limbs of both will appear elevated about 18 minutes.

Such is the explanation of a phenomenon which must take place at every central eclipse of the moon; for there is always some place of the earth where the moon is on the horizon at the middle of the eclipse.

IV. Refraction enables us to explain also a very common phenomenon, viz, the apparent elliptical form of the sun and moon, when on the horizon; for the lower limb

of the sun corresponding, we shall suppose, with the rational horizon, is elevated 33 minutes by the effect of refraction; but the upper limb being really elevated 30 minutes, (which is nearly the apparent diameter of that luminary at its mean distances,) is elevated in appearance by refraction no more than 28 minutes above its real altitude; the vertical diameter therefore will appear shortened by the difference between 33 and 28, that is to say 5 minutes; for if the refraction of the upper limb were equal to that of the lower, the vertical diameter would be neither lengthened nor shortened. The apparent vertical diameter will thus be reduced to about 28 minutes.

But there ought to be no sensible decrease in the horizontal diameter, for the extremities of this diameter are carried only a little higher in the two vertical circles passing through them, and which, as they meet in the zenith, are sensibly parallel. The vertical diameter then being contracted, while the horizontal diameter remains the same, the result must be, that the disks of the sun and moon will apparently have an elliptical form, or appear shorter in the vertical direction than in the horizontal.

V. There is always more than one half of the earth enlightened by a central illumination; that is to say by an illumination, the centre of which is visible; for if there were no refraction, the centre of the sun would not be seen till it corresponded with the plane of the rational horizon; but as the refraction raises it about 33 minutes, it will begin to appear when it is in the plane of a circle parallel to the rational horizon, and 33 minutes below it.

There is therefore a central illumination for the whole hemisphere, plus the zone comprehended between that hemisphere and a parallel distant from it 33 minutes; and there is a complete illumination from the whole disk of the sun to the same hemisphere, and the zone comprehended between the border of it, and a parallel about 16 minutes farther below the horizon.

What Ozanam therefore, or his continuator, endeavours to demonstrate, after Deschales, with so much labour and tediousness, (see *Recreations Mathematiques*, vol. II. p. 277 edit. of 1750,) is absolutely false; because no allowance is made for refraction.

PROBLEM XV.

To determine, without astronomical tables, whether there will be an eclipse at any new or full moon given.

Though the calculation of eclipses, and particularly those of the sun, is exceedingly laborious; those which took place in any given year of the 18th century, that is between 1700 and 1801, may be found, without much difficulty, by the following operation. The method of finding those of the present or 19th century, will be shown in the additional remark to this problem.

For the New Moons.

Find the complete number of lunations between the new moon proposed, and the 8th of January 1701, according to the Gregorian calendar, and multiply that number by 7361; to the product add 33890, and divide the sum by 43200, without paying any regard to the quotient. If the remainder after the division, or the difference between that remainder and the divisor, be less than 4060, there will be an eclipse, and consequently an eclipse of the sun.

Example.—It is required to find whether there was an eclipse of the sun on the first of April 1764. Between the 8th January 1701, and the 1st of April 1764, there were 782 complete lunations; if this number then be multiplied by 7361, the product will be 5756302; to which adding 33890, we shall have 5790192; and this sum divided by 43200 will leave for remainder 1392: this number being less than 4060, shows that on the 1st of April 1764 there was an eclipse of the sun, which was indeed the case; and this eclipse was annular to a part of Europe.

For the Full Moons.

Find the number of complete lunations between that which began on the 8th of January 1701, and the conjunction which precedes the full moon proposed: multiply this number by 7361; and having added to the product 37326, divide the sum by 43200: if the remainder after the division, or the difference between the remainder and the divisor, be less than 2800, it will show that an eclipse of the moon took place at that time.

Example.—Let it be required to find whether there was an eclipse at the full moon which took place on the 13th of December 1769. Between the 8th of January 1701, and the 28th of November 1769, the day of the new moon preceding the 13th of December, there were 852 complete lunations: the product of this number by 7361 is 6271572; to which if we add 37326, the sum will be 6308898. But this sum divided by 43200, leaves for remainder 1698, which being less than 2800, shows that there was an eclipse of the moon on the 13th of December 1769, as indeed may be seen by the almanacs for that year.

REMARK.—To determine the number of lunations, which have elapsed between the 8th January 1701, and any proposed day, the following method, which is attended with very little difficulty, may be employed. Diminish by unity the number of years above 1700, and multiply the remainder by 365; to the product add the number of bissextiles between 1700 and the given year, and the result will be the number of days from the 8th of January 1701 to the 8th of January of the proposed year. Then add the number of days from the 8th of January of the given year to the day of the new moon proposed, or to that which precedes the full moon proposed; and having doubled the sum, divide it by 59, the quotient will be the number of lunations required.

Let us propose, by way of example, the 13th of De-

cember 1769, the day of full moon. The preceding new moon fell on the 28th of November. If 69 be diminished by unity, the remainder is 68; which multiplied by 365, gives 24820. As in that interval there were 17 bissextiles, we must add 17, which will give 24837. Lastly, the number of days from January 8th to November 28th, 1769 was 309, which added to the above sum make 25146. This number doubled is 50292; which divided by 59, gives for quotient 852. The number of complete lunations therefore, between the 8th of January 1701 and the full moon December 13th 1769, was 852.

ADDITIONAL REMARK.—This easy method of finding eclipses was invented by M. de la Hire, a celebrated French astronomer; but as it will require some alteration to make it answer for the present century, we shall first explain the principles on which it is founded, and then show how this alteration is to be made.

1st. In regard to the full moons, we shall suppose that the sun is at present in the ascending node, and the moon in the descending: the former during the period of a lunation will move from his node 30 degrees 40 minutes 15 seconds; which expressed in quarters of a minute are equal to 7361. Hence M. de la Hire multiplies this number by that of the complete lunations, between the new moon on the 8th of January 1701, and the full moon proposed; and the product necessarily gives all the movements which the sun has made during that time, to recede from the one node and to approach the other.

2d. The sun at the time of the full moon in the month of January 1701, was distant from his node 155 degrees 31 minutes 30 seconds, which expressed in quarters of a minute, give 37326: hence according to M. de la Hire this number must be added to the product of 7361 multiplied by the lunations.

3d. The two nodes of the lunar orbit are distant from each other 180 degrees, or 10800 minutes; which multi-

plied by 4, give 43200: the distance therefore of the one node from the other is represented by 43200.

4th. To obtain the true distance of the sun from the node, 43200 must be subtracted from the sum mentioned in the example, viz 6308898, as many times as possible; and hence, according to M. de la Hire, this sum must be divided by 43200, neglecting the quotient.

5th. The remainder after the last division gives the true distance of the sun from his node, which we have hitherto supposed to be the *ascending node*; that is, the node by which the moon passes from the southern to the northern side of the ecliptic. If this remainder does not exceed 2800, there will be an eclipse, or at least it will be possible; because the sun will not be distant from his node 11 degrees 40 minutes. For 11 degrees 40 minutes are equal to 700 minutes; and 700 minutes multiplied by 4, give 2800 quarters of a minute.

6th. There may be an eclipse though the remainder after the last division exceeds 2800; but in that case the difference between this remainder and the divisor will be less than 2800. The reason of this is, that the sun is necessarily distant from one of the two nodes less than 11 degrees 40 minutes. The one node indeed being distant from the other only 43200 quarters of a minute, and as the sun cannot recede from the one node without approaching the other, if the difference between the remainder after the division, and the divisor 43200, does not exceed 2800, there will necessarily be one of the nodes from which the sun will not be distant 11 degrees 40 minutes.

But it may here be objected, as the sun during the time of a lunation does not pass over 30 degrees of the ecliptic from west to east, why have we asserted that if he be at present in the ascending node, he will remove from it in the course of a lunation, 30 degrees 40 minutes 15 seconds?

This objection will not appear of much consequence,

but to those who imagine that the nodes which the lunar orbit forms with the solar are fixed and immoveable. This is not the case; these nodes have a periodical motion, that is, they pass through the 12 signs of the zodiac in the course of almost 19 years, not from west to east, as the sun, but from east to west: at the end of a lunation then the sun must be 30 degrees 40 minutes 15 seconds distant from the node he has quitted; because he not only moves from his node, but his node moves from him.

In regard to new moons, the only difference in the operation is, that 33890 is added to the product of the lunations by 7361, instead of 37326. At the time of the new moon in January 1701, the sun was distant from his node 141 degrees 12 minutes 30 seconds; which expressed in quarters of a minute are equal to 33890. For an eclipse of the sun therefore, 33890 must be added to the product of the lunations by 7361.

It is to be observed also, that for solar eclipses, the remainder must be less than 4060, which represents the quarters of a minute contained in 16 degrees 55 minutes. A solar eclipse indeed is not impossible but when the sun and moon are at a greater distance from their nodes than 16 degrees 55 minutes: the remainder and divisor therefore must not be compared with 2800, as for eclipses of the moon, but with 4060.

To apply the above rules to the present century.

It is evident from what has been said, that to find, by the above method, the eclipses of the sun and moon in the present century, nothing will be necessary but to substitute, for the sun's distance from the node at the time of the new and full moon in the month of January 1701, the same distance at the time of the new and full moon in the month of January 1801, and to count the lunations between the new moon in January 1801, that is the 14th, and the time proposed. But the sun's distance from the node at

the time of the new moon on the 14th day of January 1801, was 280° 56′ 44″, and his distance from the node at the time of the full moon on the 29th of January 1801 was 297° 15′ 11″. The former of these reduced to quarters of a minute gives 67427, and the latter reduced in the same manner gives 71341.

Example 1st.—Let it be required to find whether there will be an eclipse at the full moon on the 18th of March 1802. Between the 14th of January 1801 and the 3d of March 1802, the day of the new moon preceding the 18th of March, there will be 14 complete lunations. The product of this number by 7361 is 103054, to which if we add 71341, the sum will be 174395. But this sum divided by 43200, leaves for remainder 1595: which, being less than 2800, shows that there will be an eclipse of the moon on the 18th of March 1802.

Example 2d.—It is required to find whether there will be an eclipse of the sun on the 3d of March 1802. Between the 14th of January 1801 and the 3d of March 1802, there will be 14 complete lunations; if this number be multiplied by 7361, the product will be 103054, to which adding 67427, we shall have 170481; and this sum divided by 43200, will leave for remainder 40881: this number is not less than 4060, but its difference from 43200, which is 2319, is less than 4060; we may conclude therefore that there will be an eclipse of the sun on the 3d of March 1802.

PROBLEM XVI.

Construction of a machine which indicates the new and full moons, with the eclipses that have happened, or that will happen, during a certain period of time.

This ingenious machine, which deserves a place in the cabinet of the Astronomer, was invented by M. de la Hire. It consists of three circular pieces of copper, wood or pasteboard, and an index (pl. 3 fig. 9), which all turn around a common centre. Towards the edge of the upper

piece, which is the least, there are two circular bands, containing small apertures; the exterior ones of which exhibit the new moons, with the image of the sun, and the interior ones the full moons, with the image of the moon.

The edge of this circular piece is divided into 12 lunar months; each consisting of 29 days 12 hours 44 minutes; but in such a manner that the end of the twelfth month, which forms the commencement of the second lunar year, surpasses the first new moon of the second, by 4 of the 179 divisions marked on the second circular piece, placed between the other two.

The edge of this piece is furnished with an index, one of the sides of which forms part of a right line, that tends to the centre of the machine; and which passes also through the middle of one of the external apertures, that shows the first new moon of the lunar year. The diameter of each of these apertures is equal to about 4 degrees.

The edge of the second circular piece is divided into 179 equal parts, corresponding to as many lunar years, each consisting of 354 days and about 9 hours. The first year begins at the number 179, where the last ends.

The complete years are each marked with the figures 1, 2, 3, 4, &c, placed at every four divisions; and go four times round, to make up the number 179, as seen in the figure. Each lunar year comprehends four of these divisions; so that in this figure they anticipate one over the other four of the 179 divisions of the edge.

On the same circular piece, and below the apertures of the former, there are spaces, coloured black, at the two extremities of the same diameter, which correspond to the external apertures; and which indicate the eclipses of the sun: other spaces coloured red, correspond to the internal apertures, and indicate the eclipses of the moon. The quantity of each colour, which appears through the openings, shows the extent of the eclipse. The middle between the two colours, which is the place of the moon's

node, corresponds on one side to the division marked 4 and $\frac{2}{3}$ of a degree; and on the other side to the opposite number. The figure of the coloured space is seen on the second circular piece; and its amplitude or extent marks the boundaries of the eclipses.

The third and largest of the circular pieces, which is below the rest, contains the days and months of the common years. The division begins at the first day of March, in order that when the year is bissextile one day may be added to the month of February. The days of the year are described in the form of a spiral; as the month of February passes beyond the month of March, because the lunar year is shorter than the solar, so that the 15th hour of the 10th day of February corresponds to the beginning of March. But, after counting the last day of February, it is necessary to move the two upper pieces backward, in the state in which they are, in order to resume the first day of March.

Thirty days are marked before the month of March, which serve for finding the epacts.

It is to be observed, that the days, as here understood, are not counted according to the usual method of astronomers, but according to the vulgar mode of computation, beginning at midnight, and ending at midnight of the day following. When we speak therefore of the first day of a month, or of any other, we mean the space of that day marked in the division; for we here count the current days, according to the vulgar usage, as already said.

In the middle of the upper circular piece are described the epochs which mark the commencement of the lunar years, in regard to the solar, according to the Gregorian calendar, and for the meridian of London. The commencement of the first year, which ought to be marked 0, and which corresponds on the division 179, took place at London on the 29th of February 1680 at 4 hours 44 mi-

nutes, new stile. The end of the first lunar year, which is the commencement of the second, corresponds to the division marked 1; and took place at London on the 17th of February 1681, at 13 hours, 33 minutes, counting 24 hours, as before said, from one midnight to another. To prevent error, in making the divisions on the edge of the second circular piece to agree with the corresponding ones of the epochs of the lunar year, the same numbers are inscribed on both.

The epochs of all the lunar years, from 1777 to 1791, are marked in succession; in order that in using this machine, the solar and lunar years may better agree. In regard to the other years of the cycle of 179 years, they may be easily completed by adding 354 days 8 hours 48¾ minutes, for each lunar year.

The index, which extends from the centre of the instrument to the edge of the largest circular piece, serves for making the divisions of one piece correspond with those of the other two. If this machine be applied to a clock, it will form an instrument perfect and complete in all its parts.

The table of epochs, which is calculated for the meridian of London, may be easily reduced to any other meridian, if the difference of meridians, in time, be added for places laying to the east of London, and subtracted for those lying to the west.

It is proper to put the table of epochs on the middle of the upper plate, in order that it may be seen with the machine.

EPOCHS OF THE LUNAR YEARS,

Corresponding to the Civil Years, for the Meridian of London.

Lunar years.		Civil years.		Months.		D.	H.	M.
179	.	1680 B	.	February	.	29	4	44
1	.	1681	.	February	.	17	13	33
2	.	1682	.	February	.	6	22	21
10	.	1689	.	November	.	11	20	50
20	.	1699	.	Ju[illegible]	.	26	12	57
30	.	1709	.	April	.	9	5	3
40	.	1718	.	December	.	21	21	10
50	.	1728 B	.	September	.	3	13	15
60	.	1738	.	May	.	18	5	21
70	.	1718 B	.	January	.	29	21	27
80	.	1757	.	October	.	12	13	35
90	.	1767	.	June	.	26	5	40
100	.	1777	.	March	.	8	21	46
101	.	1778	.	February	.	26	6	34
102	.	1779	.	February	.	25	15	22
103	.	1780 B	.	February	.	4	0	10
104	.	1781	.	January	.	24	8	58
105	.	1782	.	January	.	13	17	46
106	.	1783	.	January	.	3	2	34
107	.	1783	.	December	.	23	11	22
108	.	1784 B	.	December	.	11	20	10
109	.	1785	.	December	.	1	4	59
110	.	1786	.	November	.	21	13	47
111	.	1787	.	November	.	10	22	35
112	.	1788 B	.	October	.	30	7	24
113	.	1789	.	October	.	19	16	12
114	.	1790	.	October	.	9	1	0
114	.	1791	.	September	.	28	9	48
120	.	1796 B	.	August	.	3	5	59
130	.	1806	.	April	.	16	22	5

Lunar years.		Civil years.		Months.		D.	H.	M.
140	.	1815	.	December	.	29	14	12
150	.	1825	.	September	.	11	5	28
160	.	1835	.	May	.	25	22	24
170	.	1845	.	February	.	5	14	31
1	.	1854	.	October	.	20	6	37

Method of making the divisions on the circular pieces of the instrument.

The circle of the largest piece is divided in such a manner, that 368 degrees 2 minutes 12 seconds comprehend 354 days and somewhat less than 9 hours; hence it follows, that this circle ought to contain 346 days, and 15 hours, which may be assumed without any sensible error as two thirds of a day. But to divide a circle into 346 parts and two thirds, reduce the whole into thirds, which in this case make 1040 thirds, and then find the greatest multiple of 3 that can be easily halved, and is contained in 1040. This number will be found in the double geometrical progression, the first or least term of which is 3; as for example 3, 6, 12, 24, 48, 96, 192, 384, 768.

The ninth number of this progression, viz 768, is the one required: subtract this number from 1040; and by the rule of three, find the number of degrees minutes and seconds contained in the remainder 272, by saying, as 1040 thirds : 360 degrees :: 272 thirds : 94 degrees 9 minutes 23 seconds.

Then cut off from this circle an angle of 94° 9′ 23″, and divide the rest of the circle always into halves. When eight sub-divisions have been made, you will come to the number 3, which will be the arc of one day; and if the arc of 94° 9′ 23″ be divided by this also, the whole circle will be divided into 346 days and two thirds; for there will be 256 days in the larger arc, and 90 days two thirds in the other. Each of these spaces will correspond to 1° 2′ 18″, as may be seen by dividing 360 by 346⅔, and 10 days cor-

respond to 10° 23′. By these means a table for dividing this circular piece might be formed.

These days must afterwards be distributed to each of the months of the year, according to the number which belongs to them, beginning with March, and continuing to the fifteenth hour of the tenth of February, which corresponds to the commencement of March; and the remainder of the month of February passes beyond and above.

The circle of the second plate must be divided into 179 equal parts. For this purpose, find the greatest number that can always be halved to unity, and which is contained in 179. This number is 128, which taken from 179, leaves for remainder 51. Then find, by the rule of three, what part of the circumference is equal to this remainder, by saying, as 179 : 360 degrees :: 51 parts : 102 degrees 34 minutes 11 seconds.

Having cut off from the circle an arc of 102° 34′ 11″, divide the remainder always into halves, and after seven sub-divisions you will come to unity. This part of the circle therefore will be divided into 128 equal parts: then with the same aperture of the compasses, divide the remaining arc into 51 parts; and the whole circle will be divided into 179 equal parts, each corresponding to 2 degrees and 40 seconds, as may be easily seen by dividing 360 by 179.

In the last place, to divide the circle of the upper plate, take the fourth of its circumference, and add to it one of the 179 parts or divisions of the edge of the middle plate: if the compasses, with an aperture equal to the fourth thus increased, be then made to turn four times, it will divide the circle in the manner in which it ought to be; since by subdividing each of these quarters into three equal parts, we shall have 12 spaces for the 12 lunar months; so that the end of the twelfth month, which forms the commencement of the twelfth lunar year, surpasses the first new moon by 4 of the 179 divisions marked on the middle plate.

The method of using this machine is contained in the following problem.

PROBLEM XVII.

A lunar year being given; to find, by means of the preceding machine, the days of the solar year corresponding to it; and on which there will be new or full moon, or an eclipse of the sun or moon.

Let the proposed lunar year be the 101st in the table of epochs, which corresponds to the division of the middle circular plate marked 101. Bring the edge of the index of the upper plate to the division marked 101 of the middle plate, where the commencement of the 101st lunar year falls; and as this commencement took place, according to the table of epochs, on the 26th February 1778, at 6 hours 34 minutes, turn both the upper plates together in that state, till the edge of the index, attached to the upper plate, corresponds to the 6th hour, or a little more than the fourth of the 26th of February marked on the lower plate, at which time the first new moon of the proposed lunar year happened.

Then, without altering the situation of the three plates, extend a thread from the centre of the instrument, or turn the moveable index, till it pass through the middle of the aperture of the first full moon: the edge of the index will then correspond to the middle of the 13th of March, which ought to be the time of full moon, within a few hours; and as the aperture of this full moon does not present a red colour, there was no eclipse of the moon.

To find what took place at the following full moon, add to the new moon of the epoch 29 days 12 hours 44 minutes, and you will have the time of new moon on the 27th of March, at 19 hours 16 minutes; and by performing the same operation, it will be found that there was no eclipse either at that new moon, or at the full moon following.

But, proceeding in this progressive manner, you will come to the new moon of November, which took place on the 19th of that month, at 1 hour, 8 minutes; then performing the same operation, you will find the full moon following on the 3d of November, at about 8 in the evening, and it will be seen that there was a partial eclipse, the aperture of the full moon being in that part filled with the red colour.

The eclipses of the sun will be found in like manner: they will be indicated by the black colour which will present itself at the aperture of the new moons.

On the 24th of June 1778, for example, new moon took place at 19 hours 8 minutes, or 8 minutes past 7 in the evening; and as the aperture of this new moon will be in part occupied by the black colour which is below, we may conclude that there was a partial eclipse of the sun on the 24th June 1778 in the evening: which was indeed the case.

By such a machine however, it is not possible, as may be readily conceived, to determine the exact hour and minute of an eclipse or of a lunation. It is enough, if it indicates whether a conjunction or opposition takes place in the ecliptic; the rest must be determined by calculation, for which precepts may be found in all works that treat expressly on this subject.

To gratify the curiosity of the reader, we shall here give a table of the eclipses, both of the sun and moon, which will take place in the course of the present century; with the different circumstances attending them, such as the time of the middle of the eclipse, and its extent; and, in regard to eclipses of the moon, how many digits will be eclipsed, &c.

We must however observe, that as this table is extracted from an immense labour*, undertaken for another purpose,

* This labour is a table of the solar and lunar eclipses since the commencement of the Christian æra, to the year 1900, inserted in *l'Art de véri-*

perfect exactness must not be expected, either in extent or time, and particularly in regard to the eclipses of the sun, since it is well known that a solar eclipse, on account of the moon's parallax, varies in quantity according to the place of the earth; that an eclipse, for example, which is central and total to the regions of the southern hemisphere, may be only partial and small to the northern. The author therefore, to whom we allude, was satisfied with indicating, rather than calculating, these eclipses; and left the more exact determinations to astronomers.

To render this table however more generally useful, we shall add the following explanation. The hour marked indicates the middle of the eclipse in true time; ½ signifies one half, ¼ one fourth of an hour, morn. morning, aft. afternoon. The quantity of the eclipse is expressed in digits and divisions of a digit. A digit is one twelfth part of the diameter of the luminary eclipsed. Six digits are equal to one half of the disc; four digits to one third, &c. When an eclipse is marked 0 digits, the meaning is that it is less than a quarter, or ¼ of a digit. When the moon is within a minute of a degree or less of the centre, the eclipse is marked central; when within two minutes, almost central. The duration of eclipses is nearly proportioned to their greatness; a total lunar eclipse will continue at least 3½ hours, and at most four hours and some minutes; a partial eclipse, which exceeds six digits, may continue 2½ or 3¼ hours; eclipses of between 3 and 6 digits, are of two or three hours' duration; those of two digits will last about 1½ hours; those of 1 digit, about 1 hour; and those of ½ digit, about ¾ of an hour. The time therefore of the middle of an eclipse, and its duration being given, its beginning and end may be nearly ascertained by the following rule, viz: subtract its semi-duration from the time given, and the remainder will be the hour of the

fier les Dates, by the Abbé Pingré, a celebrated astronomer, and member of the Royal Academy of Sciences.

beginning; add the same quantity, and the sum will be the time of the end. A lunar eclipse must begin and end every where at the same time; with this difference, that so many hours must be added or subtracted as the one place is to the eastward or westward of the other. Thus, an eclipse that begins about $4\frac{1}{4}$ hours P.M. at Greenwich observatory, will begin about 12 P.M. at Pekin, as the latter is 7 hours 46 minutes eastward of the former.

In regard to solar eclipses, they are dated from the time of the conjunction of the sun and moon. Though this date be sensibly different from that of the middle of the eclipse; yet this difference will never amount to two hours, and may be nearly found by the following rules: 1st. In the morning a solar eclipse must always happen sooner, and in the evening later, than the time of the conjunction. 2d. The nearer the sun is to the horizon, the more sensible will be the difference. 3d. The acceleration in the morning will be great in proportion to the elevation of the sun at mid-day, three months before, and the retardation in the evening will be great in proportion to the sun's elevation, three months after the time proposed. It thence follows, 1st. That the difference must be greatest in the torrid zone; and 2d. That the greatest difference in the other latitudes must happen in the evening of the vernal, and in the morning of the autumnal equinox; for the greatest meridian altitudes are observed three months before and after these seasons.

The parts of the world where the eclipse is visible, are marked. If there be no limitation, the whole or the greater part of Europe or Asia must be understood. Particular divisions of these quarters are denoted by the letters E. W. N. and S. that is East, West, &c. When an eclipse is said to be visible in E. or W. of Europe, &c, the meaning is, that it is visible in all the parts of the region specified, where the sun is sufficiently elevated above the horizon at the time of conjunction. When it is marked as

visible N. or S. of any particular region, all places in every other direction are excluded. The terms small and great for the most part refer to the eclipses, and not to the places where they are visible. The latitude of those places is marked in which an eclipse is central. South latitude is indicated by the letter S. and north latitude by N. which is frequently omitted. An O, or cypher, denotes north latitude.

The course of a central eclipse is ofttimes pointed out by three numbers. The first and third show the latitude in which the eclipse is central in the planes of the 5th and 155th meridians, the second, included in crotchets, gives the latitude in which it is central at mid-day. The place where an eclipse is central at mid-day, may be easily found, when the time of the true conjunction at Paris is known. The interval between the true conjunction as given, and mid-day, nearly shows how many hours and minutes the required place is east or west of the meridian of Paris.

It is to be observed also, that the limits of eclipses are fixed to be the tropic of Cancer in Africa, and the northern extremity of Lapland, and from 5° to 6° N. lat. in Asia to the Polar circle. In longitude, the limits are the 5th and the 155th meridians, supposing the 20th to pass through Paris.

The first and third numbers above mentioned, do not always express the latitude, under the 5th and 155th meridians. Sometimes an eclipse begins before the sun has risen upon the former, and ends after it has gone down on the latter meridian. In these cases, the first number denotes the latitude in which the eclipse is central at sun-rising; and the next the latitude in which it is central at sun-set. The number included in crotchets is omitted when there is no meridian within the limits prescribed, under which the time of mid day coincides with the middle of the eclipse. It is to be observed also, that a number is

sometimes added to point out the increase or decrease of an eclipse.

A single character or number indicates the latitude in which an eclipse is central in Europe or Africa at sun-set; and towards the eastern extremity of Asia at sun-rising. An asterisk * denotes that the course of a central eclipse extends many degrees beyond the equator. A dagger † indicates that its course is beyond the pole; and the excess is sometimes added to 90. Thus 94 intimates that the eclipse referred to is central 4° beyond the pole. The sign + affixed to pen, is used to express that the penumbra is deep or strong.

An eclipse is visible from 32° to 64° north; and as far south of the place where it is central.

Table of Eclipses, from the beginning to the end of the present Century.

1801. Eclipse of the moon, total, March 30th. 5½ morn. cent. Of the sun, April 13th. 4½ morn. Europe N.E. Asia, N. dim. from W. to E. Of the sun, September 8th. 6 morn. Asia N.E. small. Of the moon, total, September 22d. 7½ morn.

1802. Of the moon, March 19th. 11½ morn. 5 dig. Of the sun, August 28th. 7¼ morn. Eur. Afr. Asia, cent. 69 (59) 23 an. Of the moon partial, September 11th. 11 aft. 9 digits.

1803. Of the sun, August 17th. 8½ morn. great part of Eur. s. Afr. Asia, s. cent. 26 (12) * an.

1804. Of the moon partial, January 26th. 9¼ aft. Of the sun, February 11th. 11½ morn. Eur. Afr. Asia, W. cent. 25 (22) 64. Of the moon partial, July 22d. 5½ aft. 10¾ dig.

1805. Of the moon total, January 15th. 9 morn. Of the sun, June 26th. 11 aft. part of Asia N.E. Of the moon total, July 11th. 9 aft.

1806. Of the moon partial, January 5th. 0 morn. 9 dig.

Of the sun, June 16th. 4 aft. Eur. Afr. w. cent. 31 – 16 tot. Of the moon partial, June 30th. 10 aft. pen. Of the sun, December 10th. 2½ morn. small, Asia, s. e.

1807. Of the moon partial, May 21st. 5½ aft. 1½ dig. Of the sun, June 6th. 5½ morn. small, Asia s. e. Of the moon partial, November 15th. 8½ morn. 3 dig. Of the sun, November 29th. merid. all Eur. Afr. Asia, w. cent. 18 (13) 9 – 25.

1808. Of the moon total, May 10th. 8 morn. Of the moon total, November 3d. 9 morn. Of the sun, November 18th. 3 morn. great part of Asia n. incr. from w. to e.

1809. Of the moon partial, April 30th. 1 morn. 10 dig. Of the moon partial, October 23d. 9½ morn. 9½ dig.

1810. Of the sun, April 4th. 2 morn. Asia, s. e. cent. * 10 an.

1811. Of the moon partial, March 10th. 6½ morn. 5 dig. Of the moon partial, September 2d. 11 aft. 7 dig.

1812. Of the moon total, February 27th. 6 morn. almost cent. Of the moon total, August 22d. 5 aft.

1813. Of the sun, February 1st. 9 morn. Eur. Afr. Asia, cent. 32 – 24 (26) 55 an. Of the moon partial, February 15th. 9 morn. 7½ dig. Of the moon partial, August 12th. 3¼ morn. 4½ dig.

1814. Of the sun, January 21st. 2½ aft. Eur. s. e. Afr. cent. * 10 an. Of the sun, July 17th. 7 morn. Eur. s. Afr. e. Asia, s. cent. 14 – 33 (31) 5 tot. Of the moon partial, December 26th. 11½ aft. 6 dig.

1815. Of the moon total, June 21st. 6½ aft. 12¼ dig. Of the sun, July 7th. 0 morn. Eur. and Asia, n. cent. 62 † tot. Of the moon partial, December 16th. 1¼ aft.

1816. Of the moon total, June 10th. 1½ morn. Of the sun, November 19th. 10½ morn. Eur. Afr. Asia,

w. cent. 59 (38) 33 – 37 tot. Of the moon partial, December 4th. 9 aft. 7¼ dig.

1817. Of the sun, May 16th. 7 morn. Asia, s. cent. * (7) 12 – 7 an. Of the moon partial, May 3d. 3½ aft. pen. +. Of the sun, November 9th. 2½ morn. Asia, E. cent. 26 – 5 s. tot.

1818. Of the moon partial, April 21st. 0½ morn. 5¾ dig. Of the sun, May 5th. 7½ morn. Eur. Afr. Asia, cent. 13 (51) 60 – 53 an. Of the moon partial, October 14th. 6 morn. 2 dig.

1819. Of the moon total, April 10th. 1½ aft. Of the sun, April 24th. merid. N. of Eur. and of Asia, dim. from W. to E. Of the sun, September 19th. 1 aft. Eur. N.E. small. Of the moon total, October 3d. 3½ aft.

1820. Of the moon partial, March 29th. 7 aft. 6 dig. Of the sun, September 7th. 2 aft. Eur. Afr. Asia, w. cent. 62 – 29 an. Of the moon partial, September 22d. 7 morn. 10 dig.

1821. Of the sun, March 4th. 6 morn. Asia, S.E. cent. * (7 s.) 24 tot.

1822. Of the moon partial, February 6th. 5½ morn. 4½ dig. Of the moon partial, Aug. 3d. 0½ morn 9 dig.

1823. Of the moon total, January 26th. 5½ aft. Of the sun, February 11th. 3 morn. great part of Asia N. small. Of the sun, July 8th. 6½ morn. Eur. and Asia, N. Of the moon total, July 23d. 3½ morn.

1824. Of the moon partial, January 16th. 9 morn. 9 dig. Of the sun, June 26th. 11½ aft. Asia, E. cent. 27 – 41 tot. Of the moon partial, July 11th. 4½ morn 1 dig. Of the sun, December 20th. 11 morn. Indies, s. small.

1825. Of the moon partial, June 1st. 0½ morn. Of the sun, June 16th. 0½ aft. Afr. small cent. * (0) *.

Of the moon partial, November 25th. 4½ aft. 2½ dig.

1826. Of the moon total, May 21st. 3½ aft. Of the moon total, November 14th. 4½ aft. Of the sun, November 29th. 11½ morn. Eur. Afr. Asia, w.

1827. Of the su ., April 26th. 3½ morn. Eur. N.E. Asia, N. cent. 49 (81) 84 an. Of the moon partial, May 11th. 8¾ morn. 11¼ dig. Of the moon partial, November 3d. 5 aft. 10 dig.

1828. Of the sun, April 14th. 9¾ morn. small part of Eur. S.E. Afr. Asia, cent. 2 s. (18) 29 – 26. Of the sun, October 9th. 0½ morn. Asia S.E. cent. 7 * an.

1829. Of the moon partial, March 20th. 2 aft. 4 dig. Of the moon partial, September 13th. 7 morn. 5¼ dig. Of the sun, September 28th 2½ morn. Asia, E. cent. 59 – 40 an.

1830. Of the sun, February 23d. 5 morn. Asia, N. dim. from W. to E. Of the moon total, March 9th. 2 aft. Of the moon total, September 2d. 11 aft. cent.

1831. Of the moon partial, February 26th. 5 aft. 8 dig. Of the moon partial, August 23d. 10½ morn. 6 dig.

1832. Of the sun, July 27th. 2½ aft. Eur. s. Afr. Asia, S.E. cent. 23 N. 3 s. tot.

1833. Of the moon partial, January 6th. 8 morn. 5¾ dig. Of the moon partial, July 2d. 1 morn. 10¼ dig. Of the sun, July 17th. 7 morn. Eur. Afr. E. Asia N. cent. 83 (80) 73 tot. Of the moon total, December 26th. 10 aft.

1834. Of the moon total, June 21st. 8½ morn. Of the moon partial, December 16th. 5¼ morn. 8 dig.

1835. Of the sun, May 27th. 1½ aft. small part of Eur. Afr. Asia, S.W. cent. 7 – 8 – 3 s. an. Of the moon partial, June 10th. 11 aft. 0½ dig. Of the sun, November 20th. 11 morn. small part of Eur. S.W. Afr. small part of Asia, S.W. cent. 4 (11 s.) * tot.

1836. Of the moon partial, May 1st. 8½ morn. 4¾ dig. Of the sun, May 15th. 2½ aft. Eur. Afr. Asia, w. cent. 53 – 54 – 44 an. Of the moon partial, October 24th. 1¾ aft. 1½ dig.

1837. Of the moon total, April 20th. 9 aft. Of the sun, May 4th. 7½ aft. small part of Eur. N. great part of Asia, N. E. Of the moon total, October 13th. 11½ aft.

1838. Of the moon partial, April 10th. 2¼ morn. 7 dig. Of the moon partial, October 3d. 3 aft. 0¾ dig.

1839. Of the sun March 15th. ½ aft. Eur. s. Afr. Asia, s. w. cent. 17 - 26 tot. Of the sun, September 7th 10½ aft. extrem. of Asia, E. cent. 37. an.

1840. Of the moon partial, February 17th. 2 aft. 4½ dig. Of the sun, March 4th. 4 morn. cent. 16 (37) 48. Of the moon partial, August 13th. 7½ morn. 7¼ dig.

1841. Of the moon total, February 6th. 2½ morn. Of the sun, February 21st. 11 morn. almost all Eur. N. Asia, N.W. dim from W. to E. Of the sun, July 18th. 2 aft. great part of Eur. N E. Asia, N.W. incr. from W. to E. Of the moon total, August 2d. 10 morn.

1842. Of the moon partial, January 26th. 6 aft. 9 dig. Of the sun, July 8th. 7 morn. Eur. Afr. Asia, cent. 35 – 50 (49) 21 tot. Of the moon partial, July 22d. 11 morn. 3 dig.

1843. Of the moon partial, June 12th. 8 morn. pen. Of the moon partial, December 7th. 0½ morn. 2¼ dig. Of the sun, December 21st. 5½ morn. Asia, cent. 25 (8) 21 tot.

1844. Of the moon total, May 31st. 11¼ aft. Of the moon total, November 25th. 0¼ morn.

1845. Of the sun, May 6th. 10½ morn. almost all Eur. N.W. Asia, N.W. cent. 90 (98) † an. Of the moon total, May 21st. 4½ aft. 12¾ dig. Of the moon partial, November 14th. 1 morn. 10½ dig.

1846. Of the sun, April 25th. $5\frac{1}{2}$ aft. Eur. and Afr. w. cent. 28 – 26. Of the sun, October 20th. $8\frac{1}{2}$ morn. Eur. s.w. Afr. Asia, s. w. cent. (18 s.) * an.

1847. Of the moon partial, March 31st. $9\frac{1}{2}$ aft. $2\frac{1}{4}$ dig. Of the sun, September 24th. 3 aft. $4\frac{1}{4}$ dig. Of the sun, October 9th. $9\frac{1}{2}$ morn. Eur. Afr. Asia, cent. 58 (31) 16 – 17 an.

1848. Of the moon total, March 19th. $9\frac{1}{2}$ aft. Of the moon total, September 13th. $6\frac{1}{2}$ morn. Of the sun, September 27th. 10 morn. Eur. N.E. Asia, N.

1849. Of the sun, February 23d. $1\frac{1}{2}$ morn. Asia, E. cent. 31 – 28 – 32 an. Of the moon partial, March 9th. 1 morn. $8\frac{1}{2}$ dig. Of the moon partial, September 2d. $5\frac{1}{2}$ aft. 7. dig.

1850. Of the sun, February 12th. $6\frac{1}{2}$ morn Asia, S.E. cent. * (11 s.) 17 N. an. Of the sun, August 7th. 10 aft. extrem. of Asia, E. cent. 14 tot.

1851. Of the moon partial, January 17th. 5 aft. $5\frac{1}{4}$ dig. Of the moon partial, July 13th. $7\frac{1}{2}$ morn. $8\frac{1}{2}$ dig. Of the sun, July 28th. $2\frac{1}{2}$ aft. Eur. Afr. Asia, w. cent. 70 – 39 tot.

1852. Of the moon total, January 7th. $5\frac{1}{2}$ morn. Of the moon total, July 1st. $3\frac{3}{4}$ aft. Of the sun, December 11th. 4 morn. Asia, E. cent. 59 (36) 35 tot. Of the moon partial, December 26th. 1 aft. 8 dig.

1853. Of the moon partial, June 21st. 6 morn. $2\frac{1}{2}$ dig.

1854. Of the moon partial, May 12th. 4 aft. 3 dig. Of the moon partial, November 4th. $9\frac{1}{2}$ aft. 1 dig.

1855. Of the moon total, May 2d. $4\frac{1}{2}$ morn. Of the sun, May 16th. $2\frac{1}{2}$ morn. great part of Asia, N. dim. from w. to E. Of the moon total, October 25th. 8 morn.

1856. Of the moon partial, April 20th. $9\frac{1}{2}$ morn. $8\frac{3}{4}$ dig. Of the sun, September 29th. 4 morn. Asia, N. cent. 84 (67) 66 an. Of the moon partial, October 13th. $11\frac{1}{2}$ aft. $11\frac{1}{2}$ dig.

1857. Of the sun, September 18th. 6 morn. Eur. and Afr. E. Asia, s. cent. 40 (12) 12 s. an.

1858. Of the moon partial, February 27th. $10\frac{1}{4}$ aft. 4 dig. Of the sun, March 15th. $0\frac{1}{2}$ aft. Eur. Afr. Asia, w. cent. (40) 68. Of the moon partial, August 24th. $2\frac{1}{2}$ aft. $5\frac{1}{2}$ dig.

1859. Of the moon total, February 17th. 11 morn. Of the sun, July 29th. $9\frac{1}{2}$ aft. small, Asia, N.E. Of the moon total, August 13th. $4\frac{1}{2}$ aft.

1860. Of the moon partial, February 7th. $2\frac{1}{2}$ morn. $9\frac{1}{2}$ dig. Of the sun, July 18th. [illegible] aft. Eur. Afr. Asia, w. cent. 19 – 16 tot. Of the moon partial, August 1st. $5\frac{1}{2}$ aft. $4\frac{3}{4}$ dig.

1861. Of the sun, January 11th. $2\frac{1}{2}$ morn. small, Asia, s.w. Of the sun, July 8th. 2 morn. Asia, s.e. cent. [illegible] 9 an. Of the moon partial, December 17th. $8\frac{1}{2}$. morn. 2 dig. Of the sun, December 31st. $2\frac{1}{2}$ aft. all Eur. Afr. cent 17 – 30 tot

1862. Of the moon total, June 12th. $6\frac{3}{4}$ morn. Of the moon total, December 6th 8 morn. Of the sun, December 21st. $5\frac{1}{2}$ morn. great part of Asia, N.

1863. Of the sun, May 17th. 5 aft. great part of Eur. N. Of the moon total, June 2d 0 morn. Of the moon partial, November 25th. 9 morn. 11 dig.

1864. Of the sun, May 6th $0\frac{1}{4}$ morn. Asia, s.E. cent. 6 – 23.

1865. Of the moon partial, April 11th. 5 morn. $1\frac{1}{2}$ dig. Of the moon partial, October 4th. 11 aft. $3\frac{3}{4}$ dig. Of the sun, October 19th. 5 aft. extrem. of Eur. and of Afr. w. cent. 16 an.

1866. Of the sun, March 16th. 10 aft. small, Asia, N.E. Of the moon total, March 31st. 5 morn. Of the moon total, September 24th. $2\frac{1}{2}$ aft. Of the sun, October 8th. $5\frac{1}{4}$ aft. Eur. w. dim. from N. to s.

1867. Of the sun, March 6th. 10 morn. Eur. Afr. Asia, cent. 31 (15) 69 an. Of the moon partial, March

20th. 9 morn. 9¼ dig. Of the moon partial, September 14th. 1 morn. 8 dig.

1868. Of the sun, February 23d. 2½ aft. Eur. s. Afr. Asia, s.w. cent. 9–21. an. Of the sun, August 18th. 5½ morn. Eur. s.E. Afr. Asia, s. cent. 14–18 (11) 0 tot.

1869. Of the moon partial, January 28th. 1¾ morn. 5½ dig. Of the moon partial, July 23d. 2 aft. 6¾ dig. Of the sun, August 7th. 10 aft. Asia, N.E. cent. 46 tot.

1870. Of the moon total, January 17th. 3 aft. Of the moon total, July 12th. 11 aft. Of the sun, December 22d. 0¾ aft. Eur. Afr. Asia, w. cent. (36) 49 tot.

1871. Of the moon partial, January 6th. 9½ aft. 8 dig. Of the sun, June 18th. 2½ morn. Asia, s.E. small. Of the moon partial, July 2d. 1½ aft. 1 dig. Of the sun, December 12th. 1½ morn. Asia, s. cent. 17 ½ tot.

1872. Of the moon partial, May 22d. 11½ aft. 1½ dig. Of the sun, June 6th. 5½ morn Asia, cent. 8 (42) 43 an. Of the moon partial, November 15th. 5½ morn. 0½ dig.

1873. Of the moon total, May 12th. 11½ morn. Of the sun, May 26th. 9½ morn. great part of Eur. N.W. Afr. w. Asia, s. dim from w. to E. Of the moon total, November 4th. 4½ aft.

1874. Of the moon partial, May 1st, 4½ aft. 9¼ dig. Of the sun, October 10th. 11½ morn. Eur. Afr. Asia, w. cent. 82 (74) 55 an. Of the moon partial, October 25th. 8 morn. 12 dig.

1875. Of the sun, April 6th. 7 morn. Asia, s.E. cent. * (1) 21 tot. Of the sun, September 29th. 1¾ aft. small part of Eur. s.w. Afr. Asia, s.w. cent. 13 (10) 13 s. an.

1876. Of the moon partial, March 10th. 6½ morn. 3½ dig. Of the moon partial, September 3d. 9½ aft. 4 dig.

1877. Of the moon total, February 27th. 7½ aft. Of the sun, March 15th. 3 morn. great part of Asia, N. dim. from W. to E. Of the sun, August 9th. 5 morn. Asia, N.E. small. Of the moon total, August 23d. 11½ aft. almost cent.

1878. Of the moon partial, February 17th. 11½ morn. 9½ dig. Of the sun July 29th. 9½ aft. extrem. of Asia, E. cent. 52 tot. Of the moon partial, August 13th 0½ morn. 6½ dig.

1879. Of the sun, January 22d. merid. small, Asia, S.W. cent. * 7 an. Of the sun, July 19th. 9 morn. Eur. S. Afr. Asia, S.W. cent. 8—16 (12) * an. Of the moon partial, December 28th. 4½ aft. 1¾ dig.

1880. Of the sun, January 11th. 11 aft. Asia, E. cent. 16 tot. Of the moon total, June 22d. 2 aft. 12¾ dig. Of the moon total, December 16th. 4 aft. Of the sun, December 31st. 2 aft. Eur. Afr. dim. from N. to S.

1881. Of the sun, May 28th. 0 morn. Asia, N. dim. from W. to E. Of the moon total, June 12th. 7¼ morn. Of the moon partial, December 5th. 5½ aft. 11½ dig.

1882. Of the sun, May 17th. 8 morn. Eur. S.E. Afr. Asia, cent. 10 (38) 42—26 tot. Of the sun, November 11th. 0 morn. Asia, S.E. cent. 2 * an.

1883. Of the moon partial, April 22d. merid. 0¼ dig. Of the moon partial, October 16th. 7½ morn. 3 dig. Of the sun, October 31st. 0½ morn. Asia, E. cent. 46 an.

1814. Of the sun, March 27th. 6 morn. small, great part of Eur. N.E. dim. in Asia, from W. to E. Of the moon total, April 10 merid. Of the moon total, October 4th. 10½ aft. Of the sun, October 19th. 1 morn. Asia, N.

1885. Of the moon partial, March 30th. 5 aft. 10 dig. Of the moon partial, September 24th. 8½ morn. 9 dig.

1886. Of the sun, August 29th. 1½ aft. extrem. of Eur. s.w. Afr. cent. 6 (4) * tot.

1887. Of the moon partial, February 8th. 10¾ morn. 5¼ dig. Of the moon partial, Aug. 3d. 9 aft. 5 dig. Of the sun, Aug. 19th. 6 morn. Eur. and Afr. E. Asia, cent. 54—62 (54) 29 tot.

1888. Of the moon total, January 28th. 11¾ aft. Of the moon total, July 23d. 6 morn. almost cent.

1889. Of the moon partial, January 17th. 5½ morn. 8¼ dig. Of the moon partial, July 12th. 9 aft. 5½ dig. Of the sun, December 22d. 1 aft. Asia, s.w. cent. * 5 tot.

1890. Of the moon partial, June 3d. 6 morn. 0¼ dig. Of the sun, June 17th. 10 morn. Eur. Afr. Asia, cent. 25 (38) 19 an. Of the moon partial, November 26th. 2 aft. 0¼ dig.

1891. Of the moon total, May 23d. 7 aft. Of the sun, June 6th. 4½ aft. great part of Eur. N. cent. †. Of the moon total, November 16th. 0¾ morn.

1892. Of the moon partial, May 11th. 11½ aft. 11¼ dig. Of the moon total, November 4th. 4½ aft 12½ dig.

1893. Of the sun, April 16th. 3 aft. Eur. s. Afr. cent. 20—18 tot.

1894. Of the moon partial, March 21st. 2½ aft. 3 dig. Of the sun, April 6th. 4½ morn. Eur. N.E. Asia, cent. 10 (43) 8. Of the moon partial, September 15th 4¾ morn. 2½ dig. Of the sun, September 29th. 5½ morn. Afr. E. small.

1895. Of the moon total, March 11. 4 morn. Of the sun, March 26th. 10 morn. almost all Eur. N.W. Asia, N. dim. from W. to E. Of the sun, August 20th. 0½ aft. Asia, N. small. Of the moon total, September 4th. 6 morn.

1896. Of the moon partial, February 28th. 8 aft. 10 dig.

Of the sun, August 9th. 4½ morn. Eur. E. Asia, cent. 60–68 (59) 49 tot. Of the moon partial, August 23d. 7 morn. 8 dig.

1897. No eclipse.

1898. Of the moon partial, January 8th. 0½ morn. 1½ dig. Of the sun, January 22d. 8 morn. Eur. E. Afr. E. all Asia, cent. 11–5 (10) 44 tot. Of the moon partial, July 3d. 9½ aft. 11 dig. Of the moon total, December 27th. 12 aft.

1899. Of the sun, January 11th. 11 aft. extrem. of Asia, E. dim. from N. to S. Of the sun, June 8th. 7 morn. Eur W. and N. Asia, N. Of the moon total, June 23d. 2¼ aft. Of the moon partial, December 17th. 1¼ morn. 11½ dig.

1900. Of the sun, May 28th. 3¾ aft. Eur. Afr. cent. 45–26 tot. Of the moon partial, June 13th. 4 morn. pen. +. Of the sun, November 22d. 8 morn. small eclipse, in Afr. cent. 3 s. E an.

PROBLEM XVIII.

To observe an Eclipse of the Moon.

To observe an eclipse of the moon, in such a manner as to be useful to geography and astronomy, it will be necessary, in the first place, to have a clock or watch that indicates seconds, and which you are certain is so well constructed as to go in a uniform manner. It ought to be regulated some days before by means of a meridian, if you have one traced out, or by some of the methods employed for that purpose by astronomers; and you must ascertain how much it goes fast or slow in 24 hours; that the difference may be taken into account at the time of the observation.

You ought to be provided also with a refracting or reflecting telescope, some feet in length; for the longer it is, the more certain you will be of discerning exactly the moment of the phases of the eclipse; and, if you are desir-

ous of observing the quantity of the eclipse, it should be furnished with a micrometer.

When you find the moment of the eclipse approaching, which may be always known either by a common Almanac, or the Ephemerides published by the astronomers in different parts of Europe, you must carefully remark the instant when the shadow of the earth touches the moon's disk. It is necessary here to mention, that there will always be some uncertainty on account of the penumbra; because it is not a thick black shadow which covers the moon's disk, but an imperfect one, that thickens by degrees. This arises from the sun's disk being gradually occulted from the moon; and hence it is difficult to fix with exactness the real limits of the shadow, and the penumbra. Here, as in many other cases, observers are enabled by habit to distinguish this boundary, or are at least prevented from falling into any great error.

When you are certain that the real shadow has touched the moon's disk, the time must be noted down; that is to say, the hour, minute, and second, at which it happened.

In this manner you must follow the shadow on the moon's disk, and remark at what hour, minute, and second the shadow reaches the most remarkable spots: all this likewise must be noted down.

If the eclipse is not total, the shadow, after having covered part of the lunar disk, will decrease. You must therefore observe in like manner the moment when the shadow leaves the different spots it before covered, and the time when the disk of the moon ceases to be touched by the shadow, which will be the end of the eclipse.

If the eclipse is total, so that the moon's disk remains some time in the shadow, you must note down the time when it is totally eclipsed, as well as that when it begins to be illuminated, and the moment when the shadow leaves the different spots.

When this is done, if the time of the commencement of

the eclipse be subtracted from that of the end, the remainder will be its duration; and if half the duration be added to the time of commencement, the result will be the middle.

To facilitate these operations, astronomers have given certain names to most of the spots with which the moon's disk is covered. The usual denominations are those of Langrenus, who distinguished the greater part of them by the names of astronomers and philosophers who were his contemporaries, or who had flourished before his time. Some others have been since added; but there was no room for the most celebrated of the moderns, such as Huygens, Descartes, Newton, and Cassini. Hevelius, far more judicious in our opinion, gave to these spots names taken from the most remarkable places of the earth: in this manner he calls the highest mountain of the moon, mount Sinai, &c. This however is a matter of indifference, provided there be no confusion. We have here subjoined a representation of the moon, pl. 4, by means of which and the following catalogue they can be easily known, on comparing the numbers in the latter with those in the former.

1 Grimaldi
2 Galileo
3 Aristarchus
4 Kepler
5 Gassendi
6 Schikard
7 Harpalus
8 Heraclides
9 Lansberg
10 Reinhold
11 Copernicus
12 Helicon
13 Capuanus
14 Bulliald
15 Eratosthenes
16 Timocharis
17 Plato
18 Archimedes
19 Isle of the middle Bay
20 Pittacus
21 Tycho
22 Eudoxus
23 Aristotle
24 Manilius
25 Menelaus
26 Hermes
27 Posidonius
28 Dionysius

29 Pliny
30 Catharina, Cyrillus, Theophilus
31 Fracastorius
32 The acute promontory
33 Messala
34 Promontory of dreams
35 Proclus
36 Cleomedes
37 Snell and Furner
38 Petau
39 Langren
40 Tarunt

A Sea of humours
B Sea of clouds
C Sea of rain
D Sea of nectar
E Sea of tranquillity
F Sea of serenity
G Sea of fecundity
H Sea of crises.

PROBLEM XIX.

To observe an Eclipse of the Sun.

1st. The same precautions, in regard to the measuring of time, must be employed in this case, as in that of lunar eclipses; that is to say, care must be taken to regulate a good clock by the sun on the day before, or even on the day of the eclipse.

2d. A good telescope must be provided, of at least three or four feet in length; which must be directed towards the sun on a convenient supporter. If you are then desirous to look at the sun without the telescope, you must employ a piece of smoked glass or rather two pieces, the smoked sides of which are turned towards each other; but are prevented from coming into contact by means of a small diaphragm cut from a card placed between them. These two bits of glass may be then cemented at the edges, so as to make them adhere. By means of these glasses interposed between the eye and the telescope, you may then view the sun without any danger to the sight.

About the time when the eclipse ought to commence, you must carefully observe the moment when the solar disk begins to be touched by the disk of the moon: this period will be the commencement of the eclipse. If there

are any spots on the solar disk, you must observe the time when the moon's disk reaches them, and also when it again permits them to appear; in the last place, you must observe, with all possible attention, the instant when the moon's disk ceases to touch the solar disk, which will be the end of the eclipse.

But if, instead of observing in this manner, you are desirous to make an observation susceptible of being seen by a great number of persons at the same time, affix to your telescope, on the side of the eye-glass, an apparatus to support a piece of very straight pa board at the distance of some feet. This paste-board ought to be perpendicular to the axis of the telescope, and, if it be not sufficiently white, you must paste to it a sheet of white paper. Make the end of the telescope, which contains the object glass, to pass through the window-shutter of a darkened room, or one rendered considerably obscure; and if the axis of the telescope be directed to the sun, the image of that luminary will be painted on the paper, and of a larger size according as the paper is at a greater distance. It is necessary here to remark, that before you begin to observe, a circle of a convenient size must be delineated on it, so that, by moving it nearer to or farther from the telescope, the image of the sun may be exactly comprehended within it. The space contained within this circle must be divided by twelve other concentric circles, equally distant from each other, so that the diameter of the largest may be divided into 24 equal parts, each of which will represent a semidigit.

It may now be readily conceived, that if a little before the commencement of the eclipse you look with attention at the image of the sun, you will see the moment when it begins to be obscured by the entrance of the moon's body; and that you may in like manner observe the end of it, and also its extent.

It must not however be expected that the same exact-

ness can be attained by employing this method, as by the former; especially if you are furnished with a long telescope, and a good micrometer.

REMARKS.—There are partial eclipses of the sun, that is, eclipses in which only a part of the solar disk seems to be covered, and these are most common. Others are total and annular.

Total eclipses take place when the centre of the moon passes over that of the sun, or nearly so; and when the apparent diameter of the moon is equal to that of the sun, or greater. In the latter case, the total eclipse may be what is called *cum mora;* that is to say, with duration of darkness: of this kind was the famous eclipse of 1706.

During eclipses which are total and *cum mora*, so great darkness prevails, that the stars are seen in the same manner as at night, and particularly Mercury and Venus. But what excites a sort of terror, is the dismal appearance which all nature assumes during the last moments of the light. Animals struck with fear, retire therefore to their habitations, sending forth loud cries; the nocturnal birds issue from their holes; the flowers contract their leaves; a coldness is felt, and the dew falls; but as soon as the moon has suffered a few rays of the solar light to escape, all is again illumination; day instantly returns, and with more brightness than when the weather is cloudy.

Some eclipses, as already said, are really annular: they take place when the eclipse is very near being central, while the apparent diameter of the moon is less than that of the sun; which may be the case if the moon at the time of the eclipse is at her greatest distance from the earth, and the sun at his nearest distance to it. The eclipse of the sun on the 1st of April 1764 was of this kind to a part of Europe.

During eclipses of this kind, when the sun is entirely eclipsed, a luminous circle of a silver colour, and as broad as the 12th part of the diameter of the sun or moon, is

often observed around the former; it is effaced as soon as the smallest part of the sun begins to shine: it appears more lively towards the sun's limb, and decreases in brilliancy the farther it is distant. Some are inclined to believe that this circle is formed by the luminous atmosphere with which the sun is surrounded, others have conjectured that it is produced by the refraction of his rays in the atmosphere of the moon, and some have ascribed it to the diffraction of the light. Those who are desirous of farther information on this subject, may consult the Memoirs of the Academy of Sciences, for the years 1715 and 1748.

PROBLEM XX.

To measure the Height of Mountains.

The height of a mountain may be measured by the common rules of geometry: for if we suppose CSD (plate 5 fig. 9) to be a mountain, the perpendicular height of which is required, the following method can be employed. If the nature of the adjacent ground will admit, measure a horizontal line AB, in the same vertical plane as the summit S of the mountain. The greater the extent of this line, the more correct will be the result. At the two stations A and B, measure the angles SAE and SBE, which are the apparent heights of the summit S, above the horizon, when seen from A and B. It will then be easy, by means of plane trigonometry, to find, in the right-angled triangle SEA, the side EA, as well as the perpendicular SE, or the elevation of the summit S above AE continued.

Now let us suppose the vertical line SFH to be drawn, intersecting BE in F. As, in dimensions of this kind, the angle ESF, formed by the vertical line and the perpendicular SE, will for the most part be exceedingly small, and much below one degree, the lines SE and SF may be considered as equal*. On the other hand, the line FH, com-

* For even in the case of this angle being a degree, they would not differ

prehended between the line AE and the spherical surface CA, is evidently the quantity by which the real level is lower than the apparent level, in an extent such as AF, or more correctly in a mean length between AF and BF: for this reason take the mean length between AE and BE, which differ very little from AF and BF; and in the table of differences between the apparent and real levels, find the height corresponding to that mean distance: if this height be then added to the height SE or SF, already found, you will have SH for the corrected height of the mountain, above the spherical surface, where the points A and B are situated.

If it be known how much this surface is higher than the level of the sea, it will be known also how much the summit S of the mountain is elevated above the same level.

Another Method.

As it may be difficult to establish a horizontal line, so as to be in the same vertical plane with the summit of the mountain, it will perhaps be better to proceed in the following manner:

Trace out your base in the most convenient manner, so as to be horizontal: we shall here suppose that it is represented by *a b* (pl. 5 fig. 10); let *s* . be the perpendicular from the summit *s* to the horizontal plane passing through *a b*; and let *c* be the point where this plane is met by the perpendicular: if the lines *a c* and *b c* be drawn to that point, we shall have the triangles *s a c* and *s b c*, right-angled at *c*, and the angles *s a c* and *s b c* may be found by measuring, from the points *a* and *b*, the apparent height of the mountain above the horizon: the angles *s a b* and *s b a*, in the triangle *a s b*, must also be measured.

Now, since in the triangle *s a b*, the angles *s a b* and *s b a*

a ten thousandth part, which would suppose the distance of the stations from the mountain to be more than 100000 yards

are known, and also the side *ab*; any one of the other sides, such for example as *sa*, may be easily determined by plane trigonometry. In the triangle *acs*, right-angled at *c*, as the angle *sac* is known, the side *ac* and the perpendicular *sc* may be found in the same manner. When this is done, the method pointed out in the preceding operation must be employed: that is, find the depression of the real level below the apparent level for the number of feet or yards comprehended in the line *ac*, and add it to the height *sc*: the sum will be the height of the point *s*, above the real level of the points *a* and *b*.

Example.—Let the horizontal length *ab* be 2000 yards, or 6000 feet; the angle *sab* 80° 30′; and the angle *sba* 85° 10′; consequently the angle *bsa* will be 14° 20′. By means of these data, the side *sa* of the triangle *asb* will be found to be 8050 yards. On the other hand, if we suppose the angle *sac* to have been measured, and to be 18°, the side *ac* will be found, by trigonometrical calculation, to be 7656 yards; and *sc*, perpendicular to the horizontal plane passing through *ab*, will be found equal to 2488. Now, as the depression of the real level below the apparent level at the distance of 7656 yards, is 12½ feet, or 4 yards 6 inches*, if this quantity be added to the height *sc*, we shall have 2492 yards 6 inches, for the real height of the mountain.

REMARK.—When either of these methods is employed, if the mountain to be measured is at a considerable distance, such as 20000 or 40000 yards, as its summit in that case will be very little elevated above the horizon, the apparent height must be corrected by making an allowance for refraction, otherwise there may be a very considerable error in the result. The necessity of this correction may be easily conceived by observing, that the summit C of the mountain BC (pl. 5 fig. 11), is seen by a ray of light ECA,

* See the table in the additional remark.

which is not rectilineal, but bent; so that the summit C is judged to be in D, according to the direction of the line AD, a tangent to the curve ACE, which in the small space AC may be considered as the arc of a circle. The angle DAB therefore, of the apparent height of the mountain, exceeds the height at which the summit would appear without refraction, by the quantity of the angle CAD; which must be determined. But it will be found that this angle CAD is nearly equal to half the refraction which would belong to the apparent height DAB. You must therefore find, in the tables, the refraction corresponding to the apparent height DAB of the mountain, and subtract the half of it from that height: the remainder will be that of the summit of the mountain, such as it would be seen without refraction.

Let us suppose, for example, that the summit of the mountain seen at the distance of 20000 yards appears to be elevated above the horizon 5 degrees; the refraction corresponding to 5 degrees is 9′ 54″, the half of which is 4′ 57″; if 4′ 57″ therefore be substracted from 5°, the remainder will be 4° 55′ 3″ which must be employed as the real elevation *.

It may thence be seen, that to proceed with certainty in such operations, it will be necessary to make choice of stations at a moderate distance from the mountain; so that its summit may appear to be elevated several degrees above the horizon; otherwise the difference of the refraction, which is very variable near the horizon, will occasion great uncertainty in the measurement.

We shall give hereafter another method for measuring the height of mountains, by means of the barometer; but

* Montucla here employs the common tables of refraction for nautical and astronomical purposes, such as that given in *Robertson's Navigation*, vol. I. p. 328. In regard to terrestrial refraction, and the allowance made for it, see the additional remark at the end of this article.

in this case it is supposed that it is possible to ascend to the summit of them. We shall also give a table of the heights of the principal mountains of the earth above the level of the sea. We shall here only observe that the highest mountains in the world, at least in that part of it which has hitherto been accessible to scientific men, are situated in the neighbourhood of the equator; and it is with justice that an historian of Peru says, that when compared with our Alps and our Pyrenees, they are like the towers and steeples of the churches in our cities, compared with common edifices. The high[illegible] yet known is Chimboraço in Peru, which rises more than 19000 feet in a perpendicular direction above the level of the sea.

As all the known mountains in Europe are scarcely two-thirds of the height of those enormous masses, the falsity of what the ancients, and some of the moderns, such as Kircher, have published respecting the height of mountains, will readily appear. According to these authors, Ætna is 4000 geometrical paces in height; the mountains of Norway 6000, Mount Hœmus and the Peak of Teneriff 10000; Mount Atlas and the Mountains of the Moon in Africa 15000; Mount Athos 20000; Mount Cassius 28000. It is asserted that these heights were found by means of their shadows; but nothing is more destitute of truth, and if ever any observer ascends to the summit of these mountains, or measures their height geometrically, they will be found very inferior to the mountains of Peru, as is the case with the Peak of Teneriff, which when measured geometrically by Father Feuillé was found not to exceed 6600 feet.

Hence it appears that the elevation of the highest mountains is very little, when compared with the diameter of the earth, and that its regular form is not sensibly altered by them; for the mean diameter of the earth is about 7951½ miles; therefore if we suppose the height of a

mountain to be 3½ miles, it will be only the 2273d part of the diameter of the earth, which is less than the elevation of half a line on a globe six feet in diameter.

ADDITIONAL REMARK.—As Montucla has not here explained the method of finding the difference between the apparent and true level, we think it necessary to add a few observations on the subject. Two or more places are said to be on a true level, when they are equally distant from the centre of the earth. One place also is higher than another, or out of level with it, when it is farther from the centre of the earth; and a line equally distant from that centre in all its parts, is called the *line of true level.* Hence, because the earth is round, that line must be a curve, or at least parallel or concentric to it. But the line of sight, given by operations of levelling, which is a tangent, or a right line perpendicular to the semi-diameter of the earth at the point of contact, always rising higher above the true curve line of level, the farther the distance, is called the *apparent line of level,* and the difference between the line of true level and the apparent, is always equal to the excess of the secant of the arch of distance above the radius of the earth. Hence it will be found that this difference is equal to the square of the distance between the places, divided by the diameter of the earth, and consequently it is always proportional to the square of the distance.

From these principles is obtained the following table, which shows the height of the apparent above the true level, for every 100 yards of distance on the one hand, and for every mile on the other

The common methods of levelling are sufficient for laying pavements of walks, or for conveying water to small distances, &c; but in more extensive operations, as in levelling the bottoms of long canals, which are to convey water to the distance of many miles, and such like, the difference between the true and apparent level must be taken into account.

Dist.	Diff. of Level
Yards.	Inches.
100	0·026
200	0·103
300	0·231
400	0·411
500	0·643
600	0·925
700	1·260
800	1·645
900	2·081
1000	2·570
1100	3·110
1200	3·701
1300	4·344
1400	5·038
1500	5·784
1600	6·580
1700	7·425

Dist.	Diff. of Level	
Miles.	Feet.	Inches.
¼	0	0½
½	0	2
¾	0	4½
1	0	8
2	2	8
3	6	0
4	10	7
5	16	7
6	23	11
7	32	6
8	42	6
9	53	9
10	66	4
11	80	3
12	95	7
13	112	2
14	130	1

By means of these tables of reductions, the difference between the true and apparent level can be found by one operation; whereas the ancients were obliged to employ a great many; for being unacquainted with the correction answering to any distance, they levelled only from one 20 yards to another, as they had occasion to continue the work to some considerable extent.

These tables will answer several useful purposes: First, to find the height of the apparent level above the true, at any distance. If the given distance be contained in the table, the correction of the level will be found in the same line with it. For example, at the distance of 1000 yards, the correction is 2·57, or nearly two inches and a half; and at the distance of ten miles, it is 66 feet 4 inches. But if the exact distance be not found in the table, multiply the square of the distance in yards by 2·57, and divide by 1000000, or cut off six places on the right for decimals, the rest will be inches; or multiply the square of the distance in miles by 66 feet 4 inches, and divide by 100.

2d. To find the extent of the visible horizon, or how far can be seen from any given height on a horizontal plane, as at sea, &c. Let us suppose the eye of an observer on the top of a ship's mast at sea, to be at the height of 130 feet above the water, it will then see about 14 miles all around; or from the top of a cliff by the sea side, the height of which is 66 feet, a person may see to the distance of nearly 10 miles on the surface of the sea. Also, when the top of a hill, or the light in a light-house, the height of which is 130 feet, first comes into the view of an eye on board a ship, the table shows that the distance of the ship from it is 14 miles, if the eye be at the surface of the water, but if the height of the eye in the ship be 80 feet, the distance will be increased by nearly 11 miles, making in all about 25 miles.

3d. Suppose a spring to be on the one side of a hill, and a house on an opposite hill, with a valley between them, and that the spring seen from the house appears, by a levelling instrument, to be on a level with the foundation of the house, which we shall suppose to be at the distance of a mile from it: this spring will be 8 inches above the true level of the house; and that difference would be barely sufficient for the water to be brought in pipes from the spring to the house, the pipes being laid all the way under ground.

4th. If the height or distance exceed the limits of this table: Then first, if the distance be given, divide it by 2, or by 3, or by 4, &c, till the quotient come within the distances in the table; then take out the height answering to the quotient, and multiply it by the square of the divisor, that is by 4, or by 9, or by 16, &c, which will give the height required. Thus, if the top of a hill be just seen at the distance of 40 miles; then 40 divided by 4, is 10, and opposite to 10 in the table will be found $66\frac{1}{3}$ feet, which multiplied by 16, the square of 4, gives $1061\frac{1}{3}$ feet for the height of the hill. But when the height is

given, divide it by one of these square numbers, 4, 9, 16, 25, &c, till the quotient come within the limits of the table, and multiply the quotient by the square root of the divisor, that is by 2, or 3, or 4, or 5, &c, for the distance sought. Thus, when the top of the peak of Teneriff, said to be about 3 miles or 15840 feet high, just comes into view at sea, divide 15840 by 225, or the square of 15, and the quotient is 70 nearly, to which in the table corresponds by proportion nearly $10\frac{2}{7}$ miles; which multiplied by 15, will give 154 miles and $\frac{2}{7}$, for the distance of the mountain.

In regard to the terrestrial refraction, which in measuring heights is to be taken into account also, as it makes objects to appear higher than they really are, it is estimated by Dr. Maskelyne at $\frac{1}{10}$ of the distance observed, expressed in degrees of a great circle. Thus, if the distance be 10000 fathoms, its 10th part 1000 fathoms is the 60th part of a degree on the earth, or 1′, which is therefore the refraction of the altitude of the object at that distance.

Le Gendre, however, says he is induced by several experiments to allow only $\frac{1}{14}$th part of the distance for refraction in altitude. So that on the distance of 10000 fathoms, the 14th part of which is 714 fathoms, he allows only 44″ of terrestrial refraction, so many being contained in the 714 fathoms.

Delambre, an ingenious French Astronomer, makes the quantity of terrestrial refraction to be the 11th part of the arch of distance. But the English measurers, Col. Ed. Williams, Capt. Mudge, and Mr. Dalby, from a multitude of exact observations made by them, determine the quantity of refraction to be the 12th part of the said distance. The quantity of this refraction however is found to vary, with the different states of the weather and atmosphere, from the 15th part of the distance to the 9th part; the medium of which is the 12th, as above mentioned.

PROBLEM XXI.

Method of knowing the Constellations.

To learn to know the heavens, you must first provide yourself with some good celestial charts, or a planisphere of such a size, that stars of the first and second magnitude can be easily distinguished. At the end of the present article we shall point out the best works on this subject.

Having placed before you one of these charts, that containing the north pole, turn your face towards the north, and first find out the Great Bear, commonly called Charles's wain (pl. 5 fig. 12). It may be easily known, as it forms one of the most remarkable groupes in the heavens, consisting of seven stars of the second magnitude, four of which are arranged in such a manner as to represent an irregular square, and the other three a prolongation in the form of a very obtuse scalene triangle. Besides, by examining the figure of these seven stars, as exhibited in the chart, you will easily distinguish those in the heavens which correspond to them. When you have made yourself acquainted with these seven principal stars, examine on the chart the configuration of the neighbouring ones, which belong to the Great Bear; and you will thence learn to distinguish the other less considerable stars which compose that constellation.

After knowing the Great Bear, you may easily proceed to the Lesser Bear; for nothing will be necessary but to draw, as seen in the annexed figure (pl. 5 fig. 13), a straight line through the two anterior stars of the square of the Great Bear, or the two farthest distant from the tail: this line will pass very near the polar star, a star of the second magnitude, and the only one of that size in a pretty large space. At a little distance from it, there are two other stars of the second and third magnitude, which, with four more of a less size, form a figure, somewhat similar to that of the Great Bear, but smaller. This is what is called the

Lesser Bear; and you may learn, in the same manner as before, to distinguish the stars which compose it.

Now, if a straight line be drawn through those stars of the Great Bear, nearest to the tail, and through the polar star, it will conduct you to a very remarkable group of five stars arranged nearly in this form M (pl. 5 fig. 14): these are the constellation of Cassiopeia, in which a very brilliant new star appeared in 1572; though soon after it became fainter, and at length disappeared.

If a line, perpendicular to the above line, be next drawn, through this constellation, it will conduct, on the one side, to a very beautiful star called Algenib, which is in the back of Perseus; and, on the other, to the constellation of the Swan (fig. 15), remarkable by a star of the first magnitude. Near Perseus is the brilliant star of the Goat, called Capella, which is of the first magnitude, and forms part of the constellation of Auriga.

After this, if a straight line be drawn through the two last stars of the tail of the Great Bear, you will come to the neighbourhood of Arcturus, one of the most brilliant stars in the heavens, which forms part of the constellation of Bootes (fig. 16).

In this manner you may successively employ the knowledge you have obtained of the stars of one constellation, to enable you to find out the neighbouring ones. We shall not enlarge farther on this method; for it may be easily conceived, that we cannot proceed in this manner through the whole heavens: but any person of ingenuity, in the course of a few nights, may learn by these means to know a great part of the heavens; or at any rate the principal stars.

The ancients were not acquainted with, or rather did not insert into their catalogues, more than 1022 fixed stars, which they divided into 48 constellations; but their number is much greater, even if we confine ourselves to those which can be distinguished by the naked eye. The abbé

de la Caille observed 1492 in the small space comprehended between the tropic of Capricorn and the south pole; a part of which he formed into new constellations. But this space is to the whole sphere, as 3 to 10 nearly; so that in our opinion the whole number of the stars visible to the naked eye may be estimated at about 6500. It is a mere illusion that makes us conclude, on the first view, that they are innumerable; for if you take a space comprehended between four, five or six stars of the second and third magnitude, and try to count those it contains, you will find that it can be done without much difficulty; and some idea may be thence formed of their total number, which will not much exceed that above stated.

The stars are divided into different classes, viz, stars of the first, second, third, &c, magnitude, as far as the 6th, which are the smallest perceptible to the naked eye. There are 20 of the first magnitude, 76 of the second, 223 of the third, 512 of the fourth, &c.

In regard to the constellations, the number of those commonly admitted is 90; of which 33 belong to the northern hemisphere, 12 to the Zodiac, and the remaining 45 to the austral or southern hemisphere. We shall here give a catalogue of them, containing the number of stars of which each is composed, together with the names of some of the most remarkable stars: the constellations which have this mark * against them, are modern ones, the others ancient. The figures placed against the principal stars denote their magnitudes.

I. CONSTELLATIONS NORTH OF THE ZODIAC.

No.	Constellations.	No. of Stars.	Chief Stars.	
1	Ursa Minor	24	Pole Star	2
2	Ursa Major	87	Dubhe	1
3	Perseus	59	Algenib	2
4	Auriga	50	Capella	1
5	*Bootes	[illegible]	Arcturus	1
6	Draco	[illegible]	Rastaber	3
7	*Cepheus	[illegible]5	Alderamin	3
8	*Canes Venatici scil. Asterion et Chara	[illegible]5		
9	*Cor Caroli	3		
10	*Triangulum	10		
11	Triangulum minus	5		
12	*Musca	6		
13	*Lynx	44		
14	*Leo Minor	24		
15	*Coma Berenices	40		
16	*Camelopardalus	58		
17	*Mons Menelaus	11		
18	Corona Borealis	21		
19	Serpens	50		
20	Scutum Sobieski	8		
21	Hercules cum Ramo et Cerbero	113	Ras Algiatha	3
22	*Serpentarius sive Ophiuchus	67	Ras Alhagus	3
23	*Taurus Poniatowski	7		
24	Lyra	22	Vega	1
25	*Vulpecula et Anser	37		
26	Sagitta	18		
27	Aquila	40	Altair	
28	Delphinus	18		
29	*Cygnus	73	Deneb Adige	1
30	*Equuleus	10		
31	*Lacerta	16		
32	*Pegasus	85	Markab	2
33	*Andromeda	66	Almaac	2

II. CONSTELLATIONS IN THE ZODIAC.

No.	Constellations.	No. of Stars.	Chief Stars.	
1	Aries	66		
2	Taurus	140	Aldebaran	1
3	Gemini	85	Castor and Pollux	1.2
4	Cancer	83		
5	Leo	95	Regulus	1
6	Virgo	110	Spica Virginis	1
7	Libra	51	Zubenich Mali	2
8	Scorpio	44	Antares	1
9	Sagittarius	69		
10	Capricornus	51		
11	Aquaries	108	Scheat	3
12	Pisces	112		

III. CONSTELLATIONS SOUTH OF THE ZODIAC.

No.	Constellations.	No. of Stars.	Chief Stars.	
1	*Phœnix	13		
2	*Officina Sculptoria	12		
3	Eridanus	70	Achernar	1
4	*Hydrus	10		
5	*Cetus	80	Menkar	2
6	*Fornax Chemica	14		
7	*Horologium	12		
8	*Reticulus Rhomboidalis	10		
9	*Xiphias	7		
10	*Celapraxitellis	16		
11	*Lepus	19		
12	*Columba Noachi	10		
13	Orion	78	Betelguese	1
14	Argo Navis	50	Canopus	1
15	Canis Major	30	Sirius	1
16	*Equuleus Pictorius	8		
17	*Monoceros	31		
18	Canis Minor	14	Procyon	1
19	*Chameleon	10		

No.	Constellations.	No. of Stars.	Chief Stars.	
20	*Pyxis Nautica	4		
21	*Piscis Volans	8		
22	Hydra	60	Cor Hydræ	1
23	*Sextans	4		
24	*Robur Carolinum	12		
25	*Machina Pneumatica	3		
26	*Crater	11	Alkes	3
27	*Corvus	9	Algorab	3
28	*Crosiers	[illegible]		
29	*Musca	[illegible]		
30	*Apis Indica	[illegible]		
31	*Circinus	4		
32	Centaurus	36		
33	*Lupus	24		
34	*Quadra Euclidis	12		
35	*Triangulum Australe	5		
36	Ara	9		
37	*Telescopium	9		
38	*Corona Australis	12		
39	*Pavo	11		
40	*Indus	12		
41	*Microscopium	10		
42	*Octans Hadleianus	43		
43	*Grus	14		
44	*Toucan	9		
45	Piscis Australis	20	Fomalhaut	1

IV. NUMBER OF STARS OF EACH MAGNITUDE.

Constellations.	Constellations.	Magnitudes.						Total Number of Stars.
		I	II	III	IV	V	VI	
In the Zodiac	12	5	16	44	120	183	646	1014
In the N. Hemisphere	33	6	24	95	200	291	635	1251
In the S. Hemisphere	45	9	36	84	190	221	323	865
	90	20	76	223	512	695	1604	3180

We shall not here enter into any physical details respecting the stars; as we reserve these for another place, where we shall speak of their distances, magnitudes, motion, and various other things relating to this subject; such as new stars, changeable or periodical stars, &c.

The best celestial charts were for a long time those of Bayer's Uranometria, a work in folio, published in 1603, and which has gone through a great many editions. But these charts have given place to the magnificent Celestial Atlas of Flamsteed, published in folio at London in 1729; a work indispensably necessary to every practical astronomer. Of the other charts or planispheres, those of Pardies, published in 1673, in six sheets magnificently engraved by Duchange, are esteemed. We have also the two planispheres of de la Hire, in two sheets. Senex, an English engraver, published likewise two new planispheres, according to the observations of Flamsteed; one of them in two sheets, where the two hemispheres are projected on the plane of the equator; and the other where they are projected on the plane of the ecliptic. Those who have not the Celestial Atlas of Flamsteed must provide themselves with either of these planispheres. The modern astronomers, and particularly la Caille, having added a great number of new constellations to the old ones in the southern hemisphere, two new planispheres have on that account been formed. One of them, by M. Robert, consists of two sheets, where the ground of the heavens is coloured blue; so that the constellations are very distinctly seen. It is constructed according to the newest observations; and it is accompanied with useful instructions respecting the method of knowing the heavens.

As it is of the greatest importance to astronomers, to be acquainted with the constellations and stars of the Zodiac, because the planets move in that circular band, Senex, before mentioned, published about half a century ago, The Starry Zodiac, from Flamsteed's Observations; and as it

was difficult to be procured at Paris, the Sieur Dheuland, engraver, gave, in 1755, a new edition of it; with such corrections as the interval between that period and the time when Senex published his edition, had rendered necessary. He was directed in this undertaking by M. de Seligny, a young officer in the service of the East-India Company. To the Zodiac of Dheuland is annexed a minute catalogue of the Zodiacal stars, with their longitudes and latitudes, reduced to the year 1755. This catalogue comprehends 924 stars; but the author, to render his work more useful in nautical observations, gives to his Zodiac ten degrees of latitude, on each side of the ecliptic. It may be readily seen, from what has been here said, that those who are not possessed of the Celestial Atlas of Flamsteed, must procure the Zodiac and Catalogue of Dheuland, or rather of Seligny, and that even possessing the former work does not supersede the necessity of the latter.

A new edition of Flamsteed's Atlas, reduced to a third of its original size, has since been published, with a planisphere of the austral stars observed by la Caille. M. Fortin, the author, reduced all the stars to the year 1780; and added a chart of the stars representing the different figures which they form, together with their relative positions.

To the above list we may add the large Celestial Atlas lately published by professor Bode, of Berlin, consisting of twenty sheets.

Remark.—Since the period when mankind began to observe the stars, various astronomers, at different times, have undertaken to exhibit in charts, their places, relative distances, and magnitudes. To the works of this kind before mentioned, we may add also the *Cœlum Stellatum* of Julius Schiller, 1627; the *Firmamentum Sobescianum* of Hevelius, 1690, in 54 sheets; and Doppelmayer's Celestial Atlas, Nuremberg 1742. In the year 1729 Flamsteed's Celestial Atlas was published in 28 sheets, containing 2919 stars, observed by that astronomer at Greenwich,

and divided into 56 constellations. In the year 1776, an edition of it, reduced to the quarto form, was published at Paris by Fortin, in 30 sheets; in the year 1796 la Lande and Mechain published the same plates, considerably improved, and enlarged with seven new constellations. In the year 1782 M. Bode published the same Atlas in 34 sheets, small folio; but he added, besides the old observations, a great many new ones, and above 2100 fixed stars and nebulæ. In the year 1748, a new *Uranographia*, of the same kind as that of Bayer, to consist of 50 sheets, was announced to be published by subscription in England. Dr. Bevis, a noted astronomer, was at the head of this undertaking, and some of the sheets were engraved, but the work was never completed*. The Atlas now published by professor Bode, in 20 sheets, is constructed according to an entirely new projection. Flamsteed's charts were each 21 inches in breadth and 28 inches in length; those of Bode's Atlas are 26 inches in breadth and 38 in length. Flamsteed's Atlas contains only 56 constellations on 28 sheets; that of Bode contains 106 on 18 sheets, together with the stars around the south pole, and two hemispheres. Of late years, by the continued assiduity of astronomers, the number of stars observed has been much increased. Dr. Herschel, with his excellent telescopes, has discovered above 2500 nebulæ, groups of stars, and double stars. Baron von Zach of Gotha constructed a new and complete catalogue of the fixed stars, from his own obervations; but professor Bode for the greatest number of his improvements was indebted to la Lande. This meritorious astronomer supplied him at different times with new stars, amounting altogether to about 6000, which were observed

* Another little known Celestial Atlas, which at least is mentioned by Lalande, is that of Corbinianus Thomas, a Benedictine and professor of mathematics at Ausburg. It is entitled *Firmamentum Firmianum*, in honour of the then bishop of the house of Firmian, and was published at Augsburg in small folio, in the year 1731. In this Atlas the northern crown is called *Corona Firmiana*.

by himself and his nephew le François, at the Military school, with a mural quadrant by Bird. But the first manuscripts transmitted by la Lande, contained the right ascensions only to minutes of time; and consequently were not accurately enough defined for the large scale on which these charts are constructed. Professor Bode therefore inserted only some of these stars into his charts, being obliged to leave out the greater part of them. La Lande sent afterwards more correct positions; and though the professor encountered many difficulties in reducing them, in consequence of errors in the transcribing or calculation, he was enabled to add to his charts some thousands of new stars, furnished by the above astronomer. The professor however found several vacuities, and being desirous that the improvement introduced into his work should be uniform, he resolved to supply these deficiencies from his own observations. He began therefore in the month of December 1796, at the royal observatory of Berlin, to search for and observe new stars, with a mural quadrant by Bird; and by these means was enabled to enrich his Atlas with some hundreds of stars, of the 6th and 7th magnitudes, not to be found in any of the catalogues.

Plate 1 and 2 represents the hemispheres of Aries and Libra according to the stereographic projection, the first has 0° ♈, and the second 0° ♎, in the centre; the poles are at the top and bottom, and the solstitial colure in the circumference. Plate 3 to 10 all the principal stars of the polar regions, and all the old and new constellations north of the Zodiac. Plate 11 to 16 the twelve constellations of the Zodiac, and some neighbouring stars. Plate 17 to 20 all the stars below the Zodiac and in the south polar regions. These charts altogether contain upwards of 17000 stars, nebulæ, groups, and double stars. Many of the sheets contain 13 or 1400 stars, nebulæ, &c; whereas those of Flamsteed do not contain above 300.

Flamsteed, for his charts, made choice of a kind of pro-

jection by which, especially under great declinations, no proper idea is given of the real figure of the circles of the sphere. In these charts the parallels to the equator are straight lines, which intersect the meridians, where the cosines of their distance from the mean meridian falls. They appear therefore as crooked lines; the meridians or great circles appear also crooked, and the parallels or less circles straight lines, entirely contrary to the real form which these circles of the sphere exhibit. Professor Bode therefore made choice of another kind of projection, namely that conical projection described by Kastner in his Geometrical Treatises, and in which the semi-diameter of the mean parallel is the cotangent of its declination. The mean meridian, on the other hand, is lengthened where these cotangents fall; and from this point as a centre are drawn the parallel circles at every 5 degrees. At this centre the value of the angle of right ascension, for example 10 degrees, is made = sin. decl. 10°; and the meridians are drawn as straight lines. By this construction the degrees of ascension are kept in the proper proportion to those of declination, in the mean zones lying between the parallels, as far as they extend east or west; and the principal stars which each sheet exhibits, fall in these mean zones. Each sheet generally contains about 75°, on the equator, of right ascension, and 54° in declination. When the equator falls in the middle of the chart, the parallels and meridians are straight lines, placed at equal distances, and intersecting each other at right angles. The polar regions are delineated according to the stereographic projection. The scale of these charts, the two polar ones excepted, is 10° declination to 4 inches English.

The names of all the constellations are given in Latin, according to the general practice; the original constellations, when they form the principal figures in the chart, are completely shaded; but in such a manner that the smallest stars and the nebulous spots are apparent. The

names are given in large Roman shaded characters. The constellations introduced in modern times are shaded in the punctured manner; and the names are added in large open Roman characters. Besides the Arabic and Latin names, already known, the old Arabian names are also added to many of the stars. The epoch of the right ascension of these stars is fixed at the 1st of January, 1801.

CHAPTER II.

A short View of the principal Facts in regard to Physical Astronomy, or the System of the Universe.

THERE is no difference of opinion at present among enlightened philosophers, in regard to the position of the planets and of the sun. All those capable of estimating the proofs deduced from astronomy and physics, admit that the sun occupies the centre of an immense space, in which the following planets revolve around him at different distances, viz, Mercury, and Venus; the earth, always accompanied by the moon; Mars, Pallas, discovered by Dr. Olbers; Ceres, discovered by M. Piazzi; Vesta, discovered by Harding; Juno, discovered also by Olbers; Jupiter, followed by his four moons or satellites; Saturn, surrounded by his ring, and accompanied by seven satellites; the Georgian planet, discovered by Dr. Herschel, together with its satellites; and lastly a great number of comets, which have been shewn to be nothing else but planets having orbits very much elongated.

The path in which each of the planets moves around the sun is not a circle, but an ellipsis more or less elongated; in one of the foci of which that luminary is placed; so that when the planet is at the extremity of the axis, beyond the centre, it is at its greatest distance from the sun; and when at the other extremity of that axis, it is at its nearest

distance. This ellipsis however is not very much elongated: that described by Mercury is the most of all of the ancient planets; for the distance of its focus from the centre is equal to a fifth part of its semiaxis. That of Venus is nearly a circle. In the orbit of the earth, the distance from the focus to the centre is only about a 57th part of the semiaxis. The last discovered planet, Pallas, it is said, has its orbit the most elongated of any, its eccentricity being about one third of its mean distance from the sun.

The motion of all these bodies around the sun is regulated by two celebrated laws, the discovery of which has rendered the name of Kepler immortal. The first of these laws, which relates to the motion of a planet in the different points of its orbit, is, that it always moves in such a manner, that the arc described by the radius vector, or the straight line drawn from the planet to the sun, increases uniformly in equal times, or is always proportional to the time; so that if a planet, for example, employs 30 days in moving from A to π (pl. 5, fig. 17), and 20 in moving from π to p, the mixtilineal area A S π, will be to the mixtilineal area π S p, as 30 to 20, or A S π is to A S p, as 30 to 50, or as 3 to 5. In double the time therefore this area is double, and so on; whence it follows, that when the planet is at its greatest distance, it moves with the least velocity in its orbit. The ancients laboured under a mistake, when they imagined that the retardation which they observed in the motion of any of the heavenly bodies, such as the sun for example, was a mere optical illusion: this retardation is partly real, and partly apparent.

The second law, discovered by Kepler, is that which regulates the distances of the planets from the sun, and their periodical times, or the times of their revolutions. According to this law, the cubes of the mean distances of two planets from the sun, around which they perform their revolutions, are always in proportion to each other as the

squares of their periodical times; thus, if the mean distances of two planets from the sun, be the one double of the other, since the cubes of these distances will be as 1 to 8, the squares of the periodical times will be as 1 to 8; consequently the times themselves will be to each other as 1 to the square root of 8, which is $2\frac{5}{6}$ nearly.

This rule holds good, not only in regard to the principal planets, those which revolve about the sun, but also in regard to the secondary planets, which revolve around a primary planet, as the four satellites of Jupiter, and the seven satellites of Saturn. If the earth had two moons, they also would observe this law in regard to each other by a mechanical necessity.

These two laws, first discovered by Kepler, from his observations and those of Tycho Brahe, were afterwards confirmed and proved by Newton, from the principles and laws of motion; so that those who deny truths so well established, must be incapable of feeling the force of a demonstration.

We shall now lay before our reader every thing most remarkable in regard to those celestial bodies of which we have any knowledge, beginning with the sun. They who can behold this sublime picture without emotion, ought to be classed among those stupid beings, whose minds are insensible to the most magnificent works of the Deity.

§ I. *Of the Sun.*

The sun, as we have already said, is placed in the middle of our system, as a source of light and heat, to illuminate and vivify all the planets subordinate to it. Without his benign influence, the earth would be a mere block, which in hardness would surpass marble and the most compact substances with which we are acquainted; no vegetation, no motion would be possible: in short, it would be the abode of darkness, inactivity and death. The first rank therefore among inanimate beings cannot be refused to

the sun; and if the error of addressing to a created object that adoration which is due to the Creator alone, could admit of excuse, we might be tempted to excuse the homage paid to the sun by the ancient Persians, as is still the case among the Guebres, their successors, and some savage tribes in America.

The sun is, or seems to be, a globe of fire, the diameter of which is equal almost to 111 times that of the earth, being about 883217 English miles; its surface therefore is 12321 times greater than that of the earth; and its mass 1367631 times. Its distance from the earth, according to the latest observations, is about 95 millions of miles.

This enormous mass is not absolutely at rest: for modern astronomers have found that it revolves round its axis, in 25 days 12 hours. This motion takes place, on an axis inclined to the plane of the ecliptic about $7\frac{1}{2}°$; so that the equator of the sun has the same inclination to the earth's orbit.—This phenomenon was discovered by means of the spots, with which the surface of the sun is covered at certain periods: with the assistance of a telescope, these spots, which are dark, and generally of a very irregular form, and which often remain some months, may be observed on the disk of this luminary. They were first discovered by Galileo, who thus gave a mortal blow to the opinion of the philosophers of that time, some of whom, treading in the steps of Aristotle, considered the celestial bodies as unalterable. He repeatedly observed, at different periods, large spots on the sun's disk; saw them always approach in the same direction, and almost in a straight line to one of the edges; then disappear and re appear afterwards, at the other edge; whence he concluded that the sun had a rotary motion about his axis. It is remarked that these spots employ 25 days 12 hours to return to the same point of the disk where they began to be observed; hence it follows that they require 25 days 12 hours, to

perform a complete revolution*; and consequently the sun employs that time in revolving about his axis.

It thence follows also, that a point in the sun's equator moves about four times and a third as fast, as a point of the terrestrial equator, during its diurnal motion; for, the circumference of a solar great circle being 111 times as great, these points would move with the same velocity if the period of the sun's revolution were 111 days: But being only 25 days and some hours, it is about four times and a third as rapid.

Astronomers have also had the curiosity to measure the extent of some of these solar spots; and have found that they are sometimes much larger than the whole earth.

In regard to the nature of these spots: some philosophers have conjectured, that they can be nothing else than parts of the nucleus of the sun which remain uncovered, in consequence of the irregular movements of a fluid violently agitated. An English Astronomer, Professor Wilson of Glasgow, revived this idea in the Philosophical Transactions for 1773, with this difference, that according to his theory the luminous matter of the sun is not fluid, but of such a consistence, that under particular circumstances, there may be sometimes formed in it considerable excavations, which discover a portion of the nucleus. The sloping sides of these excavations, according to his opinion, form the faculæ, or that border less luminous, without being black, with which these spots are generally surrounded. This theory he endeavours to establish, by examining the phenomena that ought to be exhibited by such excavations, according to the manner in which they might present themselves to an observer.

* The reason of this difference is, that while the sun performs a complete revolution on its axis, the earth, moving in its orbit, advances about 25 degrees towards the same side; on which account the spot must still pass over about 25 degrees, before it can be in the same point of view in regard to the earth.

Other philosophers have supposed these spots to be only clouds of fuliginous vapours, which remain suspended over the surface of the sun, in the same manner as the smoke that rises from Vesuvius at the time of an eruption; and which to an eye placed in the atmosphere would appear to cover a large tract of country. Some also have imagined them to consist of a kind of scum produced by the combustion of heterogeneous matters, which have fallen on the sun's surface. But, in all probability, nothing certain will ever be known on the subject. For whole years none of these spots are ever seen on the sun's disk, and sometimes a great many are observed. In 1637 it is said they were so numerous, that both the heat and splendour of that luminary were in some measure diminished by them. If the opinion of Descartes, respecting the incrustation of the stars, and their conversion into opake planets, had been then known, some apprehensions might have been entertained of seeing the sun, to the great misfortune of the human species, undergo this strange metamorphosis.

We shall here remark that a certain figure of the sun, given on the authority of Kircher, and copied in various maps of the world, ought to be considered merely as an imaginary production. No observations have ever been made by any astronomer, that can serve as the least foundation for it.

In 1683, Cassini discovered that the sun not only has a proper light of his own, but that he is accompanied by a kind of luminous atmosphere, which extends to an immense distance, since it sometimes reaches the earth. But this atmosphere is not of a form nearly spherical, like that of the earth: it is lenticular, and situated in such a manner, that its greatest breadth coincides almost with the prolongation of the solar equator. We indeed often see, during very serene weather, and a little after sunset, a light somewhat inclined to the ecliptic, several degrees

broad at the horizon, and decreasing to a point, which rises to the height of 45°. It is principally towards the equinoxes that this phenomenon is observed; and as it has been since seen, and in various places, by a great number of astronomers, these appearances cannot perhaps be accounted for, but by supposing around the sun an atmosphere such as that above mentioned.

Doctor Herschel has two ingenious papers in the Philosophical Transactions, for 1795 and 1802, containing many new and curious speculations on the nature and constitution of the sun, his light, &c. Dissatisfied with the old terms, used to denote certain appearances on the surface of the sun, Dr. Herschel rejects them; and instead of the words, spots, nuclei, penumbræ, luculi, &c. he substitutes, openings, shallows, ridges, nodules, corrugations, indentations, pores, &c. He imagines that the body of the sun is an opake habitable planet, surrounded and shining by a luminous atmosphere, which being at times intercepted and broken, gives us a view of the sun's body itself, which are the spots, &c. He conceives that the sun has a very extensive atmosphere, consisting of elastic fluids, that are more or less lucid and transparent, and of which the lucid ones furnish us with light. "This atmosphere, he thinks, is not less than 1843, nor more than 2765 miles in height: and he supposes that the density of the luminous solar clouds need not be much more than that of an aurora borealis, in order to produce the effects with which we are acquainted. The sun then, if this hypothesis be admitted, is similar to the other globes of the solar system, with regard to its solidity—its atmosphere—its surface diversified with mountains and valleys—the rotation on its axis—and the fall of heavy bodies on its surface; it therefore appears to be a very eminent, large, and lucid planet, the principal one in our system, disseminating its light and heat to all the bodies with which it is connected."

§ II.

Of Mercury.

Mercury is the smallest of all the ancient planets, and the nearest the sun: its distance from that luminary is about $\frac{37}{95}$ of that of the earth: Mercury therefore revolves about the sun at the distance of about 37 millions of miles. On account of this position, it is never more than 28° 20′ from the sun, and on this account it is very difficult to be seen. When at about its greatest elongation from the sun it appears as a crescent like the moon towards her quadratures; but to observe this configuration requires good telescopes.

It has not yet been ascertained from any observations whether Mercury has a motion round its axis, which however is very probably the case.

This planet completes its revolution round the sun in 87 days 23 hours 15 minutes, and its diameter is to that of the earth as 2 to 5; so that its bulk is to that of the earth as 8 to 125.

The distance of Mercury from the sun being no more than $\frac{37}{95}$ of that of the earth; and as heat increases in the inverse ratio of the squares of the distance; it thence follows that, *cæteris paribus*, it is nearly seven times as hot in that planet as on our earth. This heat even far exceeds that of boiling water. If Mercury therefore has the same conformation as our earth, and is inhabited, the beings by which it is peopled must be of a nature very different from those of the latter. In this there is nothing repugnant to reason; for who will dare to confine the power of the Deity to beings almost similar to those with which we are acquainted on the earth? We shall show hereafter that the conformation of the surface of Mercury, and the nature of the circumambient fluid, may be such as to make it not impossible for such beings as ourselves to exist in it.

§ III.

Of Venus.

Venus is the most brilliant of all the planets in the Heavens. This planet, as is well known, sometimes precedes the sun; and on that account is called *Lucifer*, or the morning star: sometimes it follows him, appearing the first after he is set; and on that account is distinguished also by the name of *Vesper*, or the evening star.

This planet revolves about the sun at a distance from him, which is to that of the earth from the sun, as 68 to 95; consequently its distance from the sun is about 68 millions of miles: its greatest elongation from the sun, in regard to us, is about 48°, and it exhibits the same phases as the moon.

The revolution of Venus around the sun is performed in 224 days 16 hours 49 minutes: its diameter, according to the latest and most correct observations, is nearly the same as that of the earth, and consequently it is of equal bulk also. Changeable spots have been discovered on the surface of Venus, which serve to prove the revolution of that planet about its axis; but the period of this revolution is not very fully ascertained. M. Bianchini makes it to be 24 days, and M. Cassini 23 hours, 20 minutes. For our part we are inclined to adopt the latter opinion; but unfortunately these spots, seen by Maraldi and Cassini, are no longer visible, even with the help of the best telescopes, at least in Europe: at present not a single spot can be observed in this planet; and therefore the question must remain undetermined till new ones are seen.

Venus may sometimes pass between the earth and the sun, in such a manner as to be seen on the disk of the latter, where it appears as a black spot, of about a minute apparent diameter. It was seen for the first time passing over the sun's disk in Nov. 1631; it was again observed under the like circumstances on the 6th of June, 1761,

and the same observation was made on the 3d of June, 1769. It will not be again seen passing over the sun's disk, till the 9th of December, 1874. The observation of this phenomenon, in the success of which all the states of Europe interested themselves, is attended with considerable advantages to astronomy, an account of which may be found in books that treat expressly on that subject.

§ IV.

Of the Earth.

The Earth, which we inhabit, is the third in the order of the planets hitherto known. Its orbit, the semi-diameter of which is about 95 millions of miles, comprehends within it those of Venus and Mercury. It performs its revolution about the sun in 365 days 6 hours 11 minutes; for it is necessary that a distinction should be made between the real or complete revolution of the earth, and the tropical revolution, or what is called the solar year. The latter consists of 365 days 5 hours 49 minutes; because it represents only the time which the sun employs in returning to the same point of the equinoctial; but as the equinoctial points go back every year 50″, which makes the stars seem to advance the same quantity, in the same period, when the earth has returned to the point of the vernal equinox, it must still pass over 50″ before it can attain to the point of the fixed sphere, where the equinox was the preceding year. But as it employs for this purpose about 20 minutes, these added to the tropical year will give, as the time of the complete revolution, from a point of the fixed sphere to the same point again, 365 days 6 hours 11 minutes, as mentioned above.

During a revolution of this kind, the earth, in consequence of the laws of motion, always maintains its axis

parallel to itself; and it performs its revolution around this axis, with respect to the fixed stars, in 23 hours 56 minutes; for it is in regard to the fixed stars that this revolution ought to be measured, and not in regard to the sun, which has apparently advanced in the same direction about a degree per day. This parallelism of the earth's axis produces the variation of the seasons; as it exposes sometimes the northern and sometimes the southern part to the direct influence of the sun's rays.

This parallelism however is not absolutely invariable. In consequence of certain physical causes, it has a small motion, by which it deviates from it, at each revolution, about 50 seconds; as if it had a conical motion, exceedingly slow, around the moveable and supposed axis of the ecliptic. On account of this motion, the apparent pole of the world, among the fixed stars, is not fixed; but revolves about the pole of the ecliptic, and approaches certain stars, while it recedes from others. The polar star has not always been that nearest the arctic pole; nor is it yet at its greatest degree of proximity: it will attain to this situation about the year 2100 of our æra, and its distance from the pole at that period will be 28′ or 29′; the arctic pole will then recede more and more from it, so that in the course of ages there will be another polar star, and even others after that in succession.

The axis of the earth is inclined to the plane of the ecliptic, at present, in an angle of 23° 28′, and some seconds, which causes the inclination of the ecliptic to the equator, and produces the different changes of the seasons. This inclination is also variable, and, according to modern observations, decreases about a minute every century: the ecliptic therefore slowly approaches towards the equator, or rather the equator towards the ecliptic, and if this motion takes place with the same velocity, and in the same direction, the equator will coincide with the ecliptic

in about 140,000 years; and then a perpetual spring, as well as an equality of the days and nights, will prevail all over the earth.

§ V.

Of the Moon.

Of all the celestial bodies which surround us, and by which we are illuminated, the most interesting, next to the sun, is the moon. Being the faithful companion of our globe during its immense revolution, she often supplies the place of the sun, and by her faint light consoles us for the loss we sustain when the rays of that luminary are withdrawn. It is the moon which raising, twice every day, the waters of the ocean, produces in them that reciprocal motion, known under the name of the flux and reflux; a motion which is perhaps necessary in the economy of the globe.

The mean distance of the moon from the earth is about 60½ semi-diameters of the latter, or 240,000 miles. Her diameter is in proportion to that of the earth, as 20 to 73, or nearly as 3 to 11; so that her mass, or rather bulk, is to that of the earth, nearly as 1 to $48\frac{4}{7}$.

The moon is an opake body; but we do not think it necessary to adduce here any proof of this assertion. She is not a polished body, like a mirror; for if that were the case, it would scarcely transmit to us any light, as a convex mirror disperses the rays in such a manner that an eye, at any considerable distance, sees only one point on the surface illuminated; whereas the moon transmits to us from her whole disk a light sensibly uniform.

To this we may add, that observation shows in the body of the moon asperities still greater, considering her magnitude, than those with which the earth is covered. If the moon indeed be attentively viewed, some days after her conjunction, the boundary of the shaded part will be seen as it were indented; which can arise only from the

effect of its inequalities. Besides, at a little distance from that boundary, in the part not yet illuminated, there are observed luminous points, which, increasing gradually as the luminous part approaches them, are at length confounded with it, and form the indentations above mentioned: in short, the shadow of those parts, when they are entirely illuminated, are seen to project themselves to a greater or less distance, and to change their position, according as they are illuminated on the one side or the other, and in a direction more or less oblique. It is in this manner that the summits of the mountains on our earth are illuminated, while the neighbouring vallies and plains are still in obscurity; and that their shadows are projected to a greater or less distance, on the right or the left, according to the elevation and position of the sun. Galileo, the author of this discovery, measured the height of one of these lunar mountains geometrically; and found it to be about 3 leagues, which is nearly double the height of the most elevated peaks of the Cordilleras, the highest mountains known on the earth. But later astronomers, by more accurate measurements, have not found the lunar mountains to rise above a mile or two in height.

We have already spoken of the names given by astronomers to these spots, and of their use in astronomy. We shall therefore not repeat them here, but proceed to something more interesting. On the surface of the moon there are spots of different kinds, some luminous, and others in some measure obscure. It was long considered as fully established that the most luminous parts were land, and the obscure parts sea; for it was said as water absorbs a part of the light, it must transmit a weaker splendour than the land, which reflects it very strongly. But this reasoning is not well founded; for if these spots, which are obscure in regard to the rest of the moon, consisted of water; when illuminated obliquely, as they are in respect to us during the first days after the conjunction,

they ought to transmit to us a very lively light; as a mirror which seems black to those not placed in the point to which it reflects the solar rays, appears on the other hand exceedingly bright to an eye situated in that point.

Others have hence been induced to believe that these obscure parts are immense forests, and this indeed may be more probable. We have no doubt that if the vast forests still in Europe, and those of America, were seen at a great distance, they would appear darker than the rest of the earth's surface.

But is this observation sufficient to make us conclude that these spots are really forests? We do not think it is; and the reasons are as follow:

It is in a manner proved that the moon has no atmosphere; for if she had, it would produce the same effects as ours. A star, on the moon approaching it, would change its colour: and its rays, broken by that atmosphere, would give it a very irregular motion, even at a considerable distance from the moon. But nothing of this kind is observed. A star covered by the dark edge of the moon suddenly disappears, without changing its colour, or experiencing any sensible refraction. Some astronomers indeed have imagined that they saw lightning in the moon during total eclipses of the sun; but this no doubt was an illusion, owing to their eyes being fatigued by looking too attentively at the sun. Besides, if there were clouds and vapours in the moon, they would sometimes be seen to conceal certain known parts of her surface; as an observer placed in the moon would certainly see certain pretty large portions of the earth, such as whole provinces, concealed sometimes for days, and even weeks, by those clouds, which frequently cover them, during as long a period. M. de la Hire has shown that an extent as large as Paris would be perceptible to an observer in the moon, if viewed through a telescope of 25 feet, or which magnified objects about 100 times.

But if there be no dense atmosphere, no elevation of vapours on the surface of the moon, it is difficult to conceive how there can be any kind of vegetation in it; and if this be the case, it can produce neither plants, trees, nor forests, and consequently no animals. It is therefore probable that the moon is not inhabited; besides, if it were inhabited by animals nearly similar to man, or endowed with some kind of reason, it is hardly to be supposed that they would not make some changes on the surface of that globe. But since the invention of the telescope, to the present time, no alteration has been observed in its surface.

The moon always presents to the earth very nearly the same face; and therefore she must have a rotary motion about an axis, nearly perpendicular to the ecliptic, the duration of which forms the lunar month; or in one of its hemispheres there must be some cause, which makes it incline towards the earth. The latter conjecture is the more probable; for why should this revolution of the moon around its axis be performed exactly in the period of its rotation about the earth? However, as the moon always presents the same face to the earth, it thence follows, that her whole surface is illuminated by the sun, in the course of a lunar month; the days therefore in the moon are equal to about 15 of ours, and the nights of the same duration.

But if we suppose, notwithstanding what has been said, that there are inhabitants in the moon, they will enjoy a very singular spectacle: an observer placed towards the middle of the lunar disk, for example, will always see the earth motionless towards his zenith, or having only a motion of nutation, in consequence of reasons which we shall explain hereafter. In short, each inhabitant of that hemisphere will always see the earth in the same point of his horizon; while the sun will appear to perform his revolution in a month. On the contrary, the inhabitants of

the other hemisphere will never see the earth ; and if there are astronomers in it, some of them no doubt will undertake a voyage to the hemisphere which is turned towards us, for the purpose of observing this sort of motionless moon, suspended in the Heavens like a lamp, and the more remarkable as it must appear to the lunar inhabitants of a diameter four times as large as that of the moon appears to us; with a great variety of spots performing their revolutions in the interval of 24 hours: for there can be no doubt that our earth, intersected by vast seas, large continents, and immense forests, such as those of America, must exhibit to the moon a disk variegated with a great many spots, more or less luminous.

We have said that the moon always presents the same disk to the earth; but strictly speaking this is not exactly the case; for it has been found that the moon has a certain motion, called libration, in consequence of which the parts nearest the edge alternately approach to or recede from that edge, by a kind of vibration. Two kinds of libration are in particular distinguished; one called a libration in latitude, by which the parts near the austral or the boreal poles of the moon, seem to vibrate from north to south, and from south to north, through an arc which may comprehend about 5 degrees. This, however, is a mere optical effect, produced by the parallelism of the moon's axis of rotation, which is inclined 2½ degrees to the ecliptic.

The other libration is that in longitude; which takes place around the above axis, at an angle of nearly 7½ degrees; and as both are combined, it needs excite no wonder that this phenomenon should have long been an object of research to philosophers, though without success. The causes of the latter are not yet so fully established, as to be beyond doubt. However, it is evident that the inhabitants of the moon, if there really be any, who are situated near the edge of the disk turned towards

the earth, must see our globe alternately rise and set, describing an arc of only a few degrees.

§ VI.

Of Mars.

Mars, which may be easily distinguished by its reddish splendour, is the fourth in the order of the primary planets. Its orbit incloses that of Mercury, Venus, and the earth; consequently the motions of these planets must exhibit to the inhabitants of Mars the same phenomena, as are presented by Mercury and Venus to the inhabitants of our globe.

The revolution of Mars around the sun is performed in 686 days 23 hours 30 minutes, or nearly two years. Its mean distance from the sun is more than $1\frac{1}{2}$ that of the earth, or about 144 millions of miles.

Spots are observed sometimes on the disk of Mars, by which it is proved that it revolves on an axis almost perpendicular to its orbit, and that this revolution is completed in 24 hours 39 minutes. The days therefore, to the inhabitants of Mars, if there are any, must be nearly equal to ours; and the days and nights in this planet must be of the same length, since its equator coincides with its orbit. As to the size of Mars, it is almost equal to that of our earth.

§ VII.

Of Jupiter.

The next planet to Mars, of the ancient ones, is Jupiter. Its distance from the sun is above 5 times that of the earth, being 490 millions of miles. The period of its revolution around the sun is 11 years 317 days 12 hours 20 minutes. Its diameter, compared with that of the earth, is as 11 to 1; so that its bulk is 1331 times as great as that of our globe.

This bulk does not prevent Jupiter from revolving

around his axis with much more rapidity than our earth. The spots observed on the disk of this planet have indeed shown that this revolution is performed in 9h 56m; so that it is more than twice as quick, and as any point in the equator of Jupiter is eleven times as far distant from the axis as a point of the earth's equator is from the terrestrial axis, it thence follows that this point in Jupiter moves with a velocity about twenty-four times as great.

It has therefore been observed that the body of Jupiter is not perfectly spherical: it is an oblate spheroid, flattened at the poles, and the diameter of its equator, is to that passing from the one pole to the other, according to the latest observations made with the most perfect instruments, as 14 to 13.

The axis of Jupiter is almost perpendicular to the plane of its orbit, for its inclination is only 3 degrees: the days and nights therefore in this planet must be nearly equal at all seasons.

The surface of Jupiter is for the most part interspersed with spots, in the form of bands, some of them obscure, and others luminous: at certain periods they are scarcely visible; nor are uniformly marked throughout their whole extent; so that they are as it were interrupted: their number also varies; and they can be seen only by the assistance of good telescopes, or when Jupiter is at his least distance from the earth. The year 1773 was exceedingly favourable for these observations, because Jupiter was then as near to the orbit of the earth as possible.

The distance of Jupiter from the sun being above 5 times that of the earth, it is evident that the sun's diameter must appear five times less, or about 6 minutes only; consequently the splendor of the sun at Jupiter will be 25 times less than it is to the earth. But a light 25 times less than that of the sun is still pretty strong, and more than sufficient to produce a very clear day: the inhabitants therefore

of Jupiter, for it is probable that there are some in this planet, will have no great cause to complain.

But if they are treated less favourably in this respect than the inhabitants of the earth, they possess advantages in others; for while the earth has only one moon, to make up for the absence of the sun, Jupiter has four. These moons, or satellites, were first discovered by Galileo; and they enabled him to reply to those who objected in opposition to the earth's motion, the impossibility of conceiving how the moon could accompany the earth during its revolution: Galileo's discovery reduced them to silence.

The satellites of Jupiter revolve around him in the periods, and at the distances, indicated in the following table.

Order of the Satellites.	Dist. in semi-diameters of Jupiter.	Periodical Times. D.	H.	M.
I	$5\frac{2}{3}$	1	18	27
II	9	3	13	14
III	$14\frac{23}{60}$	7	3	43
IV	$25\frac{3}{10}$	16	16	32

The inhabitants of Jupiter then, in this respect, enjoy much greater advantages than those of the earth; for having four moons, some of them must be always above the horizon which is not illuminated by the sun: they will even sometimes see the whole four, one as a crescent, another full, and a third half-full: they will see them eclipsed, as we see the moon deprived of her light from time to time, when she enters the shadow projected by the earth, but with this difference, that, being much nearer to Jupiter, considering his bulk, they cannot pass behind him, in regard to the sun, without suffering an eclipse.

Astronomers, however, not contented with establishing the existence of these moons attached to Jupiter, have

done more; for they have calculated their eclipses with as much correctness, at least, as those of our moon. The Nautical Almanac, and other astronomical Ephemerides, exhibit for each day of the month, the aspects of the satellites of Jupiter, and announce the hour at which their eclipses will commence, and whether they will be visible or not on the horizon of the place: they give also the time when any of these satellites will be hid behind the disk of Jupiter, or disappear by passing before it. These predictions are not matters of mere curiosity, since they are of great utility in determining the longitude.

§ VIII.

Of Saturn.

Saturn, which is still farther from the sun than Jupiter, exhibits a most singular spectacle, on account of his seven moons, and the ring by which he is surrounded. He performs his revolution around the sun in 29 years 174 days 6 hours 36 minutes; and his mean distance from that luminary is about 9½ times as great as that of the earth, or 900 millions of miles.

At such an immense distance the apparent diameter of the sun, to a spectator in Saturn, is no more than $\frac{2}{19}$ of what it is to us; and its light as well as heat must be 90 times less. An inhabitant of Saturn transported to Lapland, or even to the polar regions, covered with perpetual ice, would experience there an insupportable heat; and would no doubt perish sooner than a man immersed in boiling water; while an inhabitant of Mercury would freeze in the most scorching climates of our torrid zone.

It is probable that Saturn has a rotary motion around his axis; but the best telescopes have not yet shown on his surface any remarkable point, by means of which this rotation could be ascertained or determined *.

* Dr. Herschel having discovered that there are some belt-like appearances on this planet, similar to those which are seen on Jupiter, concluded

Nature seems to have been desirous to indemnify Saturn for his great distance from the sun, by giving him seven moons, which are called his satellites. Their distances from the centre of Saturn, in semi-diameters of that planet, and the periods of their revolution, are as expressed in the following table.

Satellites.	Distances.	Revolutions. D.	H.	M.
I	4[illegible]	1	21	18
II		2	17	41
III	[illegible]	4	12	25
IV	1[illegible]	15	22	41
V	54	79	7	48
VI	3[illegible]	1	8	53
VII	2[illegible]	0	22	40

Of these satellites, five were discovered by Cassini and Huygens, before the year 1685, and it was imagined there were no more, till two were discovered by Dr. Herschel in 1787 and 1788. These are nearer to Saturn than any of the other five, but to prevent confusion in the numbers, with regard to former observations, they are called the 6th and 7th satellites.

The inclination of the first four satellites to the ecliptic, is from 30 to 31 degrees. The fifth describes an orbit inclined in an angle of from 17 to 18 degrees to the orbit of Saturn. Dr. Herschel observes that this satellite turns once round its axis exactly in the time in which it revolves about Saturn, and in this respect it resembles our moon. We shall not here enlarge on the advantages which this planet must derive from so many moons; what we have said in regard to Jupiter is applicable in a greater degree to Saturn also.

that it must revolve on its axis, and with a pretty quick motion. He also thinks he has determined from some parts of these belts, which are less black than others, that this revolution is performed in 10 hours 16 minutes.

But something still more singular than these seven moons, is the ring by which Saturn is surrounded. Let the reader conceive a globe placed in the middle of a flat thin circular body, with a concentric vacuity; and that the eye is placed at the extremity of a line oblique to the plane of this circular ring. Such is the aspect exhibited by Saturn when viewed through an excellent telescope; and such is the position of a spectator on the earth. The diameter of Saturn is to that of the vacuity of the ring, as 3 to 5; and the breadth of the ring is nearly equal to the interval between the ring and Saturn. It is fully proved that this interval is a vacuity; for a fixed star has been once seen between the ring and the body of the planet; this ring therefore maintains itself around Saturn as a bridge would do concentric to the earth, and having every where an uniform gravity *.

This body, of a conformation so singular, is alternately illuminated on each side by the sun, for it makes, with the plane of Saturn's orbit, an invariable angle, of about 31° 20′; always remaining parallel to itself, in consequence of which it presents to the sun, sometimes the one face,

* This ring, according to Huygens, is about 22000 miles broad, and its greatest diameter is in proportion to that of the planet, as 9 to 4. De la Lande and De la Place inform us, that Cassini saw the edge of this ring, divided into separate parts, nearly equal in breadth. Hadley also, with an excellent 5½ feet reflector, saw the ring divided into two parts. Mr Short and some others thought they saw several divisions on the ring, but the long continued and accurate observations of Dr Herschel seem to confirm the division of the ring into only two concentric parts, almost beyond the possibility of doubt. The doctor says there is one such dark considerably broad line, belt, or zone, which he has constantly found on the north side of the ring. There have been various conjectures in regard to the nature of this ring. Some have imagined that the diameter of Saturn was once equal to the present diameter of the outer ring, and that it was hollow; the present body being contained within the former surface, as a kernel is contained within its shell. They suppose that in consequence of some concussion, or other cause, the outer shell fell down to the inner body, and left only the ring at the greater distance from the centre, as we now perceive it.

and sometimes the opposite one; the inhabitants therefore, of the two hemispheres of Saturn, enjoy the benefit of it alternately. Some observations seem to prove that it has a rotary motion around an axis perpendicular to its plane; but this has not yet been absolutely proved *.

Saturn is seen sometimes from the earth without his ring; but this phenomenon may be easily explained.

Saturn's ring may disappear in consequence of three causes. 1st. It disappears when the continuation of its plane passes through the sun; for in that case its surface is in the shade, or too weakly illuminated by the sun to be visible at so great a distance, and its edge is too thin, even though illuminated, to be seen from the earth. This phenomenon is observed when Saturn's place is about 19° 45′ of Virgo and Pisces.

2d. The ring of Saturn must disappear also, when the continuation of its plane passes between the earth and the sun; for the flat part of the ring, which is then turned towards the earth, is not that illuminated by the sun. It cannot therefore be seen from the earth, but its shadow may be seen projected on the disk of Saturn.

The nature of this singular ring affords much matter for conjecture. Some have supposed that it may be a multitude of moons, all circulating so near each other, that the distance between them is not perceptible from the earth, which gives them the appearance of one continued body. But this is very improbable.

Others have imagined that it is the tail of a comet, which passing very near Saturn, has been stopped by it. But such an arrangement of a circulating fluid would be something very extraordinary. In our opinion, while we admire this work of the sovereign Artist, the Creator of the universe, we must suspend our conjectures respecting the

* Dr. Herschel, from some spots he has seen on the exterior of the ring, has determined that it revolves in about 10½ hours.

nature of it, till a farther improvement in telescopes shall enable us to obtain new facts to support them.

The distance of Saturn from the sun is so great, that all the planets are inferior to it, or below it, as Venus and Mercury are, in regard to our earth. Nay, if it be inhabited by intelligent beings, it is very doubtful whether they have any knowledge of our existence, and much less of that of Mercury and Venus; for in regard to them, Mercury will never be farther from the sun than 2° 25′, Venus than 4° 15′, and the earth than 6°; Mars will be distant from the sun only about 9°, and Jupiter 28° 40′: it will therefore be much more difficult for the Saturnians to see the first three or four of these planets, than it is for us to observe Mercury; which can scarcely ever be seen, as it is almost always concealed among the rays of the sun.

It is however true that the light of the sun is on the other hand very weak; and that the constitution of Saturn's atmosphere, if it has one, may be of such a nature, that these planets are visible, as soon as the sun has set.

§ IX.

Of the Georgian Planet, and other New Planets.

It was long supposed that Saturn was the remotest planet of our system; but it is now well known that this is not the case, as another still farther distant from the sun was discovered by Dr. Herschel, in the year 1781. To this planet Dr. Herschel gave the name of the *Georgium Sidus*, in honour of his present majesty. The French call it Herschel, in honour of the discoverer; and professor Bode of Berlin, gave it the name of Uranus, who was the father of Saturn, as Saturn was of Jupiter. An interesting history of the discovery was presented to the Academy of Sciences at Brussels, in May 1785, by Baron von Zach of Gotha, and is inserted in the first volume of the Memoirs of that Academy.

The distance of this planet from the sun is immense;

being about 1800 millions of miles, which is double that of Saturn. It performs its annual revolution in 83 years 140 days and 8 hours of our time; and its motion in its orbit must consequently be above 7000 miles an hour. To a good eye, unassisted by a telescope, it appears like a faint star of the fifth magnitude; and it cannot readily be distinguished from a fixed star with a less magnifying power than 200. Its apparent diameter, to an observer on the earth, subtends an angle of no more than 4 seconds; but its real diameter is about 35000 miles, and therefore it must be about 80 times as big as the earth. Hence we may infer, as the earth cannot be seen under an angle of quite a second by the inhabitants of the Georgian planet, that it has never yet been discovered by them, unless their eyes and instruments are considerably better than ours. The orbit of this planet is inclined to the ecliptic at an angle of 46 minutes 26 seconds; but as no spots have been discovered on its surface, the position of its axis, and the length of its day and night, are not known.

On account of the immense distance of the Georgian planet from the sun, it was highly probable that it was accompanied with several satellites or moons; and the high powers of Dr. Herschel's telescopes indeed enabled him to discover six; but there may be some others, which he has not yet seen. The first and nearest the planet, revolves at the distance from it of 12½ of its semi-diameters; and performs its revolution in 5 days 21 hours 25 minutes; the second revolves at the distance from the primary of 16½ of its semi-diameters, and completes its revolutions in 18 days 17 hours 1 minute; the third, at the distance of 19 semi-diameters, in 10 days 23 hours 4 minutes; the fourth, at 22 semi-diameters, in 13 days 11 hours 5 minutes; the fifth, at 44 semi-diameters, in 38 days 1 hour 49 minutes; and the sixth, at 88 semi-diameters, in 107 days 16 hours 40 minutes. It is remarkable that the orbits of these satellites are almost all at right angles to the plane of the

ecliptic; and that the motion of every one of them, in their own orbits, is retrograde, or contrary to that of all the other known planets.

Besides the Georgian, four other planets have lately been discovered; a circumstance which leaves room to conjecture that there may be many more of such primary, though small planets.

The first of these four was discovered, on Jan. 1, 1801, the first day of the present century, by M. Piazzi, an ingenious astronomer at Palermo, in the island of Sicily. In order to preserve the honour of this discovery, as well as the observations, to himself, he kept it secret, till, on the 11th of February, he was compelled by sickness to discontinue his observations. This celestial phenomenon is an intermediate planet between the orbits of Mars and Jupiter, and appears as a star of the 8th magnitude. This planet has been named *Ceres Ferdinandea*, by the discoverer, though some astronomers call it *Piazzi*, after that gentleman's own name, and which perhaps would be the best way of naming and distinguishing all the new planets. This planet is but of very small size, its apparent diameter being only about a second and a half, and its real diameter about one-seventh of that of the earth, or half that of the moon, or nearly 1000 miles. Its distance from the sun is about 2 times that of the earth and three-fifths, and its periodic time, in revolving around the sun, about 4 years and 2 months. The eccentricity of its orbit is about ·0364 of its mean distance.

The second of these planets, named *Pallas*, or *Olbers*, was accidentally discovered on the 28th of March 1802, by Dr. Olbers, of Bremen, as he was looking out for the former, or Piazzi, which it much resembled when viewed with the telescope, appearing, like it, without either atmosphere or nebula, as a fixed star of the 7th or 8th magnitude. *Olbers* is much smaller however, and supposed to

be but about 140 miles in diameter. It is also thought to move in an orbit very eccentric, almost like a comet; so much so, that though at its nearest distance it be between Mars and Ceres, yet at its farthest distance it goes off much beyond the latter. Its mean distance from the sun may be about 2 and one-tenth that of the earth, which places it between Mars and Ceres.

The third of these new planets was discovered Sept. 1, 1804, at 10 o'clock in the evening, by M. Harding, astronomer at the Observatory of Lilienthal, near Bremen in Germany. It appeared very small, like a star of the 8th magnitude, and which he named Juno, though others give it more properly the name of the discoverer, after the manner of the two former. Observations have determined that the period of this planet is 5½ years; the inclination of its orbit 21°; its eccentricity a quarter of its radius, which radius, or mean solar distance, is 3 times that of the earth, or about 300 millions of miles, and consequently is a little more than the other two new planets, Piazzi and Olbers, these being about 288 millions of miles, and is also nearly of the same size as those two planets.

The fourth of these new planets was discovered at Bremen, March the 29th, 1807, by Dr. Olbers, the discoverer of the 2d new planet also. He gave it the name Vesta, but may more appropriately be called Olbers' 2d. This is another of the group of new planets, which revolve round the sun, between Mars and Jupiter, at nearly equal distances from it, and all nearly equal in size and period. It is remarkable that several astronomers have formerly imagined that some planet would be discovered in the large space between the orbits of Mars and Jupiter; a prediction which has been amply fulfilled by the discovery, not of one only, but of four planets, in that space. And probably there may even exist many more planets, not only in that space, but scattered about among or beyond

all the other planetary orbits, which may long revolve unseen and undiscovered, by reason of the smallness of their size.

§ X.

Of Comets.

Comets are not now considered, as they were formerly, to be signs of celestial vengeance; the forerunners of war, famine, or pestilence. Mankind in those ages must have been exceedingly credulous to imagine, that scourges confined to a very small portion of the globe, which itself is but a point in the universe, should be announced by a derangement of the natural and immutable order of the heavens. Neither are comets, as supposed by the greater part of the ancient philosophers, and those who trod in their footsteps, meteors accumulated in the middle of the air. Astronomical observations made at the same time, in different parts of the earth, have shown that they are always at a distance much greater than that even of the moon; and consequently that they have nothing in common with the meteors formed in our atmosphere.

The opinions entertained by some ancient philosophers, such as Appollonius the Myndian, and particularly Seneca, have been since confirmed. According to these philosophers, comets are bodies as old and as durable as the planets themselves; their revolutions are regulated in the same manner; and if they are seldom seen, it is because they perform their courses in such a manner, that in a part of their orbits they are so far distant from the earth as to become invisible; so that they never appear but when in the lower part of them.

Newton and Halley, who pursued the same path, have proved by the observations of different comets, which appeared in their time, that they describe elliptical orbits around the sun, which is placed in one of the foci; and

that the only difference between these orbits and those of the planets is, that the orbits of the latter are nearly circular, whereas those of comets are very much elongated; in consequence of which, during a part of their course, they approach near enough to our earth to become visible; but during the rest they recede so far from us, as to be lost in the immensity of space. These two philosophers have taught us also, by the help of a small number of observations, made in regard to the motion of a comet, how to determine the distance at which it has passed, or will pass, the sun; as well as the period when it is at its least distance, and its place in the heavens for any given time. Calculations made according to these principles agree in a surprising manner with observations.

The modern philosophers have even done more: they have determined the periods of the return of some of these comets. The celebrated Dr. Halley, considering that comets, if they move in ellipses, ought to have periodical revolutions, because these curves return into themselves, examined with great care the observations of three comets, which appeared in 1531 and 1532, 1607, and 1682; and having calculated the position and dimensions of their orbits, found them to be nearly the same, and consequently that these comets were only one, the revolution of which was completed in about 75 years: he therefore ventured to predict that this comet would re-appear in 1758, or 1759 at latest. It is well known that this prediction was verified at the time announced; hence it is certain that this comet has a periodical revolution around the sun, in 75 years and a half. According to the dimensions of its orbit, determined by observations, it's least distance from the sun is $\frac{585}{1000}$ of the semi-diameter of the earth's orbit; it afterwards recedes to a distance which is equal to 35½ of these semi-diameters; so that its greatest elongation from the sun, is about four times as great as that of Saturn.

The inclination of its orbit to the ecliptic, is 17° 40′, in a line proceeding from 23° 45′ of Taurus to 23° 45′ of Scorpio.

There are still two comets, the return of which is expected with some sort of foundation; viz, that of 1556, expected in 1848; and that of 1680 and 1681, which it is supposed, though with less confidence, will re-appear about 2256. The latter, by the circumstances which attended its apparition, seems to be the same as that seen, according to history, 44 years before the christian æra, also in 531, and in 1106; for between all these periods there is an interval of 575 years. There is reason therefore to suppose that this comet has an orbit exceedingly elongated, and that it recedes from the sun about 135 times the distance of the earth.

What is very remarkable also in this comet is, that in the lower part of its orbit it passed very near the sun; that is, at a distance from its surface which scarcely exceeded a sixth part of the solar diameter; hence Newton concludes, that at the time of its passage it was exposed to a heat 2000 times greater than that of red-hot iron. This body therefore must be exceedingly compact, to be able to resist so prodigious a heat, which there is reason to think would volatilize all the terrestrial bodies, with which we are acquainted.

At present there are near 100 comets, the orbits of which have been calculated; so that their position, and the least distance at which they must pass the sun, are known. When a new comet therefore shall appear, and describe the same, or nearly the same path, we may be assured that it is a comet which has appeared before: we shall then know the period of its revolution, and the extent of its axis, which will determine the orbit entirely: in short we shall be enabled to calculate the times of its return, and other circumstances of its motion, in the same manner as those of the other planets.

Comets have this in particular, that they are often accompanied by a train or tail. These tails or trains are transparent, and of greater or less extent; some have been seen which were 45, 50, 60, and even 100 degrees in length, as was the case with those of the comets which appeared in 1618 and 1680. Sometimes however the tail consists merely of a sort of luminous nebula, of very little extent, which surrounds the comet in the form of a ring, as was observed in the comet of 1585: it frequently happens that these tails cannot seen unless the heavens be exceedingly serene, and free from vapours. The celebrated comet, which returned about the end of the year 1758, seemed at Paris to have a tail scarcely 4 degrees in length; whereas some observers at Montpelier found it to be 25°; and it appeared still longer to others at the Isle of Bourbon.

In regard to the cause which produces the tails of comets, there are only two opinions which seem to be founded on probability. According to Newton, they are vapours raised by the heat of the sun, when the comet descends into the inferior regions of our system. It is therefore observed that the tail of a comet is longest when it has passed its perihelion; and it always appears longer the nearer it approaches to the sun. But this opinion is attended with considerable difficulties. According to M. de Mairan, these tails are a train of the zodiacal light, with which comets become charged in passing between the earth and the sun. It is remarked that comets which do not reach the earth's orbit, have no sensible tail; or are at most surrounded by a ring. Of this kind was the comet of 1585, which passed the sun at a distance $\frac{1}{10}$ greater than that of the earth; the comet of 1718, which passed at a distance almost equal to that of 1729, that is at a distance nearly quadruple; and that of 1747, which passed at a distance more than double. It is indeed true, that the comet of 1664, which passed at a greater distance from

the sun than that of the earth, appeared with a tail, but it was of a moderate size; and as the distance of its perihelion was very little more than that of the earth from the sun, and as the solar atmosphere extends sometimes beyond the earth's orbit, no objection of any great weight can thence be made, in opposition to the opinion of M. de Mairan.

We shall remark, in the last place, that while the other planets perform their revolutions in orbits very little inclined to the ecliptic, and proceed in the same direction, comets on the other hand move in orbits, the inclination of which to the ecliptic amounts even to a right angle. Besides, some move according to the order of the signs, and are called *direct*, others move in a contrary direction, and are called *retrograde*. These motions being combined with that of the earth, give them an appearance of irregularity, which may serve to excuse the ancients for having been in an error respecting the nature of these bodies.

It has been already said that there are some comets which pass very near the earth; and hence a catastrophe fatal to our globe might some day take place, had not the Deity, by particular circumstances, provided against any accident of the kind.

A comet, indeed, like that of 1744, which passed at a distance from the sun only greater by about a 50th than the radius of the earth's orbit, should it experience any derangement in its course, might fall against the earth or the moon, and perhaps carry away from us the latter. As a multitude of comets descend into the lower regions of our system, some of them, in their course towards the sun, might pass so near the orbit of our earth, as to threaten us with a similar misfortune. But the inclination of the orbits of comets to the ecliptic, which is exceedingly varied, seems to have been established by the Deity to prevent that effect. It would be a curious calculation to determine the least distances at which some of these comets

pass the earth: we should by these means be enabled to know those from which we have any thing to apprehend; that is, if it could be of any utility to be acquainted with the period of such a catastrophe; for where is the advantage of foreknowing a danger which can neither be retarded nor prevented?

An English astronomer, who possessed more imagination and learning, than soundness of judgment, the celebrated Whiston, entertained an opinion that the deluge was occasioned by the earth's meeting with the tail of a comet, which fell down upon it in the form of vapours and rain: he advanced also a conjecture, that the general conflagration, which according to the Sacred Scriptures is to precede the final judgment, will be occasioned by a comet like that of 1681; which returning from the sun, with a heat two or three thousand times greater than that of red-hot iron, will approach so near the earth as to burn even its interior parts. Such assertions are bold; but they rest on a very weak foundation; and in regard to a general deluge, occasioned by the tail of a comet, we need be under very little apprehension on that head; for if we consider the extreme tenuity of the ether in which the comets float, it may be readily conceived that the whole tail of a comet, even if condensed, could not produce a quantity of water sufficient for the effect ascribed to it by Whiston.

Cassini thought he observed that comets pursue their course in a kind of Zodiac, which he even denoted by the following verses:

Antinous Pegasusque, Andromeda, Taurus, Orion,
Procyon atque Hydrus, Centaurus, Scorpio, Arcus.

But the observations of a great number of comets have shown that this supposed Zodiac of comets has no reality.

§ XI.

Of the Fixed Stars.

As it now remains for us to speak of the fixed stars, we shall here collect every thing most curious in the modern astronomy on this subject.

The fixed stars may be easily distinguished from the planets. The former, at least in our climates, and when they are of a certain magnitude, have a splendour accompanied with a twinkling called *scintillation.* But one thing by which they are particularly distinguished is, that they do not change their place in regard to each other, at least in a sensible manner: they are therefore a kind of fixed points in the heavens, to which astronomers have always referred the positions of the moving bodies, such as the moon, the planets, and the comets.

We have said that the fixed stars in our climates exhibit a sort of twinkling. This phenomenon seems to depend on the atmosphere; for we are assured that in certain parts of Asia, where the air is exceedingly pure and dry, as at Bender-Abassi, the stars have a light absolutely fixed; and that the scintillation is never observed, except when the air is charged with moisture, as is the case in winter. This observation of M. Garcin, which was published in the History of the Academy of Sciences for 1743, deserves to be farther examined.

The distance between the fixed stars and the earth is so immense, that the diameter of the earth's orbit, which is 190 millions of miles, is in comparison of it only a point; for in whatever part of its orbit the earth may be, the observations of the same star show no difference in its aspect; so that it has no sensible annual parallax. Some astronomers however assert that they discovered, in certain fixed stars, an annual parallax of a few seconds. Cassini, in a memoir on this parallax, says he observed in Arcturus an annual parallax of seven seconds, and in the star called

Capella, one of eight. This would make the distance of the sun from the former of these stars equal to about 20250 times the radius of the earth's orbit, which, being 95 millions of miles, would give for that distance 1923750000000 miles. Between the fixed stars and the Georgian planet, which is the most distant of our system, there would therefore remain a space equal to more than 10000 times the distance of that planet from the sun.

Placed at such an immense distance from us, what can the fixed stars be but immense bodies, which shine by their own light; in short, suns similar to that which affords us heat, and around which our earth performs its revolutions? It is very probable also that these suns, accumulated as we may say on each other, have the same destination as ours, and are the centres of so many planetary systems, which they vivify and illuminate. It would however be ridiculous to form conjectures respecting the nature of the beings by which these distant bodies are peopled; but of whatever kind they may be, who can believe that our earth, or our system, is the only one inhabited by beings capable of enjoying the pleasure which arises from the contemplation of such noble works? Who can believe that an immense whole, a creation almost without bounds, should have been formed for an imperceptible point, a quantity infinitely small?

The apparent diameter of the fixed stars is in no manner magnified by the best telescopes; on the contrary, these instruments, while they increase their splendour, seem to diminish their magnitude so much, that they appear only as luminous points; but they show in the heavens a multitude of other stars, which cannot be observed without their assistance. Galileo, by means of his telescope, which was far inferior to those now employed, counted in the Pleiades 36 stars, invisible to the naked eye; in the sword and belt of Orion 80; in the nebula of Orion's head 21, and in that of Cancer 36. Father de Rheita says, he

counted 2000 in Orion, and 188 in the Pleiades. In that part of the Austral hemisphere, comprehended between the pole and the tropic, the Abbé de la Caille observed more than 6000 of the 7th magnitude, that is to say perceptible with a good telescope, of a foot in length, a longer telescope shows others apparently more distant, and so in progression perhaps without end. What immensity in the works of the Creator! And how much reason to exclaim with the Psalmist. "The heavens declare the glory of God, and the firmament sheweth his handy work!"

The fixed stars seem to have a common and general motion, by which they revolve around the pole of the ecliptic, at the rate of a degree in 72 years. It is in consequence of this motion that the constellations of the zodiac have all changed their positions. Aries occupies the place of Taurus, the latter that of Gemini, and so of the rest; so that the constellations or signs have advanced about 30 degrees beyond the divisions of the zodiac to which they gave names. But this motion is only apparent, and not real; and arises from the equinoctial points going back every year about 51 seconds on the ecliptic. The explanation of this phenomenon however is of such a nature, as not to come within the object of this work.

It has always been believed that the fixed stars have no real motion, or at least no other than that by which they change their longitude. But it has been discovered, by the very accurate observations of modern astronomers, that some of them have a small motion peculiar to themselves, by which they slowly change their places. Thus Arcturus, for example, has a motion by which it approaches the ecliptic about 4 minutes every 100 years. The distance between this star and another very small one, in its neighbourhood, has been sensibly changed in the course of the last century. Sirius also seems to have a motion in latitude, of more than 2 minutes per century, by which it recedes from the ecliptic. A similar motion

has been observed in Aldebaran or the Bull's Eye; in Rigel; in the eastern shoulder of Orion; in the Goat, the Eagle, &c. Some others seem to have a peculiar motion in a direction parallel to the equator, as is the case with the brilliant star in the Eagle; for in the course of 48 years it has approached one star in its neighbourhood 73″, and receded from another 48″. All the stars perhaps are subject to a similar motion; so that in a series of ages the heavens will afford a spectacle very different from what they do at present. So true it is that nothing in the universe is permanent!—In regard to the cause of this motion, however astonishing it may at first seem, it will appear less so if it be recollected that it has been demonstrated by Newton, that a whole planetary system may have a progressive and uniform motion in space, without the particular motion of the different parts being thereby disturbed. It needs therefore excite no surprise that suns, as the fixed stars are, should have a motion of their own. The state of rest being of one kind only, and that of motion in any direction being infinitely varied, we ought rather to be astonished to see them absolutely at rest, than to discover in them any movement.

But these are not the only phenomena exhibited to us by the fixed stars; for some have appeared suddenly, and afterwards disappeared. The year 1572 is celebrated for a phenomenon of this kind. In the month of November of that year, an exceedingly bright star suddenly appeared in the constellation of Cassiopeia: its splendor at first was equal to that of Venus when in its perigeum, and then to that of Jupiter when he exhibits the greatest brightness; three months after its appearance it was only like a fixed star of the first magnitude: its splendor gradually decreased till the month of March 1574, at which time it entirely disappeared.

There are other stars which appear and disappear regularly at certain periods: of this kind is that in the neck

of the Whale. When in its state of greatest brightness it is nearly equal to a star of the second magnitude; it retains this splendor for about fifteen days, after which it becomes fainter, and at length disappears: it then reappears, and attains to its greatest splendor, after a period of about 330 days.

The constellation of the Swan exhibits two phenomena of the same kind; for in the breast of the Swan there is a star which has a period of 15 years, during 10 of which it is invisible: it then appears for 5 years, varying in its magnitude and splendor. Another, which is situated in the neck near the bill, has a period of about 13 months. In the same constellation a star was observed in 1670 and 1671, which disappeared in 1672, and has never since been seen.

Hydra also has a star of the same kind, which is attended with this remarkable circumstance, that it appears only 4 months; after which it remains invisible for 20, so that its period is about two years.

In the last place, some stars seem to have become extinct since the time of Ptolemy; for he enumerates some in his catalogue which are not now to be seen: others have changed their magnitude; this diminution of size is proved in regard to several of the fixed stars; among this number may be classed the star α in the Eagle, which at the beginning of the last century was the second in splendor, but which at present is scarcely of the third magnitude. Of this kind also is a star in the left leg of Serpentarius or Ophiuchus.

It now remains that we should say a few words respecting those stars called *nebulæ*. They are distinguished by this name because, when seen by the naked sight, they appear only like a small luminous cloud. There are three kinds of them. Some consist of an accumulation of a great number of stars, crowded together, and as it were heaped upon each other; but when viewed through a telescope, they are seen distinct, and without any nebulous

appearance. Among these is the famous nebula of Cancer, or the *præsepe Cancri*, forming a collection of 25 or 30 stars, which may be counted by means of a telescope. Similar groupes may be seen in various parts of the heavens.

Other nebulæ consist of one or more distinct stars, but accompanied or surrounded by a whitish spot, through which they seem to shine. There are two of this kind in Andromeda; one in the girdle, and another smaller about a degree farther south than the former. Of this kind also is that in the head of Sagittarius; that between Sirius and Procion; that in the tail of the Swan; and three in Cassiopeia. It is probable that our sun appears under this form, when seen from the neighbourhood of those fixed stars which are situated towards the prolongation of his axis; for he has around him a lenticular and luminous atmosphere, which extends nearly to the earth. The abbé de la Caille counted in the Austral hemisphere fourteen stars, surrounded in this manner with nebulosities; but the most remarkable appearance of this kind, is that of the nebula in the sword of Orion; for, when viewed through a telescope, it is found to be formed of a whitish spot, nearly triangular, and containing seven stars, one of which is itself surrounded by a small cloud, brighter than the rest of the spot. One is almost inclined to believe that this spot has experienced some alteration since the time of Huygens, by whom it was discovered.

The third kind of nebulæ are composed of a white spot, in which no stars are seen when viewed with the telescope. Fourteen of this kind are found in the Austral hemisphere, among which the celebrated spots, near the South pole, called by sailors the *Magellanic clouds*, hold the first rank. They are like small detached portions of the milky way. But it may be thought an error to ascribe the splendor of that part of the heavens to small stars accumulated there in a greater multitude than any where else; for it does not

contain a number, visible by common telescopes, sufficient to produce that effect; and there are portions of the milky way no less brilliant than the rest, though no stars are observed in them, unless with the very highest improved instruments.

Respecting the milky way, nothing certain is known; but we may conjecture, not without probability, that it consists of some matter, similar to that of the solar atmosphere, and which is diffused throughout that celestial space*. If our whole system indeed were filled with a similar matter, it would exhibit to the neighbouring fixed stars the same apearance as the milky way. But why are all these systems, with which that part of the heavens is interspersed, filled with this luminous matter? To this question no answer certainly can be given.

We shall here remark, that the famous new star in Cassiopeia had its origin in the milky way, and was perhaps formed by a prodigious quantity of this luminous matter being precipitated on some centre. But it is more difficult to explain why, and in what manner, the star disappeared. This origin of the new star may acquire some probability, if it be true that in the part of the milky way where it was seen, there is a vacuity similar to the other parts of the heavens.

§ XII.

Recapitulation of what has been said respecting the System of the Universe.

We shall terminate this chapter with a familiar comparison, calculated to show, by known and common measures, the small space which our planetary system occupies in the immensity of the universe; and the poor figure, if we may be allowed the expression, which our earth makes in it. This consideration will no doubt serve to humble

* Unless, with Dr. Herschel, we suppose it is a far extended stratum of stars, by us seen edgeways.

those proud beings, who, though they occupy but an infinitely small portion of this atom, have the vanity to think that the universe was created for them.

To form an idea of our system as compared with the universe, let us suppose the sun to be in Hyde Park, as a globe of 9 feet 3 inches diameter: the planet Mercury will be represented by a globule of about $\frac{1}{3}$ of a line in diameter, placed at the distance of 37 feet. Venus will be a globe of little more than a line in diameter circulating at the distance of 68 feet from the same centre: if another globule, a line in diameter, be placed at the distance of 95 feet, it will represent the earth, that theatre of so many passions, and so much agitation; on the surface of which the greatest potentate scarcely possesses a point, and where a space often imperceptible excites, among the animalcula that cover it, so many disputes, and occasions so much bloodshed. Mars, which in magnitude is somewhat inferior to the earth, will be represented by a globule of a little less than a line in diameter, and placed at the distance of 144 feet; Jupiter by a globe 10 lines in diameter, 490 feet from the central globe; Saturn about 7 lines in diameter, at the distance of about 900 feet; and the Georgian planet, 4 lines in diameter, at the distance of 1800 feet.

But the distance from the Georgian planet to the nearest fixed stars, is immense. The reader may perhaps imagine that, according to the supposition here made, the first star ought to be placed at the distance of two or three leagues. This is the idea which one might form before calculation has been employed; but it is very erroneous, for the first, that is to say the nearest star, ought to be placed at the same distance as that between London and Edinburgh, which is more than 300 miles. Such then is the idea which we ought to have of the distance between the sun and the nearest of the fixed stars; and there is reason even to think that it is much greater, for we have supposed, in this calculation, that the parallax of the earth's orbit is the

same as the horizontal parallax of the sun, that is to say 8·5″. But it is probable that this parallax is much less; for it can hardly be believed that it could have escaped astronomers had it been so great.

Our solar system then, that is, the system of our primary and secondary planets, which circulate around the sun, is to the distance of the nearest fixed stars, almost as a circle of 1800 feet radius, would be to a concentric one of 300 miles radius; and in the first circle our earth would occupy a space a line in diameter, appearing like a grain of mustard seed.

Another comparison, proper to convey some idea of the immense distance between the sun, which is the centre of our system, and the nearest of the neighbouring bodies of the same nature, is as follows: It is well known that the velocity of light is so great, that it passes over the distance between the sun and the earth in about half a quarter of an hour: in a second and a half it would go to the moon and return, or rather it would go fifteen times round the earth in a second. What time would light then employ in coming to us from the nearest of the stars?—Not less than 108 days; or if the annual parallax be only 2 or 3 seconds, which appears very probable, it would require a year and more.

What immense distance then between this inhabited point and the nearest of its neighbours! Is it not probable that in this vast interval there are planets which will remain for ever unknown to the human species?

Modern astronomy indeed has discovered that this space is not entirely desert: it is now known that about a hundred comets move in it, at greater or less distances, but do not penetrate to a very great depth. Those of 1531, 1607, 1682, and 1759, the only ones the periods and orbits of which are known, do not immerge farther than about 37½ times the radius of the earth's orbit, or four times the distance of Saturn from the sun. If that of 1681

has a revolution of 575 years, as supposed, it must recede from us about 130 times the distance of the earth from the sun, or about 14 times that of Saturn from the same body; which is only a point when compared with the nearest of the fixed stars. But there are comets perhaps which perform their revolution only in 10000 years, and which scarcely approach so near the sun as Saturn: in that case these would penetrate into the immense space which separates us from the first of the fixed stars, as far as a fiftieth part of its depth.

Those desirous of seeing a great many curious conjectures respecting the system of the universe, the habitation of the planets, the number of the comets, &c, may consult a work by M. Lambert, member of the royal Academy of Berlin, entitled *Systeme du Monde*, Bouillon 1770, 8vo. Every one almost is acquainted with the *Pluralité des Mondes* of Fontenelle; the *Cosmotheoros* of Huygens, the *Somnium* of Kepler, and the *Iter extaticum* of Kircher. The first of these, the *Pluralité des Mondes*, is an ingenious and pleasing work, but a little too affected. The second is learned and profound, and, like Kepler's *Somnium*, will please none but Astronomers. In regard to the last, however much we may esteem the memory of Kircher, it can be considered in no other light, than as a production altogether pedantic and ridiculous.

CHAPTER III.

Of Chronology, and various Questions relating to that Subject.

All polished nations keep an account of the time which has elapsed, and of that which is to come, by means of periods that depend on the motions of the heavenly bodies; and this is even one of those things which distinguish man

in a state of civilization, from man in the animal and savage state: for, while the former is enabled at every moment to count that part of the duration of his existence which has elapsed; to foresee, at an assigned period, the recurrence of certain events, labours or duties; the latter, though in some measure happier, since he enjoys the present without recollecting the past, or anticipating the future, cannot tell his age, nor foresee the period of the renovation of his most common occupations: the most striking events of which he has been a witness, or in which he has had a share, exist in his mind only as past: while the civilized man connects them with precise periods and dates, by which they are arranged in their proper order. Without this invention, every thing hitherto done by mankind would have been lost to us; there would be no historical records; and men, whose existence in the social state requires the united efforts of its different members in certain circumstances, could not employ that concurrence of action which is necessary. No real civilized society therefore can exist without an agreement to count time in a regular manner; and hence the origin of chronology, and the various computations of time employed by different nations.

But, before we proceed farther, it will be proper to present the reader with some definitions, and a few historical facts, necessary for comprehending the questions which will be proposed in the course of this article.

There are two kinds of year employed by different nations; one of which is regulated by the course of the sun, and the other by that of the moon. The first is called the solar, and the second the lunar year. The solar year is measured by a revolution of the sun through the ecliptic, from one point of the equinoctial, that of the vernal equinox for example, to the same point again; and, as already said, consists of 365 days 5 hours 49 minutes.

The lunar year consists of twelve lunations; and its

duration is 354 days 8 hours 44 minutes 3 seconds. Hence it follows that the lunar year is about 11 days shorter than the solar; consequently, if a lunar and a solar year commence on the same day, at the end of three years the commencement of the former will have advanced 33 days before that of the latter. The commencement therefore of the lunar year passes successively through all the months of the solar year, in a retrograde direction. The Arabians, and Mussulams in general, count only by lunar years; and the Hebrews and Jews never employed any other.

But the most polished and enlightened nations have always endeavoured to combine these two kinds of year together. This the Athenians accomplished by means of the famous golden cycle, invented by Meto, the celebrated mathematician whom Aristophanes made the object of his satirical wit; and the same thing is done at present by the Europeans, or the Christians in general, who have borrowed from the Romans the solar year for civil uses; and from the Hebrews their lunar year for their ecclesiastical purposes.

Before Julius Cæsar, the Roman calendar was in the utmost confusion; but it is here needless to enter into any details on the subject: it will be sufficient to observe, that Julius Cæsar, being desirous to reform it, supposed, according to the suggestion of Sosigines his astronomer, that the duration of the year was exactly 365 days 6 hours. He therefore ordered that, in future, there should be three successive years of 365 days, and a fourth of 366. This last year was afterwards distinguished by the name of *bissextile*, because the day added every fourth year followed the sixth of the calends which was counted twice; and because, to avoid any derangement in the denomination of the following days, it was thence called *bis sexto calendas*. Among us it is added to the end of February, which has then 29 days instead of 28, which is the number it contains in common years. This form of year is called the *Julian*

year; and the calendar in which it is employed, is called the *Julian calendar.*

But Julius Cæsar was mistaken, when he considered the year as consisting exactly of 365 days 6 hours, as it contains only 365 days 5 hours 49 minutes; and hence it follows that the equinox always retrogrades in the Julian year 11 minutes annually; which gives precisely 3 days in 400 years. Hence it happened that the vernal equinox, which at the time of the council of Nice corresponded to the 21st of March, after the lapse of about 1200 years, that is to say in the year 1500, fell about the 11th. Pope Gregory XIII, being desirous to reform this error, suppressed, in 1582, ten consecutive days; counting after the 11th of October the 21st, and by these means brought back the vernal equinox following to the 21st of March; and, in order that it might never deviate any more, he proposed that three bissextiles should be suppressed in the course of 400 years. For this reason the years 1700 and 1800 were not bissextile, though they ought to have been so according to the Julian Calendar; the case will be the same with the year 1900, but the year 2000 will be bissextile; in like manner the years 2100, 2200 and 2300 will not be bissextile; but 2400 will; and so of the rest.

All this is sufficient, and more than sufficient, for the solar year. But the great difficulty of our calendar arose from the lunar year, which it was necessary to combine with it; for, as the Christians had their origin among the Jews, they were desirous of connecting their most solemn festival, that of Easter, with the lunar year; because the Jews celebrated their Passover at a certain lunation, viz, on the day of the full moon which immediately followed the vernal equinox. But the council of Nice, that the Easter of the Christians might not concur with the Passover of the Jews, ordained, that the former should celebrate their festival on the Sunday after the full moon which should take place on the day of the vernal equinox.

of which should immediately follow it. Hence has arisen the necessity of forming periods of lunations, that the day of the new or full moon may be found with more facility, in order to determine the paschal moon.

The council of Nice supposed the cycle of Meto, or the golden number, according to which 235 lunations are precisely equal to 19 solar years, to be perfectly exact. After the period of 19 years therefore, the new and full moons ought to take place on the same days of the month. It was thence easy to determine, in each of these years, the place of these lunations; and this was what was actually done by means of the epacts, as shall be hereafter explained.

But in reality 235 lunations are less, by an hour and a half, than 19 solar Julian years; whence it happens, that in 304 years, the new moons retrograde a day towards the commencement of the year; and consequently four days in 1216 years. On this account, about the middle of the 16th century, the new and full moons had anticipated, by four days, their ancient places; so that Easter was frequently celebrated contrary to the disposition of the council of Nice.

Gregory XIII undertook to remedy this irregularity by an invariable rule, and proposed the problem to all the mathematicians of Europe; but it was an Italian physician and mathematician, who succeeded best in solving it, by a new disposition of the epacts, and which the church adopted. This new arrangement is called the *Gregorian Calendar*. It began to be used in Italy, France, Spain, and other catholic countries, in 1582. It was soon adopted, at least in what concerns the solar year, even by the protestant states of Germany; but they rejected it in regard to the lunar, and preferred finding the day of the paschal full moon by astronomical calculation: the Roman Catholics therefore do not always celebrate Easter at the same time as the Protestants, in Germany. The English were the most obstinate in rejecting the Gregorian year,

and almost for the same reason which made them long exclude peruvian bark from their pharmacopeia; that is to say, because they were indebted for it to the Jesuits: but they at length became sensible that whatever is good in itself, and useful, ought to be received, were it even from enemies; and they conformed to the method of computing time employed in the rest of Europe. This change did not take place till the year 1752. Before that period, when the French counted the 21st of the month, the English counted only the 10th. In course of ages they would therefore have had the vernal equinox at Christmas, and the winter at Midsummer. The Russians are the only people of Europe who still adhere to the Julian Calendar: their Papas hate the Roman Catholic priests as much as the English did a Jesuit.

After this short historical sketch, we shall now proceed to the principal problems of chronology.

PROBLEM I.

To find whether a given year be Bissextile or not, that is to say, whether it consists of 366 days.

Divide the number which indicates the given year by 4, and if nothing remains the year is bissextile: if there be a remainder, it shows the number of the year current after bissextile. We shall here propose, as an example, the year 1774. As 1774 divided by 4 leaves 2 for remainder, we may conclude that the year 1774 was the second after bissextile.

To this rule however there are some limitations. 1st. If the year is one of the centenaries posterior to the reformation of the calendar by Gregory XIII, that is to say 1582, it will not be bissextile unless the number of the centuries which it denotes be divisible by 4, thus 1600, 2000, 2400, 2800 have been, or will be bissextiles; but the years 1700, 1800, 1900, 2100, 2200, 2300, 2500, 2600, 2700, were

not, or will not be bissextiles, for the reason already mentioned.

2d. If the year be centenary, and anterior to 1582, but without being below 474, it has been bissextile.

3d. Between 459 and 474 there was no bissextile.

4th. There was none among the first six years of the Christian æra.

5th. As the first bissextile after the Christian æra was the seventh year, and as the bissextiles regularly followed each other every four years till 459; when the given year is between the 7th and the 459th first subtract 7 from it, and then divide it by 4; if nothing remains, the year has been bissextile; but if there be any remainder, it will show what year after bissextile the proposed year was. Let the proposed year, for example, be 148: if 7 be subtracted, the remainder is 141, which divided by 4, leaves 1 for remainder, consequently the year 148 of the Christian æra was the first after bissextile.

Of the Golden Number and Lunar Cycle.

The golden number, or lunar cycle, is a revolution of 19 solar years, at the end of which the sun and moon return very nearly to the same position. The origin of it is as follows.

Since the solar Julian year, as already said, consists of 365 days 6 hours; and as the duration of one lunation is 29 days 12 hours 49 minutes; it has been found, by combining these two periods, that 235 lunations make nearly 19 solar years; the difference being only 1h 31m. It is therefore plain that after 19 solar years the new moons ought to take place on the same days of the month, and almost at the same hour. In the first of these solar years, if the new moon happen on the 4th of January, the 2d of February, &c, at the end of 19 years the new moons will take place also on the 4th of January, the 2d of February,

&c; and this will be the case eternally, if we suppose that 235 lunations are exactly equal to 19 solar revolutions. Hence it is sufficient to have once determined, during 19 solar years, the days of the month on which the new moons happen; and when it is known what rank a given year holds in this period, we can immediately tell on what days of each month the new moons fall.

The invention of this cycle appeared to the Athenians to be so ingenious, that, when proposed by the astronomer Meto, it was received with acclamations, and inscribed in the public square in golden letters: hence the name of the golden number. It is distinguished also by the less pompous denomination of the lunar cycle, or cycle of Meto from the name of its inventor.

PROBLEM II.

To find the Golden Number of any given year, or the rank which it holds in the Lunar Cycle

To the given year add 1, and divide the sum by 19: if nothing remains, the golden number of the given year will be 19: but if there be a remainder, which must necessarily be less than 19, it will be the golden number required.

Let the given year, for example, be 1813. If 1 be added to 1813, and if the sum 1814 be divided by 19, the remainder will be 9, which indicates that 9 is the golden number of 1813, or that this year is the 9th of the lunar cycle of 19 years.

If the year 1728 be proposed, it will be found by a similar operation, that the remainder is nothing: which shows that the golden number of that year was 19.

The reason of adding 1 to the given year, is because the first year of the Christian æra was the second of the lunar cycle, or had two for its golden number.

If any year before the Christian æra be proposed, such as the 25th for example, subtract 2 from that number, and divide 23 the remainder by 19, if 4 the remainder be then

taken from 19, the result will be the golden number of the year 25 before Jesus Christ; which in this case is 15.

REMARK.—It may be readily seen that when the golden number of any year has been found, the golden number of the following year may be obtained by adding 1 to the former. The golden number of the preceding year may be obtained also by subtracting 1 from the golden number already found. Thus, having found the golden number of the year 1802, which is 17, by adding 1 to it, we shall have 18 for that of the year 1803; and 1 subtracted from it, will give 16 for the golden number of 1801.

Of the Epact.

The epact is nothing else than the number of days denoting the moon's age at the end of a given year. The formation of it may be easily conceived by considering that the lunar year, which consists of 12 lunations, is less than a Julian year by about 11 days; therefore if we suppose that a lunar and a solar year begin together on the 1st of January, the moon at the end of the year will be 11 days old; for 12 complete lunations, and 11 days of a thirteenth, will have elapsed; and therefore the moon at the end of the second year, will be 22 days old, and at the end of the third 33. But as 33 days exceed a lunation, one of 30 days is intercalated, by which means that year has 13 lunations; and consequently the moon is only 3 days old at the end of the third year.

Such then is the progress of the epacts. That of the first year of the lunar cycle is 11; this number is afterwards continually added, and when the sum exceeds 30, if 30 be subtracted, the remainder will be the epact, except in the last year of the cycle, where the product of the addition being only 29, the same number is deducted to have 0 for epact: this announces that the new moon happens at the end of that year, which is also the beginning of the next one. The order of the epacts therefore is 11,

22, 3 14, 25, 6, 17, 28, 9, 20, 1, 12, 23, 4, 15, 26, 7, 18, 29

This arrangement would have been perfect and perpetual, if 19 solar years of 365 days 6 hours, had been exactly equal to 235 lunations, as supposed by the ancient astronomers; but unfortunately this is not the case. On the one hand, the solar year consists only of 365 days 5 hours 49 minutes; and besides, 235 lunations are less than 19 Julian years by one hour and a half; so that in 304 years the real new moons are anterior, by one day, to the new moons calculated in this manner. Hence it happened that in the middle of the 16th century, they preceded by four days those found by calculation; as four revolutions of 304 years had elapsed between that period and the Council of Nice, at which the use of the lunar cycle had been adopted for computing the time of Easter, it was therefore found necessary to correct the calendar, that this festival might not be celebrated, as was often the case, contrary to the intention of that council; and with this view some changes were made in the calculation of the epacts, which form two cases. One of them is that when the proposed year is prior to the reformation of the calendar, or to 1582: the second is when the years are posterior to that epoch. We shall illustrate both cases in the following problem.

PROBLEM III.

Any Year being given, to find its Epact.

I. If the proposed year be anterior to 1582, though posterior to the Christian æra, which forms the first case; find by the preceding problem the golden number for the given year, and having multiplied it by 11, subtract 30 from the product as many times as possible: the remainder will be the epact required.

Let the given year, for example, be 1489. Its golden number, by the preceding problem, is 8, which multiplied

by 11 gives 88; and this product divided by 30 leaves for remainder 28: the epact of the above year therefore was 28.

In like manner, if 1796 be considered as a Julian year, that is to say, if those who have not adopted the new style or reformation in the calendar wished to know the epact of that year, it would be necessary first to find the golden number, which is 11, this multiplied by 11 gives 121; and the latter divided by 30, leaves 1 for remainder. Hence it appears that the epact of 1796, considered as a Julian year, was 1.

II. We shall now suppose that the given year is posterior to the reformation of the calendar, or to the year 1582; which forms the second case. In this case, multiply the golden number by 11, and from the product subtract the number of days cut off by the reformation of Gregory XIII, that is, 10 if the year is between 1582 and 1700; 11 between 1700 and 1900, 12 between 1900 and 2200 &c; divide what remains after this deduction by 30, and the remainder will be the epact required *.

Let it be proposed, for example, to find the epact of the Gregorian year 1693, the golden number of which was 3: multiply 3 by 11, and from 33, the product, subtract 10: as the remainder 23 cannot be divided by 30, that number was the epact of the year 1693.

If the epact of the year 1796 were required, the golden number of which was 11; multiply 11 by 11, and from the product 121 subtract 11, which will leave 110: this number divided by 30, gives for remainder 20, which was the epact of the year 1796.

If the epact of the year 1813 were required, the golden number of which is 9; multiply 9 by 11, and from the product 99 subtract 11; the remainder 88 divided by 30

* When the golden number is 1, if the year be posterior to 1900 add 30 to it before you multiply by 11, and then proceed as above directed.

leaves for remainder 28, which therefore is the epact for the present year 1813.

Remarks.—The epact according to the Julian calendar may be found without division, in the following manner: Assign to the upper extremity of the thumb of the left hand, the value of 10; to the middle joint 20, and to the last or root 30, or rather 0. Count the golden number of the proposed year on the same thumb, beginning to count 1 at the extremity, 2 on the middle joint, 3 on the root; then 4 at the extremity, 5 on the joint, 6 on the root; and so on, till you come to the golden number found; to which, if it falls on the root, nothing is to be added, because the value assigned to it was 0 but if it falls on the extremity add 10 to it; and if on the middle joint 20; because these were the values assigned to them. The sum, if less than 30, will be the epact required; if greater than 30, subtract 30 from it and the remainder will be the epact.

Thus, if the epact of 1489 were required, as the golden number of that year was 8, count 8 on the thumb, as above mentioned, beginning to count 1 on the extremity, 2 on the middle joint, 3 on the root; then 4 on the extremity, and so on. Because 8, in this case, falls on the middle joint, add to it 20, and the sum 28 will be the epact of the above year 1489. In like manner, if the epact of 1726 be required, the golden number of which was 17; count 1 on the extremity of the thumb, 2 on the middle joint, &c, till you complete 17, which will fall on the joint; and if 20, the value assigned to that joint, be then added to the golden number, the sum will be 37; from which if 30 be subtracted, there will remain 7 for the epact of 1726, according to the Julian calendar.

By the same artifice the epact for any year of the 17th century might be found; provided 20 be assigned to the extremity of the thumb, 10 to the joint, and 0 to the root;

and that you begin to count 1 on the root, 2 on the joint, and so on.

PROBLEM IV.

To find the day of the New Moon, in any proposed Month of a given Year.

First find the epact of the given year, as taught in the two preceding problems; and add to it the number of months, reckoning from March inclusively: subtract the sum from 30, if less, or from 60 if greater; and the remainder will give the day of the new moon.

Let it be required, for example, to find on what day the new moon happened in the month of May 1813. The golden number of 1813 was 9, which multiplied by 11 gives 99; and if 11 be subtracted, according to the rule, we shall have for remainder 88: this divided by 30 leaves 28 = the epact of that year, as before found. Now the number of months from March, including May, is 2; and 2 added to the epact makes 30, which subtracted from 60 leaves 30: new moon therefore took place on the 30th of May 1813. Accordingly the Almanacs show it was new moon near midnight of the 29th of that month.

Remark.—In calculations of this nature, great exactness must not be expected. The irregular arrangement of the months which have 31 days, the mean numbers necessary to be assumed in the formation of the periods from which these calculations are deduced, and the inequality of the lunar revolutions, may occasion an error of nearly 48 hours.

More correctness may perhaps be obtained by employing the following table; which indicates what ought to be added to the epact for each commencing month.

January .	0	March	0
February .	2	April .	

May	3	September . . .	8
June	4	October	8
July	5	November . . .	10
August	6	December . . .	10

PROBLEM V.

To find the Moon's Age on any given day.

To the epact of the year add, according to the above table, the number belonging to the month in which the proposed day is, and to this sum add the number which indicates the day: if the result be less than 30, it will be the moon's age on the given day; if it be 30, it shews that new moon took place on that day; but if it exceeds 30, subtract 30 from it, and the remainder will be the age of the moon.

Let it be required, for example, to find what was the age of the moon on the 20th of March 1813. The epact of 1813 was 28, and the number to be added for the month of March, according to the preceding table, is 0: this added to 28 makes 28, and 20, the number of the proposed day, added to 28, makes 48, from which if 30 be subtracted, the remainder is 18 = the moon's age on the 20th of March; and this indeed is agreeable to what is indicated by the Almanacs.

Of the Solar Cycle and Dominical Letter.

The solar cycle is a perpetual revolution of 28 years, the origin of which is as follows:

1st. The seven first letters of the alphabet A B C D E F G are arranged in the calendar in such a manner, that A corresponds to the 1st of January, B to the 2d, C to the 3d, D to the 4th, E to the 5th, F to the 6th, G to the 7th, A to the 8th, B to the 9th, and so on through several revolutions of seven. The seven days of the week, called also feriæ, are represented by these seven letters.

2d. Because a year of 365 days contains 52 weeks and 1 day, and as that remaining day is the first of a 53d revolution, a common year of 365 days ought to begin and end with the same day of the week.

3d. According to this disposition, the same letter of the alphabet corresponds to the same day of the week, throughout the course of a common year of 365 days.

4th. As these letters all serve alternately to indicate Sunday, during a series of several years, they have on that account been called *dominical letters*.

5th. It hence follows that if a common year begins by a Sunday, it will end by a Sunday the 1st of January therefore of the following year will be a Monday, which will correspond to the letter A; and the 7th will be a Sunday, which will correspond to the letter G, which will be the dominical letter of that year. For the same reason, the dominical letter of the following year will be F: that of the next one E, and so on, circulating in an order retrograde to that of the alphabet. From this circulation of the letters has arisen the name of *solar cycle;* because Sunday among the pagans was called *dies solis*, the day of the sun.

6th. If there were no days to be added for bissextile years, all the different changes of the dominical letters would take place in the course of seven years. But this order being interrupted by the bissextile years, in which the 24th of February corresponds to two different feriæ of the week; the letter F, for example, which would have indicated a Saturday in a common year, will indicate a Sunday in a bissextile year · or if it indicated a Sunday in a common year, it will indicate a Sunday and a Monday in a bissextile, &c. Hence it follows that in a bissextile year, the dominical letter changes, and that the letter which marked a Sunday in the commencement of the year, will mark a Monday after the addition of the bissextile. This is the reason why two dominical letters are assigned

to each bissextile year; one which serves from the 1st of January to the 24th of February, and the other from the 24th of February to the end of the year; so that the second dominical letter would naturally be that of the following year if a day had not been added for the bissextile.

7th. All the possible varieties to which the dominical letters are subject, both in common and in bissextile years, take place in the course of 4 times 7, or 28 years; for after 7 bissextiles, the dominical letters return and circulate as before. This revolution of 28 years has been called the *solar cycle*, or the *cycle of the dominical letter.*

PROBLEM VI

To find the Dominical Letter of any proposed year.

1st. To find the dominical letter of any given year, according to the Gregorian Calendar, add to the number of the year its fourth part, or, if it cannot be exactly divided by 4, the least nearest to it; from the sum subtract 5 for 1600, 6 for the following century 1700, 7 for 1800, and 8 for 1900 and 2000, because the years 1700, 1800 and 1900 are not bissextiles; 9 for 2100, 10 for 2200, and 11 for 2300 and 2400, because the three years 2100, 2200 and 2300 will not be bissextiles; divide what remains by 7, and the remainder will be the dominical letter required, counting from the last letter G towards A the first; so that if nothing remains, the dominical letter will be A; if 1 remains, the dominical letter will be G; if 2 remains, it will be F; and so of the rest.

Thus, to find the dominical letter of the year 1813: add its fourth part 453, which makes 2266, and from this sum subtract 7; if the remainder 2259 be divided by 7, the remainder 5 will shew that the dominical letter is C, since it is the fifth, counting in a retrograde order, from the last letter G.

We must here observe, that to find with more certainty, by this operation, the dominical letter of a bissextile year, it will be necessary to find first the dominical letter of the preceding year, which will serve till the 24th of February of the bissextile year; after which the next letter in the retrograde order must be used for the remaining part of the year. Thus, if it be required to find the dominical letter of the year 1724; first find that of 1723, by adding to it its nearest less fourth part, 430; subtracting 6 from the sum 2153, and dividing the difference 2147, by 7: the remainder 5 shews that the dominical letter of the year 1723 was C; which is the fifth of the first seven letters of the alphabet, counting in the retrograde order. Since it is known that C was the dominical letter of 1723, it may be readily seen that B was the dominical letter of the following year 1724. But as 1724 was bissextile, B could be used only till the 24th of February, after which A, the letter preceding B, was employed to the end of the year: hence it is seen that A and B were the two dominical letters of the year 1724. In like manner the dominical letters of any future bissextile year may be found.

2d. To find the solar cycle, or rather the current year of the solar cycle, corresponding to a given year; add 9 to the proposed year, and divide the sum by 28: if nothing remains, the solar cycle of that year is 28; but if there be any remainder, it indicates the number of the solar cycle required.

Thus, if the solar cycle of 1813 be required; add 9, which makes 1822, and divide this sum by 28; the remainder, being 2, shews that 2 is the solar cycle of 1813.

The reason of this rule is, that the first year of the Christian æra was the 10th of the solar cycle; or in other words that at the commencement of this æra 9 years of the solar cycle were elapsed.

REMARKS.—The solar cycle of any year whatever may be found with great ease, and without division, by means of the subjoined table.

Years.	Solar Cycle	Years	Solar Cycle	Centuries	Solar Cycle	Centuries	Solar Cycle.
1	1	10	10	100	16	1000	20
2	2	20	20	200	25	2000	12
3	3	30	2	300	20	3000	4
4	4	40	12	400	8	4000	24
5	5	50	22	500	24	5000	16
6	6	60	4	600	12	6000	8
7	7	70	14	700	0	7000	0
8	8	80	24	800	16	8000	20
9	9	90	6	900	4	9000	12

The method of constructing this table is as follows:

Having placed opposite to the first ten years, the same numbers as the solar cycles of these years, and 20 for the solar cycle of the 20th; instead of setting down 30, for the 30th year, set down only 2, which is the excess of 30 above 28, or above the period of the solar cycle. For the 40th year, inscribe the numbers which correspond to 30 and to 10, that is 2 and 10, and so of the rest, always subtracting 28 from the sum when it is greater. Having thus shown the method of constructing this table, we shall now explain the use of it.

In the first place, if the proposed year, the solar cycle of which is required, be in the above table, look for the number opposite to it in the column on the right, marked solar cycle at the top, and add 9 to it: the sum will be the solar cycle required: thus if 9 be added to 12, which stands opposite to the year 2000, we shall have 21 for the solar cycle of that year.

But, if the given year cannot be found exactly in the above table, it must be divided into such parts as are con-

tained in it. If the numbers corresponding to these parts be then added, their sum increased by 9 will give the solar cycle of the required year; provided this sum is less than 28; if greater, 28 must be subtracted from it as many times as possible.

Let it be required, for example, to find by the above table the solar cycle of the year 1813. Divide 1813 into the four following parts 1000, 800, 10, 3, and find the numbers corresponding to them in the right hand columns, which are 20, 16, 10, 3; the sum of these is 49, and 9 added makes 58, from which if 28 be twice subtracted, we shall have for remainder 2, the solar cycle of 1813.

II. The reason of adding 9 to the sum of all these numbers, is because the solar cycle, before the first year of the Christian æra, was 9; consequently this cycle had begun 10 years before the birth of Christ, which may be ascertained in this manner:

Knowing the solar cycle of any year, either by tradition or in any other manner, that of the year 1693, for example, which was 22; subtract 22 from 1693, and divide the remainder 1671 by 28; then subtract 19, which remains, from 28, and the remainder 9 will be the solar cycle before the first year of the Christian æra.

III. A table to show the golden number of any proposed year might be constructed in the same manner; with this difference, that instead of subtracting 28, it would be necessary to subtract 19, because the period of that cycle is 19; and that instead of adding 9, it would be necessary to add only 1; because the golden number, before the first year of the Christian æra, was 1: consequently this cycle began two years before the birth of Christ; that is to say, the golden number for the first year of the Christian æra was 2, &c.

IV. The dominical letter of any proposed year may be found by another method; and when this letter is known,

it will serve to show the letter which corresponds to every day throughout the whole of the same year*.

Divide by 7 the number of days which have elapsed between the first of January and the proposed day inclusively; and if nothing remains, the required letter will be G; if there be any remainder, it will indicate the number of the required letter, reckoning according to the order of the alphabet, A 1, B 2, &c.

Thus, to find the dominical letter of the year 1813; take any Sunday, the 28th of February for example, and find how many days have elapsed between it inclusively, and the first of January: as the number is 59, divide this number by 7, and the remainder 3 will show that C, the third letter of the alphabet, is the dominical letter required.

The days which have elapsed between the first of January and any given period of the year, may be readily found by means of the following table; but it is to be observed that in bissextiles, the number of days must be increased by unity, after the end of February.

	Days.		Days.
From Jan. to Feb. . .	31	From Jan to August .	212
Jan. to March .	59	Jan. to Sept. . .	243
Jan. to April . .	90	Jan. to Octob. . .	273
Jan. to May . .	120	Jan. to Novemb. .	304
Jan. to June . .	151	Jan. to Decemb. .	334
Jan. to July . .	181	Jan. to Jan. . .	365

PROBLEM VII.

To find what day of the Week corresponds to any given day of the Year.

To the given year add its fourth part, or, when it cannot be found exactly, its nearest least fourth part, and to

* It is here to be observed, that when you wish to find the dominical letter, the proposed day must be a Sunday, otherwise you will find only the letter which belongs to some other day

the sum add the number of days elapsed since the first of January, the proposed day included: from the last sum subtract 14, for the present century, and divide what remains by 7: the remainder will indicate the day of the week, counting Sunday 1, Monday 2, Tuesday 3, and so on: if nothing remains, the required day is a Saturday.

Thus, if it be required to know what day of the week corresponded to the 27th of April 1813; add to 1813 its nearest least fourth part 453, and to the sum 2266 add 117, the number of days elapsed between that day inclusive and the 1st of January. If 14 be subtracted from the last sum, which is 2383, and if 2369 which remains be divided by 7; the remainder will be 3: consequently the 27th of April 1813 was a Tuesday.

REMARK.—If the proposed year be between 1582 and 1700, it will be necessary to deduct only 12 from the sum formed as above.

If the year be anterior to 1582, it will be necessary to deduct only 2; because in 1582 ten days were suppressed from the calendar. As a bissextile was suppressed in 1700, which makes an eleventh day suppressed, 13 must be subtracted if the given year be in the last century.

For the same reason 14 must be subtracted in the present century; 15 in the twentieth and twenty-first, and so on.

PROBLEM VIII.

To find Easter-day and the other Moveable Feasts.

By the reformation of the calendar, the 14th day of the paschal moon was brought back to the same season in which it was found at the time of the council of Nice, and from which it had removed more than 4 days. According to the decree of that council, Easter ought to be celebrated on the first Sunday after the 14th day of the moon, if this 14th day should happen on or after the 21st of March. Hence it is obvious that Easter cannot hap-

pen sooner than the 22d of that month, nor later than the 25th of April; which on that account have been called the paschal limits. The following is a table of these limits, from the year 1700 to 1900.

Lunar Cycle.	Paschal Limits	Lunar Cycle	Paschal Limits
1	April 13	11	March 24
2	April 2	12	April 12
3	March 22	13	April 1
4	April 10	14	March 21
5	March 30	15	April 9
6	April 18	16	March 29
7	April 7	17	April 17
8	March 27	18	April 6
9	April 15	19	March 26
10	April 4		

By means of this table Easter may be found in the following manner. First find the golden number or lunar cycle of the year, and opposite to it, in the above table, will be found the day of the month on which the paschal full moon happens in that year. The Sunday immediately following is Easter-day according to the Gregorian calendar. If the full moon happens on a Sunday, Easter-day will be the Sunday following.

Thus, if Easter-day 1813 were required, as the golden number of that year is 9, opposite to it will be found April 15th, and as the 3d following day, or the 18th, is a Sunday, Easter-day happens on the 18th of April.

Second Method.

Easter may be found also by means of the following table, which consists of nine columns, each divided into seven parts. The first column contains the dominical letters, the seven following the epacts, and the ninth the day on which Easter falls.

Table for finding Easter.

	23	22	21	20	19			26 March
	18	17	16	15	14	13	12	2 April
A	11	10	9	8	7	6	5	9 April
	4	3	2	1	*	29	28	16 April
	27	26	25	24				23 April
	23	22	21	20	19	18		7 March
	17	16	15	14	13	12	11	3 April
B	10	9	8	7	6	5	4	10 April
	3	2	1	*	29	[illegible]	27	17 April
	26	25	24					24 April
	23	22	21	20	19	18	17	28 March
	16	15	14	13	12	11	10	4 April
C	9	8	7	6	5	4	3	11 April
	2	1	*	29	28	27	26	18 April
	25	24						25 April
	23							22 March
	22	21	20	19	18	17	16	29 March
D	15	14	13	12	11	10	9	5 April
	8	7	6	5	4	3	2	12 April
	1*	29	28	27	26	25	24	19 April
	23	22						23 March
	21	20	19	18	17	16	15	30 March
E	14	13	12	11	10	9	8	6 April
	7	6	5	4	3	2	1	13 April
	*	29	28	27	26	25	24	20 April
	23	22	21					24 March
	20	19	18	17	16	15	14	31 March
F	13	12	11	10	9	8	7	7 April
	6	5	4	3	2	1	*	14 April
	29	28	27	26	25	24		21 April
	23	22	21	20				25 March
	19	18	17	16	15	14	13	1 April
G	12	11	10	9	8	7	6	8 April
	5	4	3	2	1	*	29	15 April
	28	27	26	25	24			22 April

To use this table, the epact and dominical letter for the given year must be found. Thus if 1813 were proposed, the dominical letter of which is C, and the epact 28; look in one of the cells, opposite to that inscribed C, for the epact 28, and opposite to it will be found, in the last column on the right, the 18th April, which is Easter-day.

Third Method.

If the epact of the proposed year does not exceed 23, subtract it from 44; and the remainder, if less than 31, will give the paschal limits in March; if greater than 31, the surplus will be the paschal limits in April.

But if the epact is greater than 23, subtract it from 43, or from 42 when it is 24 or 25; the remainder will be the day of the paschal limits in April, and the Sunday following will be Easter.

REMARK.—Since all the other moveable feasts are regulated by Easter, when the day on which it falls is known, it will be easy to find the rest. Septuagesima Sunday is 9 weeks or 64 days before it, both the Sundays included. Ash-Wednesday is the 47th day preceding Easter, and the Sunday following Ash-Wednesday is the first Sunday in Lent. Ascension-day is 40 days, Pentecoste or Whit-Sunday is 50 days, and Trinity Sunday is 57 days, after Easter.

PROBLEM IX.

To find on what day of the Week, each Month of the Year begins.

As it has been usual in the calendars to mark the seven days of the week with the first seven letters of the alphabet, always calling the 1st of January A, the 2d B, the 3d C, the 4th D, the 5th E, the 6th F, the 7th G, and so on throughout the year; the letters answering to the first day of every month in the year, according to this disposition, may be known by the following Latin verses:

Astra Dabit Dominus, Gratisque Beabit Egenos,
Gratia Christicolæ Feret Aurea Dona Fideli.

Or by these French verses:

Au Dieu De Gloire Bien Espere;
Grand Cœur, Faveur Aime De Faire

Or by the well known English ones:

At Dover Dwells George Brown Esquire,
Good Caleb Finch And David Fryer.

Where the first letter of each word is that belonging to the first day of each month, in the order from January to December.

Now, as these letters, when the dominical letter is A, indicate the day of the week by the rank which they hold in the alphabet, it is evident in that case that January begins on a Sunday, February on a Wednesday; March on a Wednesday, April on a Saturday, and so on. But when the dominical letter is not A, count either backwards or forwards from the letter of the proposed month, till you come to the dominical letter of the year, and see how many days are between them, for, as the dominical letter indicates Sunday, it will be easy, by reckoning back, to find the day of the week corresponding to the letter of the proposed month.

Thus, if it were required to find on what day of the week February 1813 began, as the dominical letter of 1813 is C, and as the letter corresponding to February is D, which is the one immediately following C, in the order of the alphabet, it is evident that February began on a Monday. In like manner, if April 1813 were proposed, as the letter G which belongs to that month is the third from C, the dominical letter, it may be readily seen that April 1813 began on a Thursday.

The day of the week on which any proposed month begins, may be found also by means of the following table.

MONTHS	A	B	C	D	E	F	G
January	Sunday	Satur	Friday	Thurs	Wedn	Tues	Mond
February	Wedn.	Tues	Mond	Sunday	Satur	Friday	Thurs.
March	Wedn	Tues	Mond	Sunday	Satur	Friday	Thurs
April	Satur.	Friday	Thurs	Wedn	Tues.	Mond	Sunday
May	Mond	Sunday	Satur.	Friday	Thurs	Wedn	Tues
June	Thurs	Wedn	Tues	Mond	Sunday	Satur	Friday
July	Satur.	Friday	Thurs	Wedn	Tues.	Mond	Sunday
August	Tues.	Mond	Sunday	Satur	Friday	Thurs.	Wedn
September	Friday	Thurs	Wedn	Tues	Mond	Sunday	Satur
October	Sund.	Satur.	Friday	Thurs	Wedn	Tues	Mond.
November	Wedn	Tues	Mond.	Sunday	Satur	Friday	Thurs
December	Friday	Thurs	Wedn	Tues	Mond	Sunday	Satur

To use this table, look for the dominical letter of the given year at the top, and in the column below it, and opposite to each month, will be found the day on which it begins. Thus, as the dominical letter for 1813 is c, it will be seen, by inspecting the table, that January began on a Friday, February on a Monday, March on a Monday, April on a Thursday, and so of the rest.

PROBLEM X.

To find what Months of the Year have 31 Days, and those which have only 30.

Raise up the thumb A (pl. 5, fig. 18), the middle finger C, and the little finger E, of the left hand; and keep down the other two, viz, the fore finger B, which is next to the thumb, and the ring-finger D, which is between the middle finger and the little finger. Then begin to count March on the thumb A, April on the fore finger B, May on the middle finger C, June on the ring-finger D, July on the little finger E, and continue to count August on the thumb,

September on the fore-finger, October on the middle finger, November on the ring-finger, and December on the little finger; then beginning again continue to count January on the thumb and February on the fore-finger: all those months which fall on the fingers raised up A, C, E, will have 31 days; and those which fall on the fingers kept down, viz. B and D, will have only 30, except February, which in common years has 28 days, and in bissextiles 29.

The number of the days in each month may be known also by the following memorial lines:

> Thirty days hath September,
> April, June and November;
> All the rest have thirty-one,
> Except February alone

PROBLEM XI.

To find the day of the Month on which the Sun enters into each sign of the Zodiac.

The sun enters into each sign of the zodiac about the 20th of each month of the year; viz, into Aries about the 20th of March, into Taurus about the 20th of April, and so on. To determine this day somewhat more exactly, the two following verses may be employed:

> Inclita Laus Justis Impenditur, Hæresis Horret,
> Grandia Gesta Gerens Felici Gaudet Honore.

Now, to use these two verses, assign the words which they contain to the twelve months of the year, beginning with March; to which you must assign *Inclita*, and end with February, which will correspond to *Honore*. Then consider what is the number in the alphabet of the first letter of each word; for if that number be subtracted from 30, the remainder will be the day of the month required.

For example, *Inclita* corresponds to the month of March, and to the sign Aries; its first letter *I* is the ninth in the alphabet, and if 9 be taken from 30, the remainder 21

shows that the sun enters Aries on the 21st of March. In like manner, *Grandia* corresponds to the month of September, and to the sign Libra, and its first letter *G* is the 7th in the order of the alphabet: if 7 therefore be subtracted from 30, the remainder 23 shows that the sun enters Libra on the twenty-third of January.

But this method is not always correct, erring by a day or two in some months.

PROBLEM XII.

To find the Sun's place, or in what degree and what sign he is on any given day of the Year.

First find on what day of the proposed month the sun enters into any of the signs of the zodiac, and into what sign. When this is done, if the proposed day precedes that day, it will be evident that the sun is then in the preceding sign; for this reason the difference between the day proposed and that when the sun enters a new sign, must be subtracted from 30 degrees, and the remainder will indicate that degree of the preceding sign in which the sun is.

Let the 18th of May, for example, be proposed: it will be found by the preceding problem, that in May the sun enters into the sign Gemini on the 21st; but as the 18th precedes the 21st by 3 days, subtract 3 from 30, and the remainder 27 will indicate that on the 18th of May the sun will be in the 27th degree of Taurus.

But if the proposed time of the month be posterior to the day of the same month on which the sun enters into a new sign, it will then be necessary to take the number of days by which they differ: this will be the degree of the sign in which the sun is, on the given day.

Let us suppose, for example, that the 27th of May is proposed: as the sun on the 21st of May enters into Gemini, and as the difference between 21 and 27 is 6, we may conclude that on the 27th of May the sun is in the 6th degree of Gemini.

PROBLEM XIII.

To find the Moon's place in the Zodiac, on any proposed day of the Year.

First find the sun's place in the zodiac, as taught in the preceding problem; and then the moon's distance from the sun, or the arc of the ecliptic comprehended between the sun and moon, which may be done as follows.

Having found the moon's age, by prob. 5, multiply it by 12, and divide the product by 30. the quotient will give the number of signs, and the remainder the degrees of the moon's distance from the sun. If this distance therefore be counted, according to the order of the signs in the zodiac, beginning at the sun's place, you will have the required place of the moon.

Thus, if it were required to determine the moon's place on the 28th of May 1693, the [illegible] in the 27th degree of Taurus, and the moon's age being 14, multiply 14 by 12, and divide the product 168 by 30, the quotient 5, and the remainder 18, show that the moon's distance from the sun was 5 signs 18 degrees. If 5 signs 18 degrees therefore be counted in the zodiac, from the 27th degree of Taurus, which is the sun's place, we shall fall upon the 15th degree of Scorpio, which was the mean place of the moon.

PROBLEM XIV.

To find to what Month of the Year any lunation belongs.

In the Roman calendar, each lunation is considered as belonging to that month in which it terminates, according to this ancient maxim of the computists.

In quo completur, mensi lunatio detur.

Hence, to determine whether a lunation belongs to a certain month of any given year, as the month of May 1693 for example; having found, by prob. 5, that the moon's age on the last day of May was 27; this age 27

shows that the lunation ends in the next month, that is to say in June, and consequently that it belongs to that month. It indicates also that the preceding lunation ended in the month of May, and therefore belonged to that month.

PROBLEM XV.

To determine the Lunar Years which are common, and those which are embolismic.

This problem may be readily solved by means of the preceding, from which we easily know that the same solar month may have two lunations. For two moons may end in the same month, which has 30 or 31 days, as November, which has 30, or one moon may end the first of that month, and the following moon on the last or 30th of the same month: this year then will have had 13 lunations, and consequently will be embolismic. We shall here give an example.

In the year 1712, the first moon having ended on the 8th of January, the second on the 6th of February, the third on the 8th of March, the fourth on the 6th of April, the fifth on the 6th of May, the sixth on the 4th of June, the seventh on the 4th of July, the eighth on the 2d of August, the ninth on the 1st of September; the tenth on the 1st of October, the eleventh also on the 30th of the same month, the twelfth on the 29th of November, and the thirteenth on the 28th of December; we know that this year, as it had 13 moons, was embolismic.

We know that all the civil lunar years of the new calendar, which begin on the first of January, are embolismic, when they have for epact 29, 28, 27, 26, 25, 24, 23, 22, 21, 19; and also 18, when the golden number is 19.

Thus we know, that in the year 1693, the epact of which was 3, the lunar civil year was embolismic; that is, had 13 moons: this happened because the month of August

had two lunations, one of which ended on the first, and the following one on the 30th of the same month.

PROBLEM XVI.

To find how long the light of the moon will continue during any given night.

Having found the moon's age, by prob. 5, add to it unity, and multiply the sum by 4, if it does not exceed 15; but if it exceeds 15 subtract it from 30, and then multiply the remainder by 4: if the product be divided by 5, the quotient will indicate as many twelfth parts of the night, during which the moon will afford light. These twelfth parts are called unequal hours. They must be counted after sun-set when the moon is increasing: and before sun rising when she is decreasing.

Thus, if it were required to find how long the moon shone on the night of May 21st, 1693, at which time her age was 17; add 1 to 17 and subtract 18 the sum from 30; if 12, the remainder, be multiplied by 4, and if the product 48 be divided by 5, the quotient will give 9 unequal hours and $\frac{3}{5}$, for the time during which the moon afforded light before sun-rise.

If it be required to find how long the moon gave light on the night between the 14th and 15th of February 1730; we must first find the moon's age on the 14th of February, which is 26, and having added 1 to it, the sum will be 27. This sum subtracted from 30, leaves 3 for remainder, which multiplied by 4 gives 12; and if this product be divided by 5, the quotient will be 2 $\frac{2}{5}$ unequal hours; that is to say 8 twelfth parts of the nocturnal arc, which must be reduced to equal and astronomical hours by the following remark.

Remark.—When the length of the given day or night is known, it is easy to reduce unequal hours to equal or astronomical hours, each of which is the 24th part of a natural day, comprehending the day and night. Thus, in

the first example, since the length of the night at London on the 21st of May, is 8 hours 10 minutes; if 8 hours 10 minutes be divided by 12, we shall have 40 minutes 50 seconds, for the value of an unequal hour; this multiplied by $9\frac{3}{5}$, the number of unequal hours during which the moon gave light, from the time of her rising till sun-rise, we shall have 6 equal hours and 32 minutes, as the time comprehended between the rising of the moon and that of the sun.

Corollary.—By these means the time of the moon's rising may be known, provided we know the hour at which the sun rises; for if 12 hours be added to the time of the sun's rising, which is 4 hours 5 minutes, and if from the sum, 16 hours 5 minutes, we subtract 6 hours 32 minutes, which is the time comprehended between the rising of the moon and that of the sun, the result will be 9 hours 33 minutes, for the time of the moon's rising.

PROBLEM XVII.

An easy method of finding the Calends, Nones, and Ides, of any month in the year.

The denomination of Calends, Nones, and Ides, was a singularity in the Roman Calendar; and as these terms frequently occur in classical authors, it may be useful to know how to reduce them to our method of computation. This may be easily done by means of the three following Latin verses.

Principium mensis cujusque vocato calendas
Sex Maius nonas, October, Julius et Mars,
Quatuor at reliqui dabit idus quidlibet octo.

Which have been thus translated into French:

A Mars, Juillet, Octobre et Mai
Six Nones *les gens ont donné,*
Aux autres mois quatre gardé;
Huit Ides *à tous accorde.*

The meaning of these verses is, that the first day of each month is always called *the calends;*

That in the months of March, May, July and October the *nones* are on the 7th day, and in all the other months on the 5th;

Lastly, that the *ides* are 8 days after the nones, viz, on the 15th of March, May, July and October; and on the 13th of the other months.

It must now be observed that the Romans counted the other days backwards; always decreasing, and that they gave the name of nones to those days of the month which were between the calends and nones of that month; that of ides to those days which were between the nones and ides of that month; and the name of calends to those days which remained between the ides and the end of the preceding month.

Thus, in the four months of March, May, July and October, where the nones had 6 days, the second day of the month was called *sexto nonas;* that is to say the sixth day before the nones, the preposition *ante* being here understood. In like manner the third day was called *quinto nonas;* that is to say the fifth day of the nones, or before the nones; and so of the rest. But, instead of calling the sixth day of the month *secundo nonas,* they said *pridie nonas;* that is the day preceding the nones. They said also *postridie calendas,* the day after the calends, *postridie nonas,* the day after the nones; *postridie idus,* the day after the ides.

PROBLEM XVIII.

To find what day of the calends, nones, or ides, corresponds to a certain day of any given month.

To solve this problem, attention must be paid to the remark already made, that all the days between the calends and the nones belong to the nones; that those between the nones and the ides bear the name of ides; and that those between the ides and calends of the following month, have the name of the calends of that month. This being premised, the following method must be pursued.

1st. If the day of the month belongs to the calends, add 2 to the number of the days in the month, and from the sum subtract the given number: the remainder will be the day of the calends.

Thus, for example, to find to what day of the Roman calendar the 25th of May corresponds, it is first to be observed that it belongs to the calends, since it is between the ides of May and the calends of June. As the month of May has 31 days, add 2 to this number, which will make 33; and if 25 be subtracted from the sum, the remainder 8 will show that the 25th of May corresponds to the 8th of the calends of June; that is to say, the 25th of May among the Romans was called *octavo calendas Junii.*

2d. If the day of the month belongs to the ides or the nones, add 1 to the number of days elapsed between the first of the month and the ides or nones inclusively; from this sum subtract the given number, which is the day of the month, and the remainder will be exactly the day of the nones or ides.

We shall suppose, for example, that the given day is the 9th of May, which belongs to the ides; as it is between the 7th day of the nones and the 15th day of the ides. If 1 be added to 15, and 9 be subtracted from the sum 16, the remainder 7 will show that the 9th of May corresponds to the 7th of the ides of that month; that is, the 9th of May among the Romans was called *septimo idus Maii.*

In like manner, if the proposed day be the 5th of May, which belongs to the nones, because it is between the 1st and 7th; add 1 to 7, and from the sum 8, subtract 5, or the given day of the month: the remainder 3 shows that the 5th of May corresponds to the 3d of the nones; or that the Romans called the 5th of May, *tertio nonas Maii.*

PROBLEM XIX.

The day of the calends, ides, or nones, being given, to find the corresponding day of the month.

This problem may be solved by a method similar to that employed in the preceding; but with this difference, that instead of subtracting the day of the month, to obtain that of the calends, &c, the latter is subtracted to obtain the day of the month.

Let it be required for example, to find what day of the month corresponds to the 6th of the calends of June, which the Romans expressed by *sexto calendas Junii.* As the calends are counted in a retrograde order from the 1st of June towards the ides of May, it is evident that the 6th of the calends of June corresponds to some day in the month of May, and as that month has 31 days, add 2 to 31, and from the sum 33, subtract 6, or the given day of the calends: the remainder, 27, shows that the 6th of the calends of June corresponds to the 27th of May.

The same operation must be employed, in regard to the nones and the ides.

REMARK.—The above two questions may be easily solved also by means of a table of the Calends, Nones and Ides, which will be found with other tables at the end of this part.

Of the Cycle of Indiction.

The cycle of indiction is a period of 15 years, distinguished by that name, according to some authors, because it served to indicate the year in which a certain tribute was paid to the Roman republic; and hence it is called the *Roman Indiction.*

It is called also the *pontifical indiction*, because employed by the court of Rome in its bulls, and in all its decrees. The following, it is said, is the origin of this custom. In the year 312, Constantine issued an edict, by which he au-

thorised the exercise of the Christian religion throughout the whole empire. Some years after, the council of Nice was assembled, which in 328 condemned the heresy of Arius: in the space therefore of 15 years Christianity triumphed over persecution and heresy; and on that account it was considered as a memorable period. To preserve the remembrance of it, the cycle of indiction was established; the commencement of which was fixed at the 1st of January 313, to make it begin with the solar year; though the epoch of this cycle, according to the institution of Constantine, had been fixed at the month of September 312, the date of his edict in favour of the Christians. It was the emperor Justinian however who first ordered, that the method of computing by the indiction, should be introduced into the public acts.

But, whatever may have been its origin, which Petau considers as very doubtful, it is certain that the first year of the indiction was the year 313 of the Christian æra. The year 312 therefore must have corresponded to 15 of the indiction, had this method of computation been then in use; and if 312 be divided by 15, the remainder will be 12; which shows that the 12th year of the Christian æra was the 15th of the indiction: consequently this cycle must have begun three years before the birth of Christ; or, in other words, the first year of the Christian æra corresponded to the 4th of the indiction, and hence we have a solution of the following problem.

PROBLEM XX.

To find the number of the Roman indiction which corresponds to any given year.

Add 3 to the given year, and divide the sum by 15: the remainder will indicate the current year of the indiction.

Let it be required, for example, to find the indiction of the year 1813. If 3 be added to 1813, we shall have 1816,

and if this sum be divided by 15, the remainder will be 1. Hence it appears that the indiction for 1818 is 1.

Of the Julian Period, and some other periods of the like kind.

The Julian period is formed by combining together the lunar cycle of 19 years, the solar of 28, and the cycle of indiction of 15. The first year of this period is supposed to have been that which corresponded to 1 of the lunar cycle, 1 of the solar cycle, and 1 of the cycle of indiction.

If the numbers 19, 28 and 15 be multiplied together, the product 7980 will be the number of years comprehended in the Julian period; and we are assured by the laws of combination, that there cannot be in one revolution two of these years which have at the same time the same numbers.

This period is merely an artificial one, invented by Julius Scaliger; but it is convenient on account of its extent, as we can refer to it the commencement of all known æras, and even the creation of the world, were that epoch certain; for according to the common chronology, it was only 3950 years before the Christian æra. But the commencement of the Julian period goes 4714 years beyond that æra; and hence it follows that the creation of the world corresponds to the year 764 of the Julian period.

The method by which it is found that the year of the birth of Jesus Christ was the 4711th of the Julian period, is as follows. It is shown, by a retrograde calculation, that if the three cycles, viz, the solar, lunar, and indiction, had been in use at the birth of Christ, the year in which he was born would have been the 2d of the lunar cycle, the 10th of the solar, and the 4th of the cycle of indiction. But these characters belong to the year 4714 of the above period, as will be seen in the following problem. That year therefore must be adapted to the year of the birth of

Christ; from which if we proceed backwards, calculating the intervals of anterior events, from the profane historians and sacred scriptures, it will be found that there were 3950 years between that period and the creation of Adam. If 3950 then be subtracted from 4714, the remainder will be 764; so that the Julian period is anterior to the creation of the world by 764 years.

PROBLEM XXI.

Any year of the Julian period being given; to find the corresponding year of the lunar cycle, the solar cycle, and the cycle of indiction.

Let the given year of the Julian period be 6522. Divide this number by 19, and the remainder 5, neglecting the quotient, will be the golden number; divide the same number by 28, and the remainder 26 will be the year of the solar cycle; if 6522 be then divided by 15, the remainder 12 will indicate the indiction. If nothing remains, when the given year has been divided, by the number belonging to one of these cycles, that number itself is the number of the cycle. Thus, if the year 6525 were proposed; when divided by 15 nothing remains, and therefore the indiction is 15.

But if it were required to find what year of the Christian æra corresponds to any given year of the Julian period, such for example as 6522, nothing is necessary but to subtract from it 4714; the remainder 1808 will be the number of years elapsed since the commencement of the Christian æra.

All this is so plain that it requires no farther illustration.

PROBLEM XXII.

The lunar and solar cycles and the cycle of indiction corresponding to any year being given, to find its place in the Julian period.

Multiply the number of the lunar cycle by 4200, that

of the solar cycle by 4845, and that of the indiction by 6916.

Add together all these products, and divide the sum by 7980; the number which remains will indicate the year of the Julian period *.

Let the lunar cycle be 2, the solar 10, and the indiction 4; which is the character of the first year of the Christian æra. In this case $4200 \times 2 = 8400$; $4845 \times 10 = 48450$; and $6916 \times 4 = 27664$; the sum of these products is 84514, which divided by 7980, leaves for remainder 4714. The year therefore in the Julian period, to which the above characters correspond, is the 4714th, or the origin of the Julian period is 4713 years anterior to the Christian æra.

Remarks.—I. There is another period, called the Dionysian, which is the product of the lunar cycle 19, and the solar cycle 28; consequently it comprehends 532 years. It was invented by Dionysius Exiguus, about the time of the council of Nice, to include all the varieties of

* The year of the Julian period may be found also by the following general rule. Multiply the golden number by 3780, and the indiction by 1064, subtract the sum of these products from the product of 4845 multiplied by the solar cycle; divide the difference, if it can be done, by 7980, and the remainder will be the year of the Julian period.

The reason of this rule may be found in the solution of the following algebraic problem: To find a number which divided by 28, shall leave for remainder a, divided by 19, shall leave b; and by 15, shall leave c.

Call the three quotients, arising from the division of the required number according to the terms of the problem, x, y, z. Then the number will be $= 28x + a = 19y + b = 15z + c$. From the first equation $28x + a = 19y + b$, we have $y = x + \frac{9x + a - b}{19}$. Now since $\frac{9x + a - b}{19}$ is an integer number, let us suppose it $= m$, then $m = \frac{9x + a - b}{19}$, and $x = 2m + \frac{m - a + b}{19}$, or making $\frac{m - a + b}{19} = n$, or $m = 19n + a - b$, we have by substitution, $x = 19n + 2a - 2b$. Therefore $28x + a = 532n + 57a - 56b = 15z + c$ by the third quotient; and by resolving this equation in the same manner, putting p and q to denote the successive fractions, we shall find the number sought to be $15z + c = 7980q + 4845a - 3780b - 1064c$.

the new moons and of the dominical letters; so that, after 532 years, they were to recur in the same order, which would have been very convenient for finding Easter and the moveable feasts; but as it supposed the lunar cycle to be perfectly correct, which is not the case, this period is no longer used.

II. As among the cycles of the Julian period there is one, viz, that of indiction, which is merely a political institution, that is, which has no relation to the motions of the heavenly bodies, it would have been of more utility perhaps, to substitute in its stead that of the epacts, which is astronomical, and contains 30 years: the number of years of the Julian period would, in this case, have been 15960. This period of 15960 years, was called by the inventor of it, Father John Louis d'Amiens, a capuchin friar, *the period of Louis the Great.* But it does not appear that it met with that reception from chronologists, which the author expected.

Of some Epochs or Periods celebrated in History.

I.

The first of these epochs is that of the Olympiads. It takes its name from the Olympic games, which, as is well known, were celebrated with great solemnity every four years, about the winter solstice, throughout all Greece. These games were instituted by Hercules; but having fallen into disuse, they were revived by Iphitus, one of the Heraclidæ, or descendants of that hero, in the year 776 before Jesus Christ; and after that time they continued to be celebrated with great regularity; till the conquest of Greece by the Romans put an end to them. The æra or epoch of the olympiads, begins therefore at the summer solstice of the year 776 before Christ.

PROBLEM XXIII.

To convert years of the Olympiads into years of the Christian æra, and vice versa.

1st. To solve this problem, subtract unity from the number of the olympiads, and multiply the remainder by 4; then add to the product the number of years of the olympiad which have been completed, and from the last sum subtract 775; or, if the sum be less, subtract it from 776: in the first case, the result will be the current year of the Christian æra, and in the second, the year before that æra.

Let the proposed year, for example, be the third of the 76th olympiad. Unity subtracted from 76 leaves 75, which multiplied by 4 gives for product 300. The complete years of an olympiad, while the third is current, are 2; if 2 therefore be added to 300, we shall have 302. But as 302 is less than 775, we must subtract the former from 776, and the remainder 474, will be the current year before Jesus Christ.

As a second example we shall take the 2d year of the 201st olympiad. If one be subtracted from 201, the remainder is 200; which multiplied by 4 gives 800, and 1 complete year being added makes 801. But 775 subtracted from 801 leaves 26, which is the year of the Christian æra, corresponding to the 2d year of the 201st olympiad.

2d. To convert years of the Christian æra into years of the olympiads, the number of years, if anterior to the birth of Christ, must be subtracted from 776; or, if posterior to that period, 775 must be added to them: if the result be divided by 4, the quotient increased by unity will be the number of the olympiad; and the remainder, also increased by unity, will be the current year of that olympiad.

Let the proposed year, for example, be 1715. By adding 775, the sum is 2490; and this number divided by 4, gives for quotient 622, with a remainder of 2. The year 1715 therefore was the 3d year of the 623d olympiad; or more correctly, the last six months of the year 1715, with the first six months of 1716, corresponded to the 3d year of the 623d olympiad.

II.

The æra of the Hegira is that used by the greater part of the followers of Mahomet: it is employed by the Arabs, the Turks, the various nations in Africa, &c.; consequently, it is necessary that those who study their history, should be able to convert the years of the hegira into those of the Christian æra, and vice versa.

For this purpose, it must be first observed that the years of the hegira are nearly lunar; and as the lunar year, or 12 complete lunations, forms 354 days 8 hours 48 minutes, if the year were always made to consist of 354 or 355 days, the new moon would soon sensibly deviate from the commencement of the year. To prevent this inconvenience, a period of 30 years has been invented, in which there are 19 common years, that is to say of 354 days; and 11 embolismic, or of 355 days. The latter are the 2d, 5th, 7th, 10th, 13th, 15th, 18th, 21st, 24th, 26th, and 29th.

It is to be observed also, that the first year of the hegira began on the 15th of July, 622, of the Christian æra.

PROBLEM XXIV.

To find the year of the Hegira which corresponds to a given Julian year.

To resolve this problem, it must first be observed that 288 Julian years form nearly 235 years of the Hegira.

This being supposed, let us take, as example, the year 1770 of the Christian æra. Now as 621 years complete of our æra had elapsed when the hegira began, we must first subtract these from 1770, and the remainder will be 1149. We must then employ this proportion: if 228 Julian years give 235 years of the hegira, how many will 1149 give: the answer will be 1184, with a remainder of 99 days. The year 1770 therefore, of the Christian æra, corresponded, at least in part, to the year 1184 of the hegira.

On the other hand, if it be required to find the year of the Christian æra which corresponds to a given year of the hegira, the reverse of this operation must be employed: the number thence resulting will be that of the Julian years elapsed since the commencement of the hegira; and by adding 621, we shall have the current year after the birth of Christ.

We shall say nothing further on this subject, but terminate the present article with a few useful tables. The first contains the dates of the principal events recorded in history, and of the commencement of the most celebrated æras; the second is a table of the golden numbers for every year from the birth of Christ to 5600; the third a table of the dominical letters from 1700 to 5600; the fourth a table of the index letters for the same period; the fifth a table of the epacts; and the sixth a table of the calends, nones, and ides.

A TABLE

Of the years of the most remarkable Epochs or Æras and Events.

Remarkable Events	Julian Period.	Years of the World.	Years before Christ
The creation of the world . .	706	0	4007
The deluge, or Noah's flood .	2362	1656	2351
Assyrian monarchy founded by Nimrod	2537	1831	2176
The birth of Abraham . . .	2714	2008	1999
Kingdom of Athens founded by Cecrops	3157	2451	1556
Entrance of the Israelites into Canaan	3262	2556	1451
The destruction of Troy . .	3529	2823	1184
Solomon's temple founded .	3701	2995	1012
The Argonautic expedition .	3776	3070	937
Lycurgus formed his laws .	3829	3103	884
Arbaces 1st king of the Medes	3838	3132	875
Olympiads of the Greeks began	3938	3232	775
Rome built, or Roman æra .	3961	3255	752
Æra of Nabonassar	3967	3261	746
First Babylonish captivity by Nebuchadnezzar	4107	3401	606
The 2d ditto, and birth of Cyrus	4114	3408	599
Solomon's temple destroyed .	4125	3419	588
Cyrus began to reign in Babylon	4177	3471	536
Peloponesian war began . .	4282	3576	431
Alexander the Great died .	4390	3684	323

Remarkable Events.	Julian Period.	Years of the World.	Years before Christ.
Captivity of 100,000 Jews by Ptolemy	4393	3687	320
Archimedes killed at Syracuse	4506	3800	207
Julius Cæsar invaded Britain .	4659	3953	54
He corrected the calendar .	4667	3961	46
The true year of Christ's birth	4709	4003	4

Christian Æra begins here.

Remarkable Events.	Julian Period.	Years of the World.	Years since Christ.
Dionysian, or vulgar æra of Christ's birth	4713	4007	0
Christ crucified, Friday April 3d	4746	4040	33
Jerusalem destroyed . . .	4783	4077	70
Adrian's wall built in Britain	4833	4127	120
Dioclesian epoch, or that of Martyrs	4997	4291	284
The council of Nice . . .	5038	4332	325
Constantine the Great died .	5050	4344	337
The Saxons invited into Britain	5158	4452	445
Hegira, or flight of Mohammed	5335	4629	662
Death of Mohammed . . .	5343	4637	630
The Persian yesdegird . . .	5344	4638	631
Sun, moon, and planets, seen from the earth	5899	5193	1186
Art of printing discovered .	6153	5447	1440
Constantinople taken by the Turks	6166	5460	1453
Reformation begun by Luther .	6230	5524	1517

Remarkable Events.	Julian Period.	Years of the World.	Years since Christ.
The calendar corrected by Pope Gregory	6295	5589	1582
Sir Isaac Newton born . . .	6355	5649	1642
Made president of the Royal Society	6416	5710	1703
Died, March 20th	6440	5734	1727
New planet discovered by Herschel	6494	5788	1781
New planet discovered by Piazzi	6514	5808	1801
New planet discovered by Olbers	6515	5809	1802
New planet discovered by Harding	6517	5811	1804
Second new planet discovered by Olbers	6520	5814	1807

Table of some other remarkable events, relating chiefly to the Arts and Sciences.

Event	A.D.
Use of bells introduced into churches	605
Alexandrian library destroyed and Egypt conquered by the Saracens	641
Organs first used in churches	660
Glass invented by a bishop and brought to England by a Benedictine monk	663
Arabic cyphers introduced into Europe by the Saracens	991
Astronomy and Geography brought to Europe by the Moors	1120
Algebra brought to Europe from Arabia	1200
Silk manufacture introduced at Venice from Greece	1209
Spectacles invented by a monk of Pisa	1299
The mariner's compass invented or improved by Flavio	1302

Gunpowder invented by a monk of Cologne . . 1330
The art of weaving cloth brought from Flanders to England 1331
Cannon first used in the English service by the Governor of Calais 1383
First company of linen-weavers settled in England 1386
Cards invented for the amusement of the French king 1391
Great guns first used in England at the siege of Berwick 1405
Paper made of linen rags invented 1417
Printing invented in Germany 1440
Engraving and etching invented 1459
Cape of Good Hope discovered 1488
Geographical maps and sea charts brought to England 1489
America discovered by Columbus 1492
First voyage round the world by Magellan . . . 1522
Variation of the compass discovered by Cabot . . 1540
Iron cannon and mortars made in England . . . 1543
Glass first manufactured in England 1557
First proposal of settling a colony in America . . 1583
Bomb-shells invented at Venloo 1588
Telescopes invented by Jansen, a spectacle-maker of Holland 1590
Art of weaving stockings invented by Lee in Cambridge 1590
Watches brought to England from Germany . . 1597
Thermometers invented by Drebbel, a Dutchman 1610
Galileo first observed three of Jupiter's satellites, Jan. 7th 1610
Logarithms invented by Lord Napier of Scotland . 1614
Circulation of the blood discovered by Harvey . . 1619
Gazettes first published at Venice 1630
Transit of Mercury over the sun's disk first observed by Gassendi, Nov. 17th 1631

	A.D.
Galileo condemned by the inquisition	1633
French academy established, January	1635
Transit of Mercury observed by Cassini, Nov. 11th	1636
Polemoscope invented by Hevelius	1637
Transit of Venus observed by Horrox, Nov. 24th .	1639
Barometers invented by Toricelli	1643
Royal academy of painting founded by Louis XIV.	1643
Galileo first applied the pendulum to clocks . .	1649
Air pump invented by Otto Gueric of Magdeburg	1654
Huygens first discovered a satellite of Saturn, March 25th	1655
Royal Society of London established, July 15th .	1663
Royal academy of inscriptions and belles-lettres founded	1663
Academy for sculpture established in France . .	1664
The observatory of Paris founded	1664
Magic lantern invented by Kircher	1665
Academy of sciences established in France . . .	1666
Cassini discovered 4 of Saturn's satellites in the course of a few years	1671
The royal observatory at Greenwich built . . .	1676
The anatomy of plants made known by Grew .	1680
The Newtonian philosophy was published . . .	1686
The academy of Sciences founded at Berlin . .	1701
Academy of sciences established at Petersburg .	1724
Aberration of the fixed stars discovered and accounted for by Bradley	1727
Transit of Mercury observed by Cassini, Nov. 11th	1736
Academy of sciences founded at Stockholm . .	1750
New style introduced into Great Britain, Sept. 3d being reckoned Sept. 14th	1752
British Museum established at Montague-House .	1753
Transit of Venus over the sun, June 6th . . .	1760
Royal academy of arts established at London . .	1768
Transit of Venus over the sun's disk, June 3d . .	1769

Eminent British Philosophers and Mathematicians.

	Died.
Arbuthnot, John, M.D.	1705
Bacon, Roger, philosopher	1294
Bacon, Lord, ditto	1626
Barrow, Isaac, mathematician	1677
Boyle, Robert, phil.	1691
Brerewood, Edward, phil. and math.	1613
Briggs, Henry, math.	1630
Cheyne, George, phys. and phil.	1748
Clark, Samuel, phil. and math.	1729
Cook, James, navigator	1779
Derham, William, phil.	1735
Dudley, Sir Robert, phil. and math.	1639
Evelyn, John, phil.	1706
Ferguson, James, phil. and mech.	1776
Graham, George, math. and mech.	1751
Gregory, James, prof. St. Andrew's	1675
Gregory, David, prof. Oxford, astronomy	1708
Gunter, Edmund, astron.	1626
Hales, Stephen, phil.	1761
Halley, Edmund, astron.	1742
Harriot, Thomas, math.	1621
Harrison, John, inventor of the time-keeper	1776
Harvey, William, phys. dis. circ. of the blood	1657
Horrox, Jeremiah, astron.	1641
Keil, John, math. and astron.	1721
Locke, John, phil.	1704
Long, Robert, astron.	1770
Lyons, Israel, math.	1775
Maclaurin, Colin, math.	1746
Newton, Sir Isaac, math. and phil.	1727
Pell, John, math.	1685

	Died.
Pemberton, Henry, phil.	1771
Ray, John, phil.	1705
Simpson, Thomas, math.	1761
Watts, Isaac, phil. and math.	1748
Whiston, William, astron.	1752
Wilkins, John, phil.	1672
Wren, Sir Christopher, math.	1723

TABLE OF THE

For every year since the birth of Christ,

The centenary years; that is, the last years of each century.						0	100	200	300	400	500
						1900	2000	2100	2200	2300	2400
						3800	3900	4000	4100	4200	4300
Intermediate years.						GOLDEN					
						1	6	11	16	2	7
1	20	39	58	77	96	2	7	12	17	3	8
2	21	40	59	78	97	3	8	13	18	4	9
3	22	41	60	79	98	4	9	14	19	5	10
4	23	42	61	80	99	5	10	15	1	6	11
5	24	43	62	81		6	11	16	2	7	12
6	25	44	63	82		7	12	17	3	8	13
7	26	45	64	83		8	13	18	4	9	14
8	27	46	65	84		9	14	19	5	10	15
9	28	47	66	85		10	15	1	6	11	16
10	29	48	67	86		11	16	2	7	12	17
11	30	49	68	87		12	17	3	8	13	18
12	31	50	69	88		13	18	4	9	14	19
13	32	51	70	89		14	19	5	10	15	1
14	33	52	71	90		15	1	6	11	16	2
15	34	53	72	91		16	2	7	12	17	3
16	35	54	73	92		17	3	8	13	18	4
17	36	55	74	93		18	4	9	14	19	5
18	37	56	75	94		19	5	10	15	1	6
19	38	57	76	95		1	6	11	16	2	7

GOLDEN NUMBERS,

to the year 5600.

600	700	800	900	1000	1100	1200	1300	1400	1500	1600	1700	1800
2500	2600	2700	2800	2900	3000	3100	3200	3300	3400	3500	3600	3700
4400	4500	4600	4700	4800	4900	5000	5100	5200	5300	5400	5500	5600
NUMBERS.												
12	17	3	8	13	18	4	9	14	19	5	10	15
13	18	4	9	14	19	5	10	15	1	6	11	16
14	19	5	10	15	1	6	11	16	2	7	12	17
15	1	6	11	16	2	7	12	17	3	8	13	18
16	2	7	12	17	3	8	13	18	4	9	14	19
17	3	8	13	18	4	9	14	19	5	10	15	1
18	4	9	14	19	5	10	15	1	6	11	16	2
19	5	10	15	1	6	11	16	2	7	12	17	3
1	6	11	16	2	7	12	17	3	8	13	18	4
2	7	12	17	3	8	13	18	4	9	14	19	5
3	8	13	18	4	9	14	19	5	10	15	1	6
4	9	14	19	5	10	15	1	6	11	16	2	7
5	10	15	1	6	11	16	2	7	12	17	3	8
6	11	16	2	7	12	17	3	8	13	18	4	9
7	12	17	3	8	13	18	4	9	14	19	5	10
8	13	18	4	9	14	19	5	10	15	1	6	11
9	14	19	5	10	15	1	6	11	16	2	7	12
10	15	1	6	11	16	2	7	12	17	3	8	13
11	16	2	7	12	17	3	8	13	18	4	9	14
12	17	3	8	13	18	4	9	14	19	5	10	15

TABLE OF THE
from 1700

Centenary years; that is, the last years of each century.				1700 2100 2500 2900 3300 3700 4100 4500 4900 5300
Intermediate years.				C
1	29	57	85	B
2	30	58	86	A
3	31	59	87	G
4	32	60	88	FE
5	33	61	89	D
6	34	62	90	C
7	35	63	91	B
8	36	64	92	AG
9	37	65	93	F
10	38	66	94	E
11	39	67	95	D
12	40	68	96	CB
13	41	69	97	A
14	42	70	98	G
15	43	71	99	F
16	44	72		ED
17	45	73		C
18	46	74		B
19	47	75		A
20	48	76		GF
21	49	77		E
22	50	78		D
23	51	79		C
24	52	80		BA
25	53	81		G
26	54	82		F
27	55	83		E
28	56	84		DC

DOMINICAL LETTERS,

to 5600.

1800 2200 2600 3000 3400 3800 4200 4600 5000 5400	1900 2300 2700 3100 3500 3900 4300 4700 5100 5500	2000 2100 2800 3200 3600 4000 4400 4800 5200 5600
E	G	BA
D C B AG	F E D CB	G F E DC
F E D CB	A G F ED	B A G FE
A G F ED	C B A GF	D C B AG
C B A GF	E D C BA	F E D CB
E D C BA	G F E DC	A G F ED
G F E DC	B A G FE	C B A GF
B A G FE	D C B AG	E D C BA

TABLE OF THE INDEX LETTERS,

from 1700 *to* 5600.

c	1700	Metemptosis *	p	3700	Met.
c	1800	M. proemtosis †	n	3800	Met.
B	1900	Met.	n	3900	Met. & proem.
B	2000	Bissextile	n	4000	Bissextile
B	2100	Met. & proem.	m	4100	Met.
A	2200	Met.		4200	Met.
u	2300	Met.	l	4300	Met. & proem.
A	2400	Bissex. & proem.	l	4400	Bissextile
u	2500	Met.	k	4500	Met.
t	2600	Met.	k	4600	Met. & proem.
t	2700	Met. & proem.	i	4700	Met.
t	2800	Bissextile	i	4800	Bissextile
s	2900	Met.	i	4900	Met. & proem.
s	3000	Met. & proem.	h	5000	Met.
r	3100	Met.	g	5100	Met.
r	3200	Bissextile	h	5200	Bissext. & proem.
r	3300	Met. & proem.	g	5300	Met.
q	3400	Met.	f	5400	Met.
p	3500	Met.	f	5500	Met. & proem.
q	3600	Bissext. & proem.	f	5600	Bissextile.

* Metemptosis, or the solar equation, is the suppression of a day. There was a metemptosis in the year 1800, because that year which ought naturally to have been bissextile, was not so. Since the reformation of the calendar it takes place three times in 400 years.

† Proemptosis, or the lunar equation, is the anticipation of the new moon. There is a proemptosis in about every 300 years, because the new moon takes place then a day sooner than it ought to do.

TABLE of the Epacts from 1700 to 5600.

Index Letters	GOLDEN NUMBERS. i	ii	iii	iv	v	vi	vii	viii	ix	x	xi	xii	xiii	xiv	xv	xvi	xvii	xviii	xix
	EPACTS.																		
C	*	xi	xxii	iii	xiv	xxv	vi	xvii	xxviii	ix	xx	i	xii	xxiii	iv	xv	xxvi	vii	xviii
B	xxix	x	xxi	ii	xiii	xxiv	v	xvi	xxvii	viii	xix	*	xi	xxii	iii	xiv	xxv	vi	xvii
A	xxviii	ix	xx	i	xii	xxiii	iv	xv	xxvi	vii	xviii	xxix	x	xxi	ii	xiii	xxiv	v	xvi
u	xxvii	viii	xix	*	xi	xxii	iii	xiv	xxv	vi	xvii	xxviii	ix	xx	i	xii	xxiii	iv	xv
t	xxvi	vii	xviii	xxix	x	xxi	ii	xiii	xxiv	v	xvi	xxvii	viii	xix	*	xi	xxii	iii	xiv
s	xxv	vi	xvii	xxviii	ix	xx	i	xii	xxiii	iv	xv	xxvi	vii	xviii	xxix	x	xxi	ii	xiii
r	xxiv	v	xvi	xxvii	viii	xix	*	xi	xxii	iii	xiv	xxv	vi	xvii	xxviii	ix	xx	i	xii
q	xxiii	iv	xv	xxvi	vii	xviii	xxix	x	xxi	ii	xiii	xxiv	v	xvi	xxvii	viii	xix	*	xi
p	xxii	iii	xiv	xxv	vi	xvii	xxviii	ix	xx	i	xii	xxiii	iv	xv	xxvi	vii	xviii	xxix	x
n	xxi	ii	xiii	xxiv	v	xvi	xxvii	viii	xix	*	xi	xxii	iii	xiv	xxv	vi	xvii	xxviii	ix
m	xx	i	xii	xxiii	iv	xv	xxvi	vii	xviii	xxix	x	xxi	ii	xiii	xxiv	v	xvi	xxvii	viii
l	xix	*	xi	xxii	iii	xiv	xxv	vi	xvii	xxviii	ix	xx	i	xii	xxiii	iv	xv	xxvi	vii
k	xviii	xxix	x	xxi	ii	xiii	xxiv	v	xvi	xxvii	viii	xix	*	xi	xxii	iii	xiv	xxv	vi
i	xvii	xxviii	ix	xx	i	xii	xxiii	iv	xv	xxvi	vii	xviii	xxix	x	xxi	ii	xiii	xxiv	v
h	xvi	xxvii	viii	xix	*	xi	xxii	iii	xiv	xxv	vi	xvii	xxviii	ix	xx	i	xii	xxiii	iv
g	xv	xxvi	vii	xviii	xxix	x	xxi	ii	xiii	xxiv	v	xvi	xxvii	viii	xix	*	xi	xxii	iii
f	xiv	xxv	vi	xvii	xxviii	ix	xx	i	xii	xxiii	iv	xv	xxvi	vii	xviii	xxix	x	xxi	ii

A TABLE of the CALENDS, NONES, and IDES.

Days of the Month.	Apr. June, Sept. Nov.	Jan. Aug. December.	March, May, July, Oct.	February.
1	Calendæ	Calendæ	Calendæ	Calendæ
2	IV	IV	VI	IV
3	III	III	V	III
4	Prid. Non.	Prid. Non.	[illegible]	Prid. Non.
5	Nonæ	Nonæ	III	Nonæ
6	VIII	VIII	Prid. Non.	VIII
7	VII	VII	Nonæ	VII
8	VI	VI	VIII	VI
9	V	V	VII	V
10	IV	IV	VI	IV
11	III	III	V	III
12	Prid. Id.	Prid. Id.	IV	Prid. Id.
13	Idus	Idus	III	Idus
14	XVIII	XIX	Prid. Id.	XVI
15	XVII	XVIII	Idus	XV
16	XVI	XVII	XVII	XIV
17	XV	XVI	XVI	XIII
18	XIV	XV	XV	XII
19	XIII	XIV	XIV	XI
20	XII	XIII	XIII	X
21	XI	XII	XII	IX
22	X	XI	XI	VIII
23	IX	X	X	VII
24	VIII	IX	IX	VI
25	VII	VIII	VIII	V
26	VI	VII	VII	IV
27	V	VI	VI	III
28	IV	V	V	Prid. Cal.
29	III	IV	IV	Martii.
30	Prid. Cal.	III	III	
31	Mensis sequentis	Prid. Cal. Mens. seq.	Prid. Cal. Mens. seq.	

USE OF THE FOREGOING TABLES.

1st. *Table of the Golden Numbers.*

This table contains the centenary years, that is to say, the last years of each century, arranged in cells at the top, and the intermediate years in the ten cells on the left hand. The centenary years which have the same golden number, are placed in different cells, but below each other in a line, as 1800, 3700, 5600. The golden numbers belong, some to the centenary years, and others to the intermediate years. The former are placed in a row by themselves below the centenary years, and are as follow: 1, 6, 11, 16, 2, 7, 12, 17, 3, 8, 13, 18, 4, 9, 14, 19, 5, 10, and 15. The latter will be found in a line with the intermediate years distributed in 30 different cells.

I. Now to find the golden number of a centenary year, for example 1800; first look for the centenary year in the cell to which it belongs, and immediately below it, in the row at the bottom standing by itself, will be found 15, which was the golden number of that year.

II. To find the golden number of an intermediate year, 1813 for example. Find the centenary year 1800 in its proper cell, and the intermediate year 13 in the cells on the left hand; then on a line with 13, and exactly below 1800, will be found 9, the golden number of 1813. It is here to be observed that this table, though printed on two separate pages, for the sake of convenience, forms only one, the lines in each page being so arranged as to correspond. The case is the same with the following table of the dominical letters.

2d. *Table of the Dominical Letters.*

The centenary years are arranged in this table, as in the preceding, in the four cells at the top, and the intermediate years in the seven cells on the left. All the centenary years which have the same dominical letter, are e-

ranged together in one cell. Those which have C for dominical letter in the first; those which have E in the second, those which have G in the third, and those which have BA in the fourth. As in 40 centenary years, the number comprehended in this table, there are 10 bissextiles, these 10 years have been placed in the fourth cell, and the other 30 in the first three. The intermediate years placed horizontally in the same cell differ by 28 years, because the solar cycle contains only that number. Thus the difference between 1 and 29 in the first cell, is 28, and the case is the same with 29 and 57, &c. Each collateral cell contains four perpendicular rows, consisting each of four numbers, because a bissextile recurs every four years. The four first dominical letters, in the four upper cells, viz, B, D, F, G, correspond to the numbers 1, 29, 57, 85, in the first cell of intermediate years; the case is the same with the dominical letters in the next row, A, C, E, F, in regard to the numbers 2, 30, 58, 86; and so on throughout the table.

I. To find the dominical letter of a centenary year, 1800 for example. Look for 1800, which stands in the second cell at the top, and immediately below it will be found the letter E.

II. To find the dominical letter of an intermediate year, as 1813. First find the centenary year 1800 in its proper cell; then look for 13 among the intermediate years, on a line with which, and below the cell containing 1800, will be found the letter C.

3d. *Table of the Index Letters, and Table of the Epacts.*

The use of the first of these tables will readily appear, when we have explained the nature of the second. The table of the epacts contains the golden numbers in the horizontal column at the top: the index letters are arranged in the first perpendicular column, and the epacts in columns parallel to it. Now if the epact of any year be re-

quired; first find the golden number of the proposed year, and, in the table of index letters, the letter corresponding to the century; then look for the same letter in the table of the epacts, and also for the golden number at the top; and on a line with the index letter, and directly below the golden number, will be found the epact required.

Let it be proposed, for example, to find the epact of 1813, the golden number of which is 9. Look in the table of the index numbers, and it will be found that the letter corresponding to 1800 is c; then find c in the first column on the left of the table of epacts; and on a line with it, and directly below ix among the golden numbers, will be found xxviii, the epact of the year 1813. The epact of any other year, till the year 5600, may be found in like manner.

4th. *Table of the Calends, Nones, and Ides.*

This table requires little explanation. look for the given month at the top, and in the column below it, and opposite to the proposed day, will be found the corresponding day of the Roman calendar. The day of our calendar, corresponding to any given day of the Roman calendar, may be found with the same ease.

MATHEMATICAL

AND

PHILOSOPHICAL

RECREATIONS.

PART SEVENTH.

Containing the most useful and interesting Problems in Gnomonics or Dialling.

GNOMONICS or Dialling is the art of tracing out on a plane, or even on any surface whatever, a sun dial; that is, a figure, the different lines of which, when the sun shines, indicate by the shadow of a style the different hours of the day. This science depends therefore on geometry and astronomy, or at least on a knowledge of the sphere.

As many people construct sun-dials without having a clear idea of the principle which serves as a basis to this part of the mathematics, it may not be improper to begin with an explanation of it.

The General Principle of Sun-Dials.

Conceive a sphere, with its 12 horary circles or meridians, which divide the equator, and consequently all its parallels, into 24 equal parts. Let this sphere be placed in a situation suited to the position of the dial; that is, let

its axis be directed to the pole of the place for which the dial is constructed, or elevated at an angle equal to the latitude. Now if we suppose a horizontal plane cutting the sphere through its centre, the axis of the sphere will represent the style, and the different intersections of the horary circles with that plane will be the hour-lines; for it is evident, that if the planes of these circles were infinitely produced, they would form in the celestial sphere the horary circles, which divide the solar revolution into 24 equal parts. When the sun therefore has arrived at one of these circles, that of 3 in the afternoon for example, he will be in the plane of the similar circle of the sphere above mentioned; and the shadow of the axis or style will fall upon the line of intersection, which that circle forms with the horizontal plane: this line then will be the line of 3 o'clock; and so of the rest.

All this is illustrated in fig. 1 plate 1, which represents a part of the sphere, with six of the horary circles. P *p* is the axis, in which all these circles intersect each other; AHB *h* the horizontal plane, or horizon of the sphere, indefinitely continued; AB the meridian; DE the diameter of the equator, which is in the meridian; and DHE *h* the circumference of the equator, of which DHE is a half, and DH a quarter. This quarter of the equator is divided into six equal parts, D 1, 1 2, 2 3, 3 4, 4 5, 5 6, and through these pass the horary circles, the planes of which evidently cut the horizon in the lines C 1, C 2, C 3, C 4, C 5, C 6: these are the hour-lines; and if we suppose them continued to AF, which is perpendicular to the meridian CA, they will give the hour-lines C I, C II, C III, C IV, C V, C VI. The style will be a portion CS of the axis of the sphere; which consequently ought to form with the meridian, and in its plane, an angle SCA, equal to the height of the pole or PCA.

Should this reasoning appear too dry and tedious, another method may be employed to acquire a clear idea of

the principles of dialling. Construct a solid sphere, divided by its 12 horary circles, and cut it in such a manner that one of its poles shall form with the plane of the section an angle equal to the height of the pole of the given place.

If the sphere, cut in this manner, be then made to rest on a horizontal plane, with its pole directed towards the pole of the world, the points where the horary circles intersect the horizontal plane, will be readily seen; and the common section of all the circles, which is the axis, will show the position of the style.

For the sake of illustration, we have here supposed the section of the sphere to be formed by a horizontal plane; but if the plane were vertical, the case would be similar, and the lines of intersection would be the hour-lines of a vertical dial. If the plane be declining or inclining, we shall have a declining or inclining dial: it may even be easily seen that this holds good in regard to every surface, whatever be its form, convex, concave, or irregular, and whatever may be its position.

The style is an iron rod, generally placed in an inclined direction, the shadow of which serves to point out the hours: as before said, it is a portion cs of the axis of the sphere; and in that case it shows the hour by the shadow of its whole length.

An upright style, however, such as SQ, is sometimes given to dials; but in that case it is only the shadow of the summit s that indicates the hour, because this summit is a point of the axis of the sphere.

The centre of the dial is the point c where all the hour-lines meet. It sometimes happens, however, that these lines do not meet. This is the case in dials which have their plane parallel to the axis of the sphere; for it is evident that in such dials the intersections of the horary circles must be parallel lines. These dials are called *dials without a centre.* Vertical east and west dials, and dials turned directly towards the south, and inclined to the ho-

rizon at an angle equal to the latitude, or which if produced would pass through the pole, are of this number.

The meridian line, as is well known, is the intersection of the plane of the meridian with the plane of the dial; when the plane of the dial is vertical, it is always perpendicular to the horizon.

The substylar line, is that marked out by the plane perpendicular to the plane of the dial, and passing through the style. As this line is of great importance in declining dials, it is necessary to have a very distinct idea of it. For this purpose, conceive a perpendicular let fall on the plane of the dial, from any point in the style; and that a plane is made to pass through the style and the perpendicular: this plane, which will necessarily be perpendicular to that of the dial, will cut it in a line passing through the centre, and through the bottom of the perpendicular, and this line will be the substylar line.

This line is the meridian of the plane; that is, it shows the moment at which the elevation of the sun above that plane is greatest. Care however must be taken not to confound this meridian with the meridian of the place, or the south line of the dial; for the latter is the intersection of the plane of the dial with the meridian of the place, which is the plane passing through the zenith of the place and the pole; whereas the meridian of the plane of the dial, is the intersection of that plane with the meridian, or the horary circle passing through the pole and the zenith of the plane.

In the horizontal plane, or any other which has no declination, the substyle and the meridian of the place coincide; but in every plane not turned directly towards the south or the north, these lines form greater or less angles.

Lastly, the equinoctial is the intersection of the plane of the equator with the dial: it may easily be seen that this line is always perpendicular to the substyle.

PROBLEM I.

To find the Meridian Line on a Horizontal Plane.

To find the meridian line, is the basis of the whole art of constructing sun-dials; but as it is at the same time the basis of all astronomical operations, and as we have already treated of it at full length in that part of this work which relates to astronomy, it would be needless to repeat here what has been already said on the subject. We shall therefore confine ourselves to one ingenious and little-known operation.

We shall give also hereafter a method of determining the position of the meridian line at all times, and in all places, provided the latitude be known.

PROBLEM II.

To find the Meridian by the Observation of three Unequal Shadows.

The meridian line on a horizontal plane is found generally by means of two equal shadows of a perpendicular style; the one observed in the forenoon and the other in the afternoon. For this purpose, several concentric circles are described from the bottom of the style; but notwithstanding this precaution, it may happen that it will be impossible to have two shadows equal to each other. This inconvenience however may be remedied by three observations, instead of two. For this ingenious method, we are indebted to a very old author on Gnomonics, named *Muzio oddi da Urbino,* who published it in a treatise entitled *Gli Orologi solari nelle superficie piane.* This author was exceedingly devout; for he piously thanks Our Lady of Loretto for having communicated to him, by inspiration, the precepts he has taught in his work. The operation is as follows.

Let P (pl. 2 fig. 2) be the bottom of the style, and PS its height; and let three shadows projected by it be PA, PB

and PC; which suppose to be unequal, and let PC be the shortest of them. From the point P draw PD, PE and PF perpendiculars to PA, PB and PC, and all equal to each other, as well as to PS. Draw also the lines DA, EB and FC, on the two largest of which, viz, DA and EB, assume DG and EH equal to PC; then from G and H draw GI and HK perpendiculars to PA and PB, and join the points I and K by an indefinite line: make IM and KL perpendicular to IK, and equal to GI and KH; and draw ML, which will meet IK in the point N: if through N and C the line CN be drawn, it will be perpendicular to the meridian; consequently by drawing, from P, the line PO, perpendicular to CN, it will be the meridian required.

As the demonstration of this problem would be too long, we must refer the reader to the fifth book of a work by Schooten, entitled *Exercitationes Mathematicæ.*

PROBLEM III.

To find the Meridian on a Plane, or the Substylar Line.

After what has been already said in regard to the substylar line, this operation will be easy; for since this line is the meridian of the plane, nothing is necessary but to consider it as if it were horizontal, and to trace out on it the meridian by the same method: the line resulting will be the substyle, the determination of which is very necessary for constructing inclined or declining dials, and those which are both at the same time.

PROBLEM IV.

To describe an Equinoctial Dial.

From any point C (pl. 2, fig. 3), as a centre, describe a circle AEDB; and having drawn the two diameters intersecting each other at right angles in the centre C, divide each quadrant into six equal parts; and draw the radii C1, C2, C3, and so on as seen in the figure. These radii will show the hours by means of a style perpendicu-

lar to the plane of the dial, which must be placed in the plane of the equator; that is, in such a manner as to form with the horizon an angle equal to the complement of the latitude. The line AD must coincide with the plane of the meridian, and the point A must be directed towards the south.

REMARKS.—I. When this equinoctial dial is erected, if the hour-lines look towards the heavens, it is called a *superior* dial, but if they are turned towards the earth, an *inferior*.

II. A superior equinoctial dial shows the hours of the day only in the spring and summer; and an inferior one only during the autumn and winter; but at the equinoxes, when the sun is in the equator, or very near it, equinoctial dials are of no use, as at those periods they are never illuminated by the sun.

III. It is well known that at London the elevation of the plane of the equator is 38° 29′, which is the complement of the elevation of the pole: the angle therefore which the plane of an equinoctial dial at London should form with the horizon, ought to be 38° 29′.

IV. It hence appears that it is easy to construct a universal equinoctial dial, which may be adjusted to any elevation of the pole whatever. For this purpose, join together two pieces of ivory, or copper, or any other matter, ABCD and CDEF, (pl. 2, fig. 4), by means of a hinge at CD: then describe on the two surfaces of the piece ABCD, two equinoctial dials; and in the centre I, place a style extending both ways in a direction perpendicular to ABCD. At G, in the middle of the piece CDEF, fix a magnetic needle, covered with a plate of glass, and towards the edge of the same piece apply a quadrant HL divided into degrees, and passing through an aperture H, made to receive it in the upper piece ABCD. The degrees and minutes must begin to be counted from the point L.

When this dial is to be used, place the needle in the

meridian, making a proper allowance for the declination; and cause the two pieces ABCD and CDEF to form an angle BCF, equal to the elevation of the equator at the given place; that is, equal to the complement of the latitude. If care be then taken to turn the quadrant towards the south, either of these equinoctial dials will show the hours at that place, except on the day of the equinox.

PROBLEM V.

To find the divisions of the hour-lines on a horizontal dial, with only two extents of the compasses.

Draw the meridian SM, (pl. 2, fig. 5), and from the point C, assumed towards the middle, as a centre, describe the circle ETOP with the radius CE, the first opening of the compasses; then from O as a centre, with a radius equal to the diameter OE of the first circle, describe the circle EAMB; and from the point E as a centre, with the same radius, the circle AOBS: these two circles will cut each other in A and B, which will be the centres of two other equal circles, XIEF and ZLEG. Through the points of intersection F and G, draw the lines EG and EF; and through the points A and B the straight line XACBZ. This line, which will be the equinoctial, will be cut both by the above circles, and by the lines EG and EF, and the centre C of the first circle, in 11 points, which will be those of the hours; they must therefore be marked with the numbers 7, 8, 9, 10, 11, 12, 1, 2, 3, 4, 5.

The next thing is to find the centre of the dial, of which the above points are the horary divisions: this is to be done in the following manner:

In the circle ETOP assume, towards T or P, an arc EK equal to the complement of the latitude or elevation of the pole, that is, equal to 38° 29′ if the latitude be 51° 31′, and draw CK: if KV be then drawn perpendicular to CK, it will cut the meridian in V, which will be the centre of the dial; so that by drawing, from the point V, the lines V 7,

v 8, v 9, &c, we shall have the hour-lines from 7 in the morning till 5 in the afternoon. If a line be drawn through the point v parallel to the equinoctial, it will be the line of 6 o'clock. The hour-lines of 7 and 8 in the morning, continued beyond the centre v, will give those of 7 and 8 in the evening; and those of 4 and 5 in the evening, if continued in the same manner, will give 4 and 5 in the morning. In the last place, if from the point v, or any other taken at pleasure, two circles be described, they will serve to terminate the hour-lines, and to contain the numbers belonging to the different hours.

PROBLEM VI.

To construct the same Dial with one Opening of the Compasses.

Through the point C (pl. 3, fig. 6) draw two lines SM, 7 5, perpendicular to each other, and, from the same point as a centre, describe the circle EIOP, with any opening of the compasses whatever: then, keeping the opening the same, place one point of the compasses in O and the other in Q, from Q turn to the point 4, and making two turns from 4 to 5, proceed back from 5 to 11 by four turns.

Then, placing the compasses on O and N, turn from N to 8, and making two turns from 8 to 7, proceed from 7 to 1 by four turns. If the lines EN and EQ be then drawn, which will give, on the line 7 5, the hours of 2 and 10, the dial will be constructed. The centre of it may be found by the operation described in the preceding problem.

PROBLEM VII.

Construction of the most important of the other Regular Dials.

Regular dials are those which have the hour-lines, forming equal angles on each side of the meridian: these dials therefore are, the equinoctial, the horizontal, the north and south vertical, and the polar. Having already spoken

of the equinoctial and horizontal, we shall now proceed to the north and south vertical dials.

Of the South Vertical Dial.

If the vertical dial be turned directly towards the south; then make the angle ECK or the arc EK (pl. 2, fig. 5) equal to the height of the pole; if CKV be then made a right angle, the point V will be the centre of the dial; and the angle CVK, which will then be equal to the complement of the latitude or of the elevation of the pole, will denote the angle which the style, in the plane of the meridian, ought to form with the plane of the dial.

Of the North Vertical Dial.

If the vertical dial be north; make, as before, the angle OC*k* (pl. 2, fig. 5) equal to the height of the pole, and the angle C*k*H a right angle: the point H will be the centre of the dial; and the angle CH*k* will be that which the style forms with the meridian. The style, instead of being inclined downwards, must be turned in a contrary direction, as may be readily conceived when we consider the position of the pole in regard to a vertical plane turned directly towards the north.

Of Polar Dials.

To make a polar dial, draw, as before directed, the meridian XII XII, (pl. 4, fig. 8), and XZ perpendicular to it. From the point M, in this line, make on each side the same construction as that taught at prob. V; if parallel lines be then drawn through the points of division, they will be the hour-lines. For it may be easily seen that, as the pole is in the continuation of this plane, they cannot meet but at an infinite distance, or that the centre of the dial is at an infinite distance; whence it follows that the lines must be parallel.

The style must be placed in a perpendicular direction in the point M; and in height must be equal to the distance

between 12 and 3; or if an iron spike be placed at that distance from the meridian XII XII, and parallel to that line, it will show the hour by its whole length.

PROBLEM VIII.

Of Vertical East and West Dials.

Next to the dials already described, the simplest are those which directly front the east or the west. The method of constructing them is as follows:

Draw the horizontal line HR, (pl. 3, fig. 7, n°. 1), and assume in it any point P, for the bottom of the style, the upper extremity of which is intended to show the hours. At the point P, make, towards the left for an east dial, and towards the right for a west one, the angle HPE, equal to the complement of the latitude, or the elevation of the pole above the horizon, and continue EP to N. The line EN will be the equinoctial. Then through the point P draw the line CA, in such a manner as to form with the line HR the angle APH, equal to the elevation of the pole; then AC, which will intersect the equinoctial EN at right angles, will be the hour-line of VI in the morning, and also the substylar line.

When these lines have been traced out, the hour-lines may be drawn in the following manner. In the substylar line AC, assume a point A, at any distance from the point P, according to the intended size of the dial; and from A, as a centre, describe a semicircle of any radius at pleasure. Divide this semicircle into 12 equal parts, beginning at the point P, and then from the centre A draw dotted lines through each of the points of division in the semicircle, till they meet the equinoctial EN· if lines parallel to the substylar line be then drawn through the points where these dotted lines cut the equinoctial, they will be the hour-lines required, the substylar line being that of VI in the morning. The parallels above the substylar line, in

the east dial, will correspond to IV and V in the morning; those below it to VII, VIII, &c, in the afternoon.

The style, the figure of which is seen in the plate, is placed parallel to the line of VI, on two supports raised perpendicular to the plane of the dial, and at a distance above it equal to that of VI hours from III or from IX. It is here evident that a west is exactly the same as an east dial; only in a contrary situation (see pl. 4, fig. 7, n°. 2); but instead of marking on it the morning hours, as IV, V, VI, &c, you must inscribe on it those of the afternoon, as I, II, III, IV, &c. If an east dial be traced out on a piece of oiled paper, and if the paper be then inverted, but not turned upside down, on holding it between you and the light, you will see a west dial.

It may be easily seen that these dials cannot show the hour of noon: for the sun does not begin to illuminate the latter till that hour, and the former ceases to be illuminated at the same period.

PROBLEM IX.

To describe a horizontal, or a vertical south dial, without having occasion to find the horary points on the equinoctial.

Let the line AB, pl. 5, fig. 9, be the meridian of the dial, which we suppose a horizontal one; and let C be its centre: make the angle HCB equal to the elevation of the pole, in order to find the position of the style; and from the point B, assumed at pleasure, but in such a manner that CB shall be of a proper length, draw BF perpendicular to CH. If we conceive the triangle BFC raised vertically above the plane of the dial, it will represent the style.

From the point C, with the radius CB, describe a circle BDAE; and from the same centre, with the radius BE, describe another circle MQNP.

Divide the whole circumference of the first circle into 24 equal parts, BO, OO, OO, &c, and then divide the second

circle into the same number of equal parts, NR, RR, &c: from the points of division O, of the great circle, draw lines perpendicular to the meridian; and from the corresponding points R of the less circle, draw lines parallel to that meridian. These parallels and perpendiculars will meet in certain points, which will serve to determine the hour-lines. For example, the lines O 3, R 3, which proceed from the third of the corresponding points of division, will meet in the point 3; through which if C 3 be drawn, it will be the position of the line of 3 o'clock; and so of the rest.

It is evident that the larger the circles, the more distinct will be the intersections, formed by the lines drawn through the points of division O and R.

It is remarkable that all these points of intersection are found in the circumference of an ellipse, the greater axis of which is equal to twice CB; and the less PQ to twice CN, or twice BF.

The reason of this construction will be easily discovered by geometricians.

PROBLEM X.

To trace out a dial on any plane whatever, either vertical or inclined, declining or not, on any surface whatever, and even without the sun shining.

This problem, as may be seen, comprehends the whole of Gnomonics; and the operation may be practised by any person who knows how to find the meridian, and to construct an equinoctial dial. The solution of it is as follows.

Having made the necessary preparation, pl. 5 fig. 10, trace out a meridian line on a table, according to the method taught in the first problem; and, by means of this meridian, place an equinoctial dial in such a situation, that the plane of it shall be raised at the proper angle; that is, at an angle equal to the elevation of the equator, or complement of the latitude, and that its south line shall coincide with the above meridian. Adjust along the axis a piece of

packthread, which being stretched shall meet the plane on which the dial is to be described: the point where it meets this plane is that where the style or axis ought to be placed, so as to form one straight line with the packthread and the style of the equinoctial dial.

When this is done, and when the axis of the dial has been fixed, hold a candle or taper before the equinoctial dial, in such a manner, that the style shall show noon; the shadow projected, at the same time, by the packthread, or the axis of the dial about to be constructed, will be the south line. You must therefore assume a point which, together with the centre, will determine that line. If you then change the position of the taper, so that the equinoctial dial shall show one o'clock, the shadow projected by the packthread, or the axis of the proposed dial, will be the hour-line of 1; and so of the rest.

REMARKS.—I. If the plane, on which the dial is to be described, be situated in such a manner that it cannot be met by the axis continued, according to the preceding method, two supporters must be affixed to the plane, for the purpose of receiving a rod of iron, so as to make one line with the packthread; and the operation may then be performed as above described.

II. Instead of an equinoctial dial, a horizontal one may be employed; provided it be placed in such a manner, that the south line corresponds with the meridian which has been traced out.

III. This operation may be performed in the day-time when the sun shines. In this case you must employ a mirror, the reflection of which will produce the same effect as the taper or candle.

PROBLEM XI.

To describe a horizontal dial in a parterre, by means of plants.

A horizontal dial might be described by the usual

methods in a parterre, the hour-lines being formed of box, &c. and a very straight tree terminating in a point, such as the cypress or sycamore, planted on the meridian line being employed as a style.

Instead of a tree, a person might act the part of a style, by standing in a very erect position, in a place marked out on the meridian, proportioned to his height; because according to this height the place must vary. For a short person, it will be near the centre of the dial; and for a tall one, at a greater distance from it. A figure placed on a pedestal might serve at the same time as a style, and as an ornament to the parterre.

PROBLEM XII.

To describe a vertical dial on a pane of glass, which will show the hours without a style, by means of the solar rays.

Ozanam relates that he once constructed a vertical declining dial on a pane of glass in a window, which had no style; and by which the hours could be known when the sun shone.

I detached, says he, from the window frame on the outside a pane of glass, and described on it a vertical dial, according to the declination of the window and the height of the pole above the horizon; taking as the height of the style the thickness of the window frame. I then fixed the pane of glass against the frame in the inside; having given to the meridian line a situation perpendicular to the horizon, as it ought to have in vertical dials. I then cemented to the window frame on the outside, opposite to the dial, a piece of strong paper, not oiled, in order that the surface of the dial might be more obscure. And that I might be able to know the hours without the shadow of a style, I made a small hole in the paper with a pin, opposite to the bottom of the style, which I had marked out. As this hole represented the extremity of the style, the rays of the sun passing through it formed on the glass a luminous

point; which, while the rest of the dial was obscure, indicated the hours in an agreeable manner.

PROBLEM XIII.

To describe three, and even four dials, on as many different planes, on which the hours may be known by the shadow of me axis.

Provide two rectangular planes, ABCD and CBEF, (pl. 6, fig. 11 and 12), equal in size, and join them together by the line CB; so as to form with each other a right angle, the one being horizontal, and the other vertical.

Then divide their common breadth BC into two equal parts in I; and draw the perpendiculars IG, IH, as the meridians of the two planes. Assume the point G at pleasure, as the centre of the horizontal dial, and if GI be made the base of a right-angled triangle GIH, in which the angle G is equal to the height of the pole, the point H will be the centre of a south vertical dial for that latitude. Describe these two dials, viz, a horizontal and a south vertical one, which will have the same points of division in their common section BC; and extend a piece of iron wire, as an axis, from the point H to the point G; this wire will be the common axis and style of the two dials.

Lastly, having with any radius at pleasure described a circle, trace out on it an equinoctial dial, which must be placed on the axis GH, in such a manner that it shall pass through its centre, and be perpendicular to its plane, and that the line of 12 o'clock shall be in the plane of the triangle GIH.

If this triple dial be exposed to the sun, so that the line GI shall be horizontal and in the plane of the meridian, it is evident that the shadow of the axis GH will show the hour on the three dials at the same time.

If it be required to have a fourth dial, to show the hour by means of the same style; in the plane of the triangle GIH draw a line parallel to GH, and through that line a

plane perpendicular to the plane of the meridian, which will cut the vertical plane in the line KL, and the horizontal plane in MN: the hour-lines of both dials will be cut by these two lines in points, every two corresponding ones of which must be joined by transversal lines; for example, the point of section of 11 hours on the one, with the point of section of 11 hours on the other, which will give on that plane parallel hour-lines, such as ought to be on a polar dial that has no declination: these four dials will show the hour at the same time, and by means of the same style or axis GH.

Another method.

Provide a cube ABCED, (pl. 6, fig. 13), and having divided the sides AB, CE, and FD, into two equal parts, in the points H, G, and I, draw the lines GH and GI; then assuming these lines as the meridians of the horizontal plane CD, and of the vertical one CA, and the point G as the centre, describe on the former a horizontal dial, and on the latter a vertical dial, each adapted to the latitude of the place. Assume the lines EM and EN, in such a manner, that the angle ENM shall be equal to the latitude of the place; make CO and CP equal to them; and let a plane pass through MN and OP, so as to cut off that angle of the cube: this plane will intersect the hour-lines of the two dials already traced out in certain points, the corresponding ones of which will give the hour-lines of a third dial.

Nothing then remains but to fix the style, which will not be attended with any difficulty. For this purpose, having drawn EQ perpendicular to MN, fix in a perpendicular position on the meridian KL, and in its plane, two supporters equal in height to EQ, bearing the style RS, prolonged towards each end, and parallel to KL: the shadow of this style will show the hours on the three dials at the same time.

PROBLEM XIV.

In any latitude, to find the meridian by one observation of the sun, and at any hour of the day.

Provide an exact cube, each side of which is about 8 inches; and describe on the upper face a horizontal dial, adapted to the latitude of the place. On the vertical face, which stands at right angles to the meridian of this dial, describe a vertical one; on the adjacent face to the left an east dial, and on the opposite one a west dial, each of which must be furnished with the proper style.

When you are desirous of finding the meridian on a horizontal plane, place this quadruple dial on it, so that the vertical one shall nearly face the south; and gradually turn it till three of these dials all show the same hour: when this takes place, you may be assured that the three dials are in their proper position. If a line be then drawn with a pencil, or other instrument, along one of the lateral sides of the cube, it will be in the true direction of the meridian.

It is indeed evident that these three dials cannot show the same hour unless they are all placed in a proper position in regard to the meridian; their concurrence therefore will show that they are properly placed, and that their common meridian is the meridian of the place.

PROBLEM XV.

To cut a stone into several faces, on which all the regular dials can be described.

Let the square ABCD (pl. 7, fig. 14), be the plane of the stone, which is to be prepared so as to receive all the regular dials. If we suppose the stone to represent an imperfect cube, or any other irregular solid, after all its faces have been smoothed, it must be squared, and reduced to an uniform thickness. When this is done, proceed as follows: On the plane ABCD describe the circle HELF,

with as large a radius as the stone will admit; and draw at right angles the two diameters FE and HL. Then make the angle FOI equal to 38½ degrees, which is the complement of the latitude of London nearly, and draw the diameter IOM; make the angle EOG equal to the latitude 51½ degrees, and draw the diameter GOK; then through the points I, G, M, K, draw tangents to the circle HELF, which will meet the other tangents passing through the points H, E, L, F, and form part of the sides of the square ABCD, that represents the plane of the stone. Cut the stone square, according to these tangents, in order to obtain planes or faces perpendicular to the plane of the stone ABCD, and the stone will then be prepared for receiving on all its faces the dials which belong to them.

On the face or plane which passes through the line VX, describe a horizontal dial; on that passing through XN, an upper equinoctial dial, and on the opposite plane, passing through SR, an inferior equinoctial dial. An upper polar dial must be described on the plane passing through TV, and an inferior one on the plane passing through QP. On the plane passing through TS make a south vertical dial, and on the opposite plane NP a north vertical one: Lastly, if an east vertical dial be described on the side of the stone IM, and on the opposite side a west vertical one, the whole will be complete.

If it be required to have the stone hollow, or rather cut through, nothing will be necessary but to draw lines parallel to these tangents, and to cut the stone square according to these lines, which will give, in the inside of the stone, surfaces parallel to those on the outside. On these internal surfaces, dials similar to those on the opposite external surfaces may then be described.

It is here to be observed, that when the stone is thus made hollow, neither an east nor a west dial can be described on it; but if it be placed on a pedestal in the form of a regular octagon, having one of its faces turned directly

towards the south, different kinds of vertical dials may be described on this pedestal, viz, a south, a north, an east, and a west dial, together with four vertical declining dials; so that on this stone and its pedestal there may be 20 or 25 dials.

If the south vertical dial be placed directly south, and if the horizontal one be perfectly level, all these dials together will show the same hour.

PROBLEM XVI.

To construct a dial on the convex surface of a globe.

This dial, which is the simplest and most natural of all, is formed by dividing the equatorial circle into 24 parts. If a globe be placed on a pedestal, in such a manner that its axis shall be in the plane of the meridian, and exactly elevated according to the height of the pole of the place, nothing then will be necessary to complete the dial, but to divide its equator into 24 equal parts.

The globe, pl. 7, fig. 15, in this state, may be used without any farther apparatus, for one half of it being enlightened by the sun, the boundary of the illumination will exactly follow on the equator, the motion of the sun from east to west. At noon, it will fall on those points of the equator turned directly to the east and west. At one o'clock, it will have advanced 15°, and so on. To render this globe then fit for being employed as a dial, VI must be inscribed at the division which corresponds with the meridian; VII at the following one, and so of the rest; so that the 12th will be exactly in the point turned towards the west; then I, II, III, &c, will be under the horizon. Nothing then will be necessary, but to observe what division corresponds with the boundary of the light and shadow; for the number belonging to that division will be the hour.

This dial however it attended with a very great inconvenience: as the boundary between the light and shadow is always badly defined, it cannot be precisely known where

it terminates; it will therefore be better to employ this dial in the following manner.

Adapt to this globe a half meridian, made of a piece of flat wire, 7 or 8 lines in breadth, and half a line in thickness, and moveable at pleasure around its axis, which must be the same as that of the globe. Then, when you wish to know the hour, move the half meridian in such a manner, that it shall project the least shadow possible, and this shadow will show the hour on the equator. In this case however it is evident that the numbers naturally belonging to the points of division in the meridian, should be inscribed on them; that is, XII at the meridian, I at the following division, towards the west, and so on.

PROBLEM XVII.

Another kind of dial, in an armillary sphere.

This dial is equally simple as the preceding, and is attended with this advantage, that it may serve by way of ornament in a garden.

Conceive an armillary sphere, pl. 7, fig. 16, consisting only of its two colures, its equator, and zodiac, and furnished with an axis passing through it. If we suppose this sphere to be placed on a pedestal, in such a manner that one of its colures shall supply the place of a meridian, and that its axis shall be directed towards the pole of the place, it is evident that the shadow of this axis, by its uniform motion, will show the hours on the equator. If the equator therefore be divided into 24 equal parts, and if the numbers belonging to the hours be inscribed at these divisions, the dial will be constructed.

But as the equator, in general, is not of sufficient thickness, the hours must be marked on the inside of the zone which represents the zodiac, and which on that account should be painted white. But in this case, care must be taken not to divide each quarter of the zodiac into equal parts; for the shadow of the axis, which passes over equal

arcs on the equator, will pass over unequal ones on the zodiac: these divisions will be narrower towards the points of the greatest declination of that circle; so that the division in the zodiac nearest to the solstitial colures, instead of 15°, which are equal to the interval of an hour on the equator, ought to comprehend only 13° 45′; the second 14° 15′; the third 15° 20′; the fourth 15° 25′; the fifth 15° 55′; and the sixth, or that nearest the equinoxes, 16° 20′. It is in this manner that the zodiacal band, on which the hours are marked, must be divided; otherwise there will be several minutes of error; but each interval may be divided into 4 equal parts for quarters, without any sensible error. Transversal lines may then be drawn through the breadth of the zodiac, taking care to make them concur in the pole. We have seen dials of this kind constructed by ignorant artists, who paid no attention to the above remark, and which therefore were very incorrect.

PROBLEM XVIII.

To construct a solar dial, by means of which a blind person may know the hours.

This may appear a paradox; but we shall show that a sun-dial might be erected near an hospital for the blind, by which its inhabitants could tell the hours of the day.

If a glass globe, 18 inches in diameter, be filled with water, it will have its focus at the distance of 9 inches from its surface; and the heat produced in this focus will be so considerable, as to be sensible to the hand placed in it. This focus also will follow the course of the sun, since it will always be diametrically opposite to it; and therefore, to construct the proposed dial, we may proceed as follows.

Let the globe be surrounded by a portion of a concentric sphere, 9 inches distant from its surface, and comprehending only the two tropics, with the equator, and the two meridians or colures; and let the whole be ex-

posed to the sun in a proper position; that is, with the axis of the globe parallel to that of the earth.

Let each of the tropics and the equator be divided into 24 equal parts; and let the corresponding parts be connected by a small bar, representing a portion of the hour circle comprehended between the two tropics. By these means all the horary circles will be represented in such a manner, that a blind person can count them, beginning at that which corresponds to noon, and which may be easily distinguished by some particular form.

When a blind person then wishes to know the hour by this dial, he will first put his hand on the meridian, and count the hour circles on the bars which represent them; when he comes to the bar on which the focus of the solar rays fall, he will readily perceive it by the heat, and consequently will know how many hours have elapsed since noon; or how many must elapse before it be noon.

Each interval between the principal bars, that indicate the hours, may be easily divided by smaller ones, in order to have the half-hours and quarters.

PROBLEM XIX.

Method of arranging a horizontal dial, constructed for any particular latitude, in such a manner as to make it show the hours in any place of the earth.

Every dial, for whatever latitude constructed, may be disposed in such a manner as to show the hour exactly in any given place; but we shall here confine ourselves to a horizontal dial, and show how it may be employed in any place whatever.

1st. If the latitude of the place be less or greater than that of the place for which the dial has been constructed, after exposing it in a proper manner, that is, with its meridian in the meridian of the place, and its axis turned towards the north, nothing will be necessary but to incline it till its axis forms with the horizon an angle equal to the

latitude of the place in which it is to be used. Thus, for example, if it has been constructed for the latitude of Paris, which is 49° 50′, and you wish to employ it at London, in latitude 51° 31′; as the difference of these two places is 1° 41′, the plane of the dial must make with the horizon an angle of 1° 41′, as seen in the figure, pl. 8 fig. 17, where SN is the meridian, ABCD the plane of the dial, and ABE, or *abe*, the angle of the inclination of that plane to the horizon. If the latitude of the primitive place of the dial be less than that of the place for which it is used, it must be inclined in a contrary direction.

2d. When the second method of rendering a horizontal dial universal is employed, the hour-lines must not be described on it, but only the points of division in the equinoctial line, as taught in the 5th problem. In regard to the style, it must be moveable in the following manner. Let ABC, pl. 8 fig. 18, represent the triangle in the plane of the meridian, where NBC is the axis or oblique style, and AB the radius of the equator. The style must be moveable, though it always remain in the plane of the meridian, so that the radius AB of the equator, having a joint in the point A, may form the angle BAC equal to a given angle; that is, equal to the complement of the latitude. For this reason a groove must be formed in the meridian, so as to admit this triangle to be raised up or lowered, always remaining in the plane of the meridian.

When every thing has been thus arranged, to adapt the dial to any given latitude, such as that of 51° 31′, for example, take the complement of 51° 31′, which is 38° 29′, and make the angle BAC=38° 29′. The style then will be in the proper position, and the dial being exposed to the sun, with its meridian corresponding to the meridian of the place, the shadow of the style, which ought to be pretty long, will show the hour at the place where it intersects the equinoctial.

PROBLEM XX.

Method of constructing some tables necessary in the following problems.

There are three tables frequently employed in Gnomonics, and which we shall have occasion to make use of hereafter. These are,

1st. A table of the angles which the hour-lines form with the meridian on an horizontal dial, according to the different latitudes.

2d. A table of the angles which the azimuth circles, passing through the sun at different hours of the day, form with the meridian, according to the different latitudes, and the sun's place in the ecliptic.

3d. A table of the sun's altitude at different hours, on a given day, and in a place of a given latitude.

From the latter is deduced the sun's zenith distance, at different hours of the day in a given place, and on a given day for the sun's zenith distance is always the complement of his altitude.

The first of these tables may be easily calculated by means of the following proportion:

As radius,
Is to the sine of the latitude of the given place,
So is the tangent of the angle which measures the sun's distance from the meridian, at a given hour,
To the tangent of the angle which the hour-line forms with the meridian.

By means of this analogy, we have calculated the following table, which we conceive will be sufficient; as it comprehends the whole extent of Great Britain; and particularly the latitude of London.

A TABLE

Of the angles which the hour-lines form with the meridian on a horizontal dial, for every half degree of latitude, from 50° to 59° 30′.

Latitude.	A. M. I. XI	A. M. II. X	A. M III. IX	A M. IV. VIII	A. M. V. VII	A. M. VI. VI.
° ′	° ′	° ′	° ′	° ′	° ′	° ′
50	11 38	23 51	37 27	53 0	70 43	90 0
50 30	11 41	24 1	37 40	[illegible]	70 51	90 0
51	11 46	24 10	37 51	53 21	70 58	90 0
51 31	11 51	24 19	38 4	53 36	71 6	90 0
52	11 55	24 27	38 14	53 46	71 13	90 0
52 30	12 0	24 36	38 25	53 58	71 20	90 0
53	12 5	24 45	38 37	54 8	71 27	90 0
53 30	12 9	24 54	38 48	54 19	71 34	90 0
54	12 14	25 2	38 58	54 29	71 40	90 0
54 30	12 18	25 10	39 8	54 39	71 47	90 0
55	12 23	25 19	39 19	54 49	71 53	90 0
55 30	12 28	25 27	39 29	54 59	71 59	90 0
56	12 32	25 35	39 40	55 8	72 5	90 0
56 30	12 36	25 43	39 50	55 18	72 12	90 0
57	12 40	25 51	39 59	55 27	72 17	90 0
57 30	12 44	25 58	40 9	55 37	72 22	90 0
58	12 48	26 5	40 18	55 45	72 27	90 0
58 30	12 52	26 13	40 27	55 54	72 33	90 0
59	12 56	26 20	40 36	56 2	72 39	90 0
59 30	13 0	26 27	40 45	56 10	72 44	90 0

We have not marked, in this table, the angles formed by the lines v hours in the morning and vii hours in the evening, iv hours in the morning and viii in the evening, because these lines are only a continuation of others; for example, that of iv hours in the morning, is the continuation of iv in the evening; that of viii hours in the evening, is the continuation of viii in the morning; and so of the rest.

The use of this table may be easily comprehended. If

the place for which a horizontal dial is required, corresponds with any latitude of the table, such as 52 for example, it may be seen at one view, that the hour-lines of XI and I must form, with the meridian, an angle of 11° 55′, at the centre of the dial; that of X and II an angle of 24° 27′; and so of the rest.

If the latitude be not contained in the table, the proportional parts may be taken without any sensible error. Thus, if it were required to find the angle which the hour-line of I or XI forms with the meridian, on a dial for the latitude of 54° 15′; as the difference of the horary angles, for 54° and 54° 30′, is 4′, take the half of 4, and add it to 12° 14′, which will give 12° 16′ for the horary angle between the hours of I or XI and the meridian, on a dial for the latitude of 54° 15′. The same operation may be employed for the other horary angles.

It is necessary to observe that this table, though constructed for horizontal dials, may be used also for vertical south or north dials for it is evident that a south vertical dial, for any particular place, is the same as a horizontal dial for another, the latitude of which is the complement of the former. Thus a south vertical dial for the latitude of London 51° 31′, is the same as a horizontal dial for the latitude of 38° 29′, and vice versa.

It is in the construction of these vertical dials that the utility of such tables will be most apparent; for as these dials are in general very large, the common rules of Gnomonics cannot easily be applied to them. To remedy this inconvenience, when the centre and equinoctial of the dial have been fixed, assume, as radius, that part of the meridian comprehended between the equinoctial and the centre, and divide it into 1000 parts; then find in some table, or by calculation as above shown, for the given latitude, that is, for its complement if a vertical dial is to be constructed, the tangents of the angles which the hour-lines form with the meridian, at I, II, III, IV, &c, and lay them off on both sides on the equinoctial: the points where they terminate

will be the horary points of I and XI hours, II and X hours, &c.

Let us suppose, for example, that a south vertical dial is to be constructed for the latitude of 51° 31′, the complement of which is 38° 29′. A vertical south dial for lat. 51° 31′, may be considered as a horizontal dial for the latitude of 38° 29′. But the angles which the hour-lines form with the meridian on a horizontal dial, for that latitude, are 9° 28′; 19° 46′; 31° 53′, 47° 9′, 66° 42′; 90° 0′, the tangents of which, radius being divided into 1000 parts, are 166, 359, 622, 1078, 2321, *infinite*. If the portion of the meridian therefore, comprehended between the centre and the equinoctial, be divided into 1000 parts, and if 166 of these parts be set off on each side of the meridian, we shall have the points of XI and I hours, if 359 parts be then laid off in the same manner, we shall have the points of X and II hours; and so of the rest. Straight lines drawn from the centre, to each of these points, will be the hour-lines.

The last tangent, which corresponds to VI hours, being infinite, indicates that the hour-line corresponding to it must be parallel to the equinoctial.

In order to give an idea of the construction of the second table, let the circle MBND, pl. 9 fig. 19, represent the horizon of the place, Z its zenith, P the pole, ZB the azimuth circle passing through the sun, and PSA the horary circle in which the sun is at any proposed time of the day; it is here evident, that if the hour be given, the angle ZPS is known; that the day of the year being given, the sun's distance from the equator is known, and consequently the arc PS, which in our hemisphere, is the 4th part of a great circle minus the sun's declination, if it be north, or plus that declination if it be south; and lastly, that if the elevation of the pole be given, the arc PZ, which is its complement, is also known. In the spherical triangle ZPS, we have therefore given the arcs ZP and PS, with the included angle ZPS; and hence we may find the angle PZS, which

subtracted from 180 degrees, will leave the angle MZB or MCB, the sun's azimuth from the south.

In the same triangle, we can find the side ZS, the complement of the sun's altitude at the same time; and consequently the altitude itself.

By these means the following tables have been constructed, for the latitude of London 51° 31′. Those who are tolerably versed in spherical trigonometry may easily construct similar tables for any other latitude.

A TABLE *of the Sun's azimuth from the South, at his entrance into each of the twelve signs, and at each hour of the day, for the latitude of London 51° 31′.*

Hours.		♋ °	′	♊ ♌ °	′	♉ ♍ °	′	♈ ♎ °	′	♓ ♏ °	′	♒ ♐ °	′	♑ °	′
XI.	I	[illegible]	2	[illegible]	[illegible]	22	1[illegible]	19	13	1[illegible]	1[illegible]	1[illegible]	4[illegible]	1[illegible]	9
X.	II	[illegible]	[illegible]	[illegible]	[illegible]	42	9	[illegible]0	2[illegible]	[illegible]1	[illegible]9	28	[illegible]	2[illegible]	4[illegible]
IX.	III	[illegible]8	[illegible]	[illegible]	[illegible]	[illegible]	1[illegible]	51	[illegible]	4[illegible]	3	42	7	40	39
VIII.	IV	82		7[illegible]	27	[illegible]2		[illegible]		[illegible]59	0	1	2[illegible]		
VII.	V	9[illegible]	[illegible]	[illegible]1	2[illegible]	[illegible]	2	7[illegible]	[illegible]0	[illegible]1	8				
VI.	VI	1[illegible]5	[illegible]	102	[illegible]	97	[illegible]	0							
V.	VII	110	[illegible]	111	[illegible]										
IV.	VIII	127	2[illegible]												

A TABLE *of the Sun's altitude at his entrance into each of the twelve signs, and at each hour of the day, for the latitude of London 51° 31′.*

Ho[illegible]		[illegible]		♊ ♌		♉ ♍		♈ ♎		♓ ♏		♒ ♐		♑	
XII.		61	57	[illegible]	11	[illegible]9	[illegible]1	38	29	2[illegible]	4[illegible]	18	1[illegible]	15	2
XI.	I	[illegible]9	40	[illegible]	[illegible]1	[illegible]8	[illegible]	30	7	2[illegible]	[illegible]0	1[illegible]	32	13	54
X.	[illegible]	[illegible]53	[illegible]1	[illegible]0	[illegible]	13	[illegible]	32	37	21	12	13	40	10	32
IX.	III	45	41	13	[illegible]	35	52	20	7	1[illegible]	[illegible]0	8	15	5	17
VIII.	IV	30	40	31	11	27	21	18	[illegible]	8	1[illegible]	1	16		
VII.	V	27	22	24	50	18	12	9	17	1	[illegible]7				
VI.	VI	18	10	15	41	8	5[illegible]								
V.	VII	9	26	6	50										
IV.	VIII	1	31												

PROBLEM XXI.

Another method of constructing a universal horizontal Sun-dial.

In one of the two preceding constructions, the equinoctial line was divided in such a manner, as to be calculated for showing the hours in every latitude, by removing to a greater or less distance the centre of the dial: but in the present case, we suppose this centre to be fixed, and that the inclination of the style, which ought always to be directed to the pole, can be varied in that point. The method of constructing a sun-dial of this kind is as follows.

Through the point C, assumed as the centre of the dial, pl. 9 fig. 20, draw the two perpendiculars AB and EF; the first of which being made to represent the line of 6 hours, the other will represent the meridian; from the point B, assumed at pleasure, set off, on the meridian, as many equal parts as you choose; for example 6, and through the points of division describe 7 concentric circles, which will represent the circles of latitude for every 5 degrees, from 30° to 70°, in order that the dial may answer for the greater part of Europe. This division at every 5 degrees, will be sufficient; because the intermediate points may be easily distinguished by the eye. We shall suppose then that the smallest circle passing through the point D, represents that of the latitude of 60°. Set off on that circle, counting from each side of the meridian, the angles formed by the hour-lines of I and XI hours, II and X hours, &c, on a horizontal dial corresponding to the latitude of 60°.

Perform the same operation on the next circle, which corresponds to the latitude of 55°, and thus in succession for all the rest. Then join the similar points of division by a curved line, and the dial will be constructed.

Having placed the dial properly, that is in such a manner, that its meridian may coincide with the meridian of the place, and that its axis be directed to the north, ele-

vate the style at an angle equal to the latitude, and then examine where the shadow of the style falls on the circle corresponding to that latitude: the point where it falls will indicate the hour.

REMARK.—That these portable dials may be easily placed in the proper position, a small compass is generally adapted to them; but those who think it sufficient to make the needle coincide with the meridian of the dial, will be deceived; for there is scarcely a place on the earth where the needle does not decline more or less towards the East or West. At London for example it declines at present about 24 degrees and a quarter to the west side.

To place a dial, therefore, of this kind in its proper situation at London, it must be disposed in such a manner, that the needle of the small compass shall form with the meridian an angle of $24\frac{1}{4}$ degrees nearly, and be on the west side of it; the meridian of the dial will then coincide with that of London. This example will be sufficient to show what method must be pursued in other places, where the declination is greater or less, or in a contrary direction; that is, to the East, as it was at London about two centuries ago.

PROBLEM XXII.

The Sun's altitude, the day of the month, and the elevation of the pole, being given; to find the hour by a geometrical construction.

We give this construction merely as a geometrical curiosity; for it is certain that the same thing can be performed with much greater accuracy by calculation. However, as the solution of this problem forms a very ingenious example of the graphic solution of one of the most complex cases of spherical trigonometry, we have no doubt that it will afford gratification to our readers; or at least to such of them as are sufficiently versed in geometry to comprehend it.

Let us return then to fig. 19 pl. 9,* in which PZ represents the complement of the latitude or elevation of the pole; ZS the complement of the sun's altitude, which is known, being given by the supposition; and PS the sun's distance from the pole, which is also given, since the declination of the sun, or his distance from the equator each day, is known. In the triangle ZPS therefore, there are given the three sides, to find the angle ZPS, the hour angle, or angle which the horary circle, passing through the sun, forms with the meridian. This case then is one of those in spherical trigonometry, where the three sides of an oblique triangle being given, it is required to find the angles; and which may be solved geometrically in the following manner.

In the circumference of a circle, which must be sufficiently large to give quarters of degrees, pl. 9 fig. 19 and 21, assume an arc equal to PZ, and draw the two radii CP and CZ. On the one side of this arc make PS equal to the arc PS, and on the other ZR equal to the arc ZS: from the points R and S let fall, on the radii PC, CZ, two perpendiculars ST and RV, which will intersect each other in some point X: then, if ST be radius, we shall have TX for the cosine of the required angle, which may be constructed in the following manner:

From the centre T, with the radius TS, or T*s*, which is equal to it, describe a quadrant, comprehended between TP and TX continued; if XY be then drawn parallel to TP, the arc Y*s* will be the one required, or the measure of the hour angle SPZ; therefore YTX will be equal to that angle.

By a similar construction we might find the angle Z, the complement of which is the sun's azimuth; but this is sufficient in regard to an operation which is rather curious than useful.

This construction is much simpler, and far more elegant, than that given by Ozanam, for the solution of the same problem.

PROBLEM XXIII.

To construct a horizontal dial, to show the hours by means of a vertical immoveable style in the centre.

In the construction of this dial, the table of the sun's azimuths, given in prob. 20, must be employed.

Along the bottom of the style, pl. 10 fig. 22, draw the meridian line AB, of any length at pleasure; and from the centre C describe, through the extremity B, the arc of a circle, which must be assumed as the tropic of cancer ♋. Having then made CD equal to about a third of CB, divide the interval DB into 6 equal parts; and from the centre describe, through the points of division, circles concentric to the first: the smallest will represent the tropic of capricorn ♑; the rest the parallels of the intermediate signs.

In the exterior circle, beginning at the point B, assume the angles or arcs BI, B XI, equal to those given in the table for the hours of I and XI, when the sun is in ♋, and mark these points with I and XI hours; do the same in regard to II and X hours, and so of the rest.

Take, from the same table, the angles or arcs corresponding to the hours XI and I, X and II, IX and III, &c, when the sun enters gemini and leo, ♊ ♌. Do the same thing on the third circle, which corresponds to the sun's entrance into taurus and virgo, ♉ ♍, and so of the rest. By these means, you will have the hour points on each circle; and if the points of the similar hours be then joined by a curved line, the dial will be completed. The hour may be known by observing the shadow on the circle which denotes the sun's place in the zodiac on the given day. For the greater exactness, the small intervals between these circles may be divided into 3 equal parts; through which if dotted circles be described, they will serve for those days when the sun occupies mean positions in the zodiac.

REMARK.—By this method, the edge of the shadow of

one of the upright bars of a window, might be employed to show the hours in a room; for if the bar be exactly perpendicular, it will represent an indefinite vertical style; and circles corresponding to the sun's place in the zodiac, and the hour-lines, may by the above process be traced out on the floor. The hour will be known by observing, on the circle corresponding to the sun's place, the point where it is intersected by the shadow.

PROBLEM XXIV.

To construct a moveable horizontal dial, to show the hours merely by the Sun's altitude.

This dial seems to be very ingenious, and convenient, as it requires neither a meridian line nor a compass, and as nothing farther is necessary to be known, but the sign and degree of the sun's place in the zodiac: this however we shall render much easier by substituting, for the sun's place, the day of the month. It is attended with one inconvenience, which is, that the hours near the rising and setting of the sun cannot be marked upon it; but we shall show how this defect may be remedied.

Having assumed the point A, pl. 10 fig. 23, as the place of the style AB, which we shall here suppose to be an inch in height, draw the indefinite line DAC, and AG perpendicular to it: draw also the lines AI, AH, AF and AE, making the equal angles CAI, IAH, HAG, &c. Having then assumed the line AC, as that corresponding to the 21st of December, the day of the winter solstice, take, from the third table, the sun's zenith distance for each hour of the day, when he enters capricorn, and make the angles AB 12, AB 11, AB 10, &c, equal to those found in the table.

On the line AD, destined to represent the 21st of June, the day of the summer solstice, assume A 12, A 1, A 2, A 3, A 4, A 5, &c, of such a length, that the angles AB 12, AB 1, AB 2, AB 3, &c. may be equal to the sun's zenith distances at the hours of 12 at noon, 1 or 11, 2 or 10, and so on.

In like manner, having raised, on the line AI, a perpendicular, AK, equal to the height of the style AB, make the angles AKL, AKM, AKN, &c, equal to the sun's zenith distances at the hours of 12, 1, 2, &c, when the sun enters aquarius or sagittarius; and on that line mark the points L, M, N, &c: these points will be those of noon, the hours of 1 or 11, 2 or 10, and so on.

On each of the lines AH, AG, AF, &c, do the same thing: which will give on these lines the hours of the day; and if the similar horary points, such as those of noon, those of 1 or 11, 2 or 10, &c, be joined by a curved line, the dial will be constructed.

The method of knowing the hour on this kind of dial, is as follows. Let us suppose, for example, that the given day is the 21st of October; expose the dial to the sun on a horizontal plane, so that the shadow of the style may fall on the line AH, or that marked 21 October, and observe where the shadow terminates; for that point will indicate the hour.

If the proposed day be different from those corresponding to the lines AC, AI, AH, &c, the intermediate line, on which the shadow of the style ought to fall, may be easily found, by counting the number of days elapsed between the given time, and the 21st of the nearest month. Let the proposed time, for example, be the 10th of April. Between the 10th of April and the 21st of March, there are 19 days; consequently the line of the shadow ought to form with the line AG an angle of 19 degrees. From A then as a centre describe a semicircle, and having divided it into degrees, draw dotted lines through every 5 of these divisions. The shadow may then be made to fall on the proper line without much difficulty.

REMARKS.—I. It may be readily seen, that in regard to the hours near sun-rise or sun-set, the length of the shadow will make them fall without the dial. But this inconvenience may be remedied in the following manner:

Adapt to the dial a circular rim, concentric with the style, and of the same height: it will be easy to find on this rim, the points where the shadow terminates at the different hours till sun-set.

II. This dial may be made concave, so as to form a portion of a spherical surface, pretty deep, that the summit of the style may be on a level with the edge. The horary points may be found by the method above described; those near sun-set or sun-rise excepted, for it is evident that the shadow of the style will never go beyond the extent of this spherical concave surfac

PROBLEM XXV.

To construct a horizontal dial, to show the hours by means of the sun, without the shadow of any style.

The invention of this dial is very ingenious; but Ozanam did not attend to one very essential circumstance; namely, the declination of the magnetic needle, which in his time was considerable, and which being at present 24¼ degrees at London, would occasion a very great error, without employing the expedient which we shall here apply to the construction of it. But we shall first suppose the needle to have no declination.

Describe, on a moveable horizontal plane, pl. 11, fig. 24, the right-angled parallelogram ABCD; and having divided each of the two opposite sides, AB and CD, into two equal parts, in the points E and F, join these points by the straight line EF, which will be the meridian. On this line assume at pleasure the point G, as the place of the style, and the points F and H as the solstitial points of cancer and capricorn; through which, from the point G as a centre, describe two circles, representing the tropics, or the commencement of these signs.

Then divide the space HF into 6 equal parts; and through the extremities of these parts describe 5 other circles representing, in order, the circles of declination

the commencement of the other signs, taken two and two; for the sun's declination corresponding to the first degree of leo, is the same as that corresponding to the first degree of gemini; that corresponding to the first degree of taurus, the same as that corresponding to the first degree of virgo; and so of the rest.

Then, on the circle representing the tropic of cancer, set off, on each side of the line GH, arcs equal to the sun's azimuth, as given in the above table, at the hours of 11 and 1, 10 and 2, 9 and 3, &c; do the same thing in regard to the circle representing the commencement of gemini and leo; and so of the rest: if the similar hour points be then joined by a line, which must necessarily be a curve, if the lines are equally spaced, the dial will be constructed.

To supply the place of a style, fix a small pin in G, and suspend from it a magnetic needle, so as to play freely, and be able to assume its natural direction.

To know the hour, expose the dial to the sun in such a manner, that the side AB shall be opposite to the sun, and that the sides CB and DA shall project no shadow: the point where the magnetic needle intersects the arc, corresponding to the sign in which the sun's place then is, will indicate the hour. According to the figure, if we suppose the sun in the beginning of cancer, it would indicate about three quarters after 9 in the morning.

Remark.—We have already observed, that this would be true only in case the magnetic needle had no declination; but as its declination at present at London is 24¼ degrees west, the following correction will be necessary.

As the needle will always be 24¼ too far towards the west, instead of making the angles C, B, A, and D, right angles, cut the board in such a manner, that the angles B and D shall be 114° 15′, and the angles C and A 65° 45′. This will rectify the error in declination; and nothing then will be necessary, but to expose the dial, as above

mentioned, in such a manner that the sides CB and AD shall project no shadow.

PROBLEM XXVI.

To construct a dial to show the hours by reflection.

A dial to show the hours by reflection may be described in the following manner, on a dark wall or ceiling. Describe a dial on a horizontal plane, that can be illuminated by the rays of the sun, such, for example, as the bottom of a window; but in such a manner, that the centre of the dial may be towards the north, and the equinoctial towards the south; which will give to the hour-lines a position contrary to that which they ought to have in common horizontal dials. When the dial has been thus constructed, and furnished with a small upright style, apply a piece of thread to any point at pleasure, of one of the hour-lines, and extend it over the end of the style, till it reach any point of the wall or ceiling: this point will be one of those of the hour-line to which the end of the thread was applied. If four or five points be determined, in the same manner, for each hour-line, by then drawing lines through these points, the required dial will be constructed.

To know the hours by reflection; adapt a small mirror, an inch or two in diameter, to the summit of the style, and let it be fixed in a position exactly horizontal: the light reflected from it will indicate the hour.

Instead of a mirror, a small goblet, an inch or two in diameter, may be applied to the summit of the style, and be filled with water till its surface be exactly on a level with the extremity of the style: the light reflected from it will indicate the hours in the same manner, and will be more easily observed in cloudy weather, when the sun scarcely appears; because the surface of the water will generally have a small movement, which by making the light tremulous, will render it perceptible, notwithstanding its weakness.

Another Method.

Place, in any part of the bottom of a window, a small goblet; and fill it with water to a given height. Place also, on the bottom of the window, a sun-dial, and when the shadow of the style falls on the hour of noon, mark on the ceiling or wall, which receives the reflected light of the sun, the central point of the image of that luminary: do the same thing in regard to all the other hours, and mark these points with the hours to which they correspond.

Two or three months after, when the sun's declination has considerably changed, if the same operation be performed, you will have two points of each hour-line, and if the surface, on which they are traced out, be a plane, to obtain the required hour-line, nothing will be necessary but to join them by a straight line.

But if the surface, which receives the reflected light, be curved or irregular, to obtain the hour-line a greater number of points will be necessary. To trace it out exactly, the operation of finding a point for each hour-line ought to be repeated for five or six months, from the one solstice to the other: if these points be then joined by a curve, they will give the hour-lines required.

Third Method.

Having described the hour-lines, in the usual manner, on a horizontal plane ABCD, pl. 11 fig. 25, turn the dial in a direction contrary to that which it ought to have, and from a point E of the meridian raise a perpendicular style of such a height, as it ought to have to indicate the hours: to this style apply a small mirror, so as to be exactly vertical, having its plane perpendicular to that of the meridian, and its centre corresponding to the summit of the style, as seen in the figure; the reflected light of the sun will then indicate the hours on the dial.

Fourth Method.

By a similar method, a sun-dial might be traced out on a wall exposed to the north, so as to show the hours by the reflection of the sun's rays from a small vertical mirror, placed against a wall exposed to the south. This would be attended with no great difficulty; and such of our readers as are curious in dialling may exercise their ingenuity on the execution of it.

GNOMONICAL PARADOX.

Every sun-dial, however accurately constructed, is false, and even sensibly so, in regard to the hours near sun-set.

The truth of what is here asserted, will be readily perceived by astronomers, who are acquainted with the effects of refraction. The following observations will make it sensible to our readers.

It is a fact, now well known to all philosophers, that the heavenly bodies always appear more elevated than they really are, except when they are in the zenith. This phenomenon is produced by the refraction, which the rays of light, proceeding from them, experience in the atmosphere; and the effect of it is very considerable in the neighbourhood of the horizon; for when the centre of the sun is really on the horizon, he still appears to be elevated more than half a degree, or 33 minutes, which in our latitudes is the quantity of the horizontal refraction. The centre of the sun then is really on the horizon, and astronomically set, when his lower limb does not touch the horizon, but is still distant from it an apparent semidiameter of the sun.

Let us suppose then, that on the day of the equinox, for example, the hour indicated by a vertical west-dial, near the time of sun-setting, has been observed at the moment when a well-regulated clock strikes 6: the shadow of the style ought to be on the hour of 6, and it would

indeed be so, if the sun were on the horizon; but being elevated 33 minutes above the horizon, the shadow of the style will be within 6 hours, for it is by the apparent image of the sun that this shadow is formed: it will even not reach that line till the sun has still descended 33′, for which he will employ, in the latitude of London, about 3^m 28^s of time. But, in a sun-dial, an error of 3^m 28^s is more than sensible.

If the sun be at the summer solstice; as he employs in the latitude of London more than 4^m to descend vertically 33 minutes on the horizon, on account of the obliquity with which the tropic cuts that circle, the difference will be more sensible as the space passed over by the shadow between the hours of 7 and 8, is sufficiently great to suffer an error of a 12th or a fifteenth to be very perceptible. We have seen, on a dial of this kind, the point of the shadow, which ought to have fallen on the line of 7 o'clock, more than an inch distant from it; though at all the other hours of the day the dial was very exact, and corresponded with an excellent watch which was compared with it. We shall therefore describe a method of constructing a sun-dial, by which this inconvenience may be obviated.

PROBLEM XXVII.

To construct a sun-dial which, notwithstanding the effect of refraction, shall indicate the hour exactly.

We shall here confine ourselves to the example of a vertical dial without declination, turned directly south, and for the latitude of 48° 50′; but the same expedient may be easily applied to any other vertical dial, and even to a declining one for any other latitude.

Let C be the centre of the dial (pl. 12, fig. 26) to be constructed, and C XII the south line. In any point P of that line, fix an upright style, consisting of an iron pin placed perpendicular to the plane of the dial, and ter-

minating in a round button, 7 or 8 lines in diameter, so that the centre of this button shall form with that of the dial a line parallel to the celestial axis.

Then set off the length of this style, taken from the centre of the button, from P to A; and through the point P draw the horizontal line QR.

Let it now be required to trace out, for example, the line of 4 o'clock in the afternoon. Consider AP as radius, and from A, as a centre, with the distance AP, describe a quadrant. Then find the sun's azimuth at 4 o'clock in the afternoon when he enters capricorn, for the latitude of 48° 50′, and the same azimuth at the same hour when he enters aquarius or sagittarius, libra or aries, taurus or virgo: these four azimuths will serve to give four points for the line of 4 hours, which will be sufficient. The sun's azimuth at 4 in the afternoon when he enters capricorn, for lat. 48° 50′, will be found to be 52° 35′; for this reason draw AK, in such a manner, that the angle KAP shall be equal to 52° 35′; that is, lay off an angle equal to that quantity by means of a protractor, or make the arc P *k* equal to that number of degrees and minutes. Draw, in like manner, for the other three signs, the lines AL, AM, and AN, making the angles PAL, PAM, PAN, respectively equal to 54° 28′, 60° 30, 74° 21′, and then draw the indefinite verticals KF, LG, MH, and NI.

Next find the sun's altitude at 4 in the afternoon when he enters capricorn; this altitude, for lat. 48° 50′, will be found to be 40′, the tangent corresponding to which is 1153, radius being supposed equal to 100000 parts of the same kind. But as 1153 is the 86th part of 100000, divide AK into 86 parts, and set off one from K to *f*; the point *f* will be one of the required points of the hour-line of 4 o'clock.

In like manner, to determine the point *g*, find the sun's altitude at the same hour when he enters aquarius, which is 3° 10′, and as the tangent corresponding to this altitude

is .5589, which is the 18th part of the radius, if AL be divided into 18 parts and one of them be set off from L to g, you will have the second point required.

Having found the other two by the like process, draw through these four points a line somewhat curved, and you will have the hour-line of 4 o'clock.

If a similar operation be performed for all the other hour-lines, the dial will be constructed.

If a curved line be made to pass through the points of each hour-line, corresponding to the commencement of each sign; you will have what are called the arcs of the signs, traced out much more exactly than by the common method; as the shadow of the summit of the style, when the sun is near the horizon, must deviate from the track marked out for it.

REMARK.—It will be best to begin by tracing out the hour-lines according to the usual method, but only with a pencil; because the difference between the hour-lines, as described by both methods, can by these means be better observed.

PROBLEM XXVIII.

To describe a dial on the convex surface of a fixed cylinder, perpendicular to the horizon.

This dial, which is exceedingly ingenious, is attended with this peculiarity, that the hour is shown, not by the shadow of a style, but by that of a horizontal circle, which intersects the sun's parallel. It may be employed as an ornament in a court or garden, or may serve as a pedestal to a statue, or to another dial, such as the spherical one described in prob. 16. This dial is represented fig. 27, pl. 13. Matters may be so arranged, that the circular cornice, which surrounds this pedestal, shall perform the part of a circular style: this will produce a much better effect, than could be produced by a detached horizontal circle or hoop. A dial of this kind, constructed with great

care, was seen formerly in the garden of the Benedictines, at the abbey of Saint-Germain-des-Prés. It was the work of Father Quesnet, a monk of that order, who made many improvements in what Kercher and Benedict had before taught, in regard to dials of this kind.

The tables of the azimuths and apparent altitudes of the sun, already given, are employed in the construction of this dial. We here make use of the term apparent altitudes, because it is evident that what we have said, respecting refraction, is applicable in the present case; and besides, the apparent altitudes may be employed with the same ease as the real altitudes, as has hitherto been done.

Let AB, pl. 14 and 15 fig. 27, be the diameter of the cylinder, on which the dial is to be described. Having drawn, from one of its extremities A, the tangent AE, equal to the semi-diameter AC, draw the secant CE, which will intersect the cylinder in D: the line DE will be the length of the style. The style however might be longer or shorter: but this length appears to be the most convenient. Then from the centre C describe, through the point E, a circle concentric to the first, and which will represent the extremities of all the styles supposed to be implanted quite round the cylinder. An iron circle of the same size, placed around the cylinder, in such a manner as to be kept at an equal distance from it by means of spikes, will serve to indicate the hours; but it will be better to crown the cylinder with a circular piece of marble, having such a projection as may render it fit for the same purpose.

Then on KF, fig. 28, made equal to the line DE describe the quadrant FN, and having divided it into degrees, count from F towards N the sun's greatest altitude above the horizon of the place, which being at Paris 64° 39', will give the arc FM, equal to that number of degrees and minutes. Through the point M, draw the secant KM, which meeting the cylinder in the point I, will give FI the tangent of 64° 39', as the height of the dial, which however ought

to be made somewhat greater, in order to leave, between the lowest shadow and the bottom of the cylinder, sufficient room for inscribing the hours and the signs. The cylinder also ought to be of such a size, that the hours may be distinctly marked on its surface.

As the operation on the body of the cylinder, though performed in the same manner, is attended with inconvenience, it may be supposed expanded into a rectangle FHLI, the length of which is equal to its circumference ADBE, and the height LH to the above tangent at least.

Having divided FH into two equal parts at G; through that point draw G XII perpendicular to it; then divide each of the two spaces HG, GF into 180 parts or degrees, reckoning on both sides from the point G, which is the south point: the points of 90 degrees, which divide each of the intervals HG, GF into two equal parts, are the points of 6 in the morning and 6 in the evening, which on the cylinder will be diametrically opposite; as the south line G XII is diametrically opposite to the line FI or HL, which we must suppose to be joined, and on the cylinder to form only one line.

Then through each degree of the arc FM draw secants, which will mark out in succession, on FI, the tangents of 1, 2, 3, &c, degrees, to 64° 39′, beyond which it is needless to go, as a greater number cannot be employed.

To inscribe the hours on the dial, and to mark, for example, the point of X in the morning and II in the afternoon, for the time when the sun enters the sign ♋, look in the table of the sun's azimuths, and opposite to X and II, you will find 53° 49′, the sun's azimuth at X or II, when he enters into ♋. In the table of altitudes, look also for the sun's altitude at the same period and hour, which will be found to be 55° 22′. Then count, on the horizontal line FH of the dial, from the south point G towards F, 53° 49′ for the sun's azimuth, and on the vertical line FI, count from F the altitude 55° 22′; then through the points where

these numbers terminate, draw two lines parallel to the respective sides of the rectangle, and the point where they intersect each other will give the hour-point required.

It is here to be observed, that the evening hours must be on the right of the south line, and the morning ones on the left.

That the reader may be better enabled to comprehend this operation, we shall suppose, for example, that it is required to find the point corresponding to VII in the morning, or V in the afternoon, when the sun enters the signs ♉ or ♍. By inspecting the before-mentioned tables, it will be found that the sun's azimuth, at VII in the morning and V in the afternoon, is 86° 23′, and that his altitude at the same time is 18° 29′. Count therefore on FG, from G, 86° 23′ for the sun's azimuth, and on the line FI, from F, 18° 29′ for his altitude: the point where the two lines drawn parallel to the sides of the rectangle, through these divisions, intersect each other, will be that of VII in the morning or V in the evening, when the sun enters ♉ or ♍.

If the points thus found, for each hour, at the sun's entrance into each of the signs, be then joined, which will require only seven operations, the lines that join them will be the hour-lines; and if all the hours of the day, when the sun enters each sign, be joined also by curved lines, these seven lines will intersect the hour-lines, and be the parallels of the commencement of the signs.

To know the hour on this dial, it will be first necessary to find in which parallel the sun is, and to observe where that parallel is intersected by the shadow: the hour-line passing through the point of intersection will indicate the hour. Let us suppose, for example, that the shadow of the style, on the day when the sun enters virgo, intersects the parallel of that sign PQR, in the point Q, which is the mean distance between the points where that parallel is cut by the lines of the hours VIII and IX: we may therefore conclude that it is half an hour past 8 o'clock.

The hour may be known also by observing, as taught by Ozanam, where the line of the shadow of the cylinder intersects the parallel of the sun; but as this line is never well terminated, as already mentioned in regard to dials constructed in the form of a globe, this is not to be recommended.

REMARKS.—I. The use of this dial will be more commodious, if, instead of the signs of the zodiac, the months of the year be employed; for every one knows the day of the month, but few except astronomers know the sign corresponding to each month, or to what third or quarter of a sign any day belongs. For this purpose it is necessary to consult an almanac.

This change on dials of this kind may be easily made; for we may assume as true, without any sensible error, that the tenth degree of each sign corresponds to the first day of each month, as the equinox falls, for the most part, on the 21st of March. Instead then of taking the sun's azimuth and altitude at the commencement of the signs, nothing will be necessary but to take it at every tenth degree of each sign. Then by performing the same operation as that above taught, and joining the points belonging to the first of each month, you will have the parallels of the commencement of each month, and the hour may be known with great ease.

II. Small portable cylindrical dials, which show the hour by means of a style affixed to the moveable top of the cylinder, are also used. The style is placed on the current sign; and being turned directly to the sun, the length of the shadow on the azimuth, parallel to the axis of the cylinder, shows the hour. As this dial may be easily constructed, we shall say nothing farther on the subject. A description of it may be seen in most books on Gnomonics.

PROBLEM XXIX.

To describe a portable dial on a quadrant.

As the construction of this dial depends also on the sun's altitude at each hour of the day, in a determinate latitude, according to his place in the zodiac, the tables before mentioned must be employed here also.

Let ABC then, pl. 15 fig. 29, be a quadrant, the centre of which is A. From the centre A describe, at pleasure, seven quadrants equally distant from each other, to represent the commencement of the signs of the zodiac; the first and last being assumed as the tropics, and that in the middle as the equator. Mark on each of these parallels of the signs, the points of the hours, according to the altitude which the sun ought to have at these hours, which may be found in the table above mentioned. To determine for example, the point of II in the afternoon, or X in the morning, for the latitude of London, when the sun enters leo; as the table shows that the sun's altitude is at that time 50° 56′, make in the proposed quadrant the angle BAO equal to 50° 56′, and the place where the parallel of the commencement of leo is intersected by the line AO, will be the required point of II in the afternoon and X in the morning.

Having made a similar construction for all the other hours, on the day of the sun's entrance into each sign, nothing will be necessary but to join, by curved lines, all the points belonging to the same hour, and the dial will be completed. Then fix a small perpendicular style in the centre A, or place on the radius AC, or any other line parallel to it, two sights, the holes of which exactly correspond; and from the centre A suspend a small plummet by means of a silk thread.

When you use this instrument, place the plane of it in such a manner as to be in the shade; and give such a direction to the radius that the shadow of the small style

shall fall on the line AC, or that the sun's rays shall pass through the two holes of the sights: the thread from which the plummet is suspended will then show the hour, by the point where it intersects the sun's parallel.

To find the hour with more convenience, a small bead is put on the thread, but in such a manner as not to move too freely. If this bead be shifted to the degree and sign of the sun's place, marked on the line AC, and if the instrument be then directed towards the sun, as above mentioned, the bead will indicate the hour on the hour-line which it touches.

REMARK.—To render this dial more commodious, and for reasons already mentioned in describing the cylindric dial, it will be better, instead of the signs, to mark the days of the month on which the sun enters them. For example, instead of marking the small circle with the sign ♑, mark December 21; close to the second place on one side January 21, instead of ♒, the sign of aquarius; and on the other November 21, instead of ♐, the sign of sagittarius, &c; for if we suppose the equinoxes invariably fixed at the 21st of March and the 21st of September, the days on which the sun enters the different signs of the Zodiac will be nearly the 21st of each month: to use the dial, nothing will then be necessary but to know the day of the month.

PROBLEM XXX.

To describe a portable dial on a card.

This dial is generally called the Capuchin, because it resembles the head of a Capuchin friar with the cowl inverted. It may be described on a small piece of pasteboard, or even a card, in the following manner.

Having described a circle, pl. 15 fig. 30, at pleasure, the centre of which is A, and the diameter B 12, divide the circumference into 24 equal parts, or at every 15 degrees, beginning at the diameter B 12. If each two points of division, equally distant from the diameter B 12, be then

joined by parallel lines, these parallels will be the hour-lines; and that passing through the centre A, will be the line of six o'clock.

Then at the point 12, make the angle B 12 ♈ equal to the elevation of the pole, and having drawn through the point ♈, where the line 12 ♈ intersects the line of 6 o'clock, the indefinite line ♋ ♑, perpendicular to the line 12 ♈, draw from the extremities of the line ♋ ♑, the lines 12 ♋ and 12 ♑, which will each make with the line 12 ♈, an angle of 23½ degrees, which is the sun's greatest declination.

The points of the other signs may be found on this perpendicular ♋ ♑, by describing from the point ♈, as a centre, through the points ♋, ♑, the circumference of a circle, and dividing it into 12 equal parts, or at every 30 degrees, to mark the commencement of the 12 signs. Join every two opposite points of division, equally distant from the points ♋, ♑, by lines parallel to each other, and perpendicular to the diameter ♋ ♑: these lines will determine, on this diameter, the commencement of the signs; from which, as centres, if circular arcs be described through the point 12, they will represent the parallels of the signs; and therefore must be marked with the appropriate characters as seen in the figure.

A slit must be made along the line ♋ ♑, to admit a thread furnished with a small weight, sufficient to stretch it; and in which it must glide, but not too freely; so that its point of suspension can be shifted to any point of the line ♋ ♑ at pleasure.

These arcs of the signs will serve to indicate the hours when the sun shines, in the following manner: Having drawn at pleasure the line C ♑, parallel to the diameter B 12, fix at its extremity C a small style in a perpendicular direction, and turn the plane of the dial to the sun, so that the shadow of the style shall cover the line C ♑; the thread and plummet being then freely suspended from the sun's

place marked on the line $\approx \gamma$, will indicate the hour on the arc of the same sign at the bottom.

The thread may be furnished with a small bead, to be used as in the preceding problem.

REMARK.—This dial originated from a universal rectilineal dial constructed by Father de Saint-Rigaud, a jesuit, and professor of mathematics in the college of Lyons, under the name of *Analemma Novum*. But though Ozanam has given a conspicuous place to it in his Recreations, as well as another universal rectilineal analemma, it appeared to us that his description of them was too complex to be admitted into a work of this kind.

PROBLEM XXXI.

Method of constructing a Ring-Dial.

Portable ring-dials are sold by the common instrument makers; but they are very defective. The hours are marked in the inside on one line, and a small moveable band, with a hole in it, is shifted till the hole correspond with the degree and sign of the sun's place marked on the outside. Such dials however, as already said, are defective; for as the hole is made common to all the signs of the zodiac, marked on the circumference of the ring, it indicates justly none of the hours but noon: all the rest will be false. Instead of this arrangement therefore, it will be necessary to describe, on the concave surface of the ring, seven distinct circles, to represent as many parallels of the sun's entrance into the signs; and on each of these must be marked the sun's altitude on his entrance into the sign belonging to the parallel to which the circle corresponds. When these points are marked, they must be joined by curved lines, which will be the real hour-lines, as has been remarked by Deschales.

Having provided a ring, pl. 16 fig. 31, or rather described a circle of the size of the ring which is to be divided; and having fixed on B as the point of suspension,

make BA and BO, on each side of B, equal to 51° 31′, for the latitude of the place, suppose London, that is, equal to the distance of the zenith from the equator: then through the points A and O draw the chord AO, and AP perpendicular to it: if the line A 12 be then drawn through A and the centre of the circle, the point 12 will be the hour of noon on the day of the equinox.

To find the other hour-points for the same day, at the commencement of aries and libra; from the centre A describe the quadrant OD; and from the point O set off toward P the sun's altitude at the different hours of the day, as at 1 and 11, 2 and 10, &c; the lines drawn from the centre A through these points of division, if continued to the circumference of the circle B 12 A, will give the hour-points for the day of the equinox.

To obtain the hour-divisions on the circles corresponding to the other signs, first set off, on both sides of the point A, pl. 16 fig. 32, the sun's declination when he enters each of the signs, viz, the arcs AE and AI of 23 degrees, for the commencement of taurus or virgo; of scorpio or pisces; AF of 40° 26′ for the commencement of gemini and leo; AK equal to it for the commencement of sagittarius and aquarius; and AG and AL of 47° for the commencement of cancer and capricorn.

Now to find the hour-points on the circle, that corresponding to the commencement of aquarius, for example, through the point K, which corresponds to the sun's entrance into that sign, draw KP parallel to AO, and also the line K 12: from the same point K describe, between K 12 and the horizontal line KP, the arc QR; on which set off, from R towards Q, the sun's altitude at the different hours of the day, when he enters sagittarius and aquarius, as seen in the figure; and if lines be then drawn from K to these points of division, you will have the hour-points of the two circles corresponding to the commencement of sagittarius and aquarius. By proceeding in the

same manner for the sun's entrance into the other signs, you will have the hour-points of the circles which correspond to them.

Then trace out, on the concave surface of the circle, seven parallel circles, pl. 16, fig. 33, that in the middle for the equinoxes; the two next on each side for the commencement of the signs taurus and virgo, scorpio and pisces; the following two on the right and left for gemini and leo, sagittarius and aquarius; and the last two for cancer and capricorn: if the similar hour-points be then joined by a curved line, the ring-dial will be completed.

The next thing to be done, is to adjust properly the hole which admits the solar rays; for it ought to be moveable, so that on the day of the equinox it may be at the point A; on the day of the summer solstice at G; on the day of the winter solstice at L; and on the other days of the year in the intermediate positions. For this purpose the exterior part of the ring CBD must have in the middle of it a groove, to receive a small moveable ring or hoop, with a hole in it. The divisions L, K, I, A, I, F, G, must be marked on the outside of this part of the ring by parallel lines, inscribing on one side the ascending signs, and on the other the descending: when this construction has been made, it will be easy to place the hole of the moveable part A on the proper division, or at some intermediate point; for if the ring be pretty large, each sign may be divided into two or three parts.

To know the hour; move the hole A to the proper division, according to the sign and degree of the sun's place; then turn the instrument in such a manner, that the sun's rays, passing through the hole, may fall on the circle corresponding to the sign in which the sun is: the division on which it falls will show the hour.

REMARKS.—I. To render the use of this instrument easier, instead of the divisions of the signs, the days corresponding to the commencement of the signs might be

marked out on it: for example, June 21 instead of ♋; April 20, August 20, instead of ♉ and ♍, and so on.

II. The hole A might be fixed, and the most proper position for it would be that which we originally assigned to the day of the equinox; but in this case, the hour of noon, instead of being found on a horizontal line, for all the circles of the signs, according to the preceding method, would be a curved line; and all the other hour-lines would be curved lines also. As this would be attended with a considerable degree of embarrassment and difficulty, it will be better, in our opinion, that the hole A should be moveable.

PROBLEM XXXII.

How the shadow of a style, on a Sun-dial, might go backwards, without a miracle.

This phenomenon, which on the first view may appear physically impossible, is however very natural, as we shall here show. It was first remarked by Nonius or Nugnez, a Portuguese mathematician, who lived about the end of the sixteenth century. It is founded on the following theorem.

In all countries, the zenith of which is situated between the equator and the tropic, as long as the sun passes beyond the zenith, towards the apparent or elevated pole, he arrives twice before noon at the same azimuth, and the same thing takes place in the afternoon.

Let Z, pl. 17 fig. 34, be the zenith of any place situated between E the equator, and T the point through which the sun passes on the day of the summer solstice; let the circle HAQBKH represent the horizon; REQ one half of the equator; TF the eastern part of the tropic above the horizon, and GT the western part. It is here evident, that from the zenith Z there may be drawn an azimuth circle, such as ZI, which shall touch the tropic in a point O, for example; and which shall fall on the horizon in a

point I, situated between the points Q and F, which are those where the horizon is intersected by the equator and the tropic; and, for the same reason, there may be drawn another azimuth, as ZH, which shall touch in *o* the other part of the tropic.

Let us now suppose that the sun is in the tropic, and consequently rising in the point F; and let a vertical style, of an indefinite length, be erected in C. Draw also the lines ICK, and FCN; it is evident that at the moment of sun-rise the shadow of the style will be projected in CN; and that when the sun has arrived at the point of contact O, the shadow will be projected in CK. While the sun is passing over FO, it will move from CN to CK, but when the sun has reached the meridian, the shadow will be in the line CB; it will therefore have gone back from CK to CB: from sun-rising to noon then it will have gone from CN to CK and from CK to CB; consequently it will have moved in a contrary or retrograde direction; since it first moved from the south towards the west, and then from the west towards the south.

Let us next suppose that the sun rises between the points F and I. In this case the parallel he describes before noon will evidently cut the azimuth ZI in two points; and therefore, in the course of a day, the shadow will first fall within the angle KCL; it will then proceed towards CK, and even pass beyond it, going out of the angle; but it will again enter it, and, advancing towards the meridian, will proceed thence towards the east, even beyond the line CL, from which it will return to disappear with the setting of the sun within the angle LCB.

It is found by calculation, that in the latitude of 12 degrees, when the sun is in the tropic on the same side, the two lines CN and CK form an angle of 9° 48′; to pass over which the shadow requires 2 hours 7 minutes.

PROBLEM XXXIII.

To construct a dial, for any latitude, on which the shadow shall retrograde or move backwards.

For this purpose incline a plane, turned directly south, in such a manner, that its zenith shall fall between the tropic and the equator, and nearly about the middle of the distance between these two circles: in the latitude of London, for example, which is 51° 31′, the plane must make an angle of about 38°. In the middle of the plane, fix an upright style of such a length, that its shadow shall go beyond the plane; and if several angular lines be then drawn from the bottom of the style towards the south, about the time of the solstice the shadow will retrograde twice in the course of the day, as above mentioned.

This is evident, since the plane is parallel to the horizontal plane having its zenith under the same meridian, at the distance of 12 degrees from the equator towards the north: the shadows of the two styles must consequently move in the same manner in both.

PROBLEM XXXIV.

To determine the line traced out, on the plane of a dial, by the summit of the style.

We here suppose that the sun, in the course of a diurnal revolution, does not sensibly change his declination; for if he did, the curve in question would be of too complex a nature, and very difficult to determine.

Let the sun then be in any parallel whatever. It may be easily seen that the central solar ray, drawn to the point of the style, describes a conical surface, unless the sun be in the equator; consequently the shadow projected by that point, which is always directly opposite to it, passes over, in its revolution, the surface of the opposite cone, which is united to it by its summit. Nothing then is necessary but to know the position of the plane

which cuts the two cones; for its intersection with the conical surface, described by the shadow, will be the curve required.

Those therefore who have the least knowledge of conic sections will be able to solve the problem. For, 1st, If the proposed place be under the equator, and the plane horizontal; it is evident that this plane intersects the two opposite cones at the summit: consequently, the track of the shadow will be an hyperbola BCD, fig. 17 pl. 35, having its summit turned towards the bottom of the style.

But it may be easily seen, that as the sun approaches the equator, this hyperbolic line becomes flatter and flatter; and at length, on the day of the equinox, is changed into a straight line; that it afterwards passes to the other side, and always becomes more and more curved, till the sun reaches the tropic, &c.

We shall here add, that the sun rises every day in one of the asymptotes of an hyperbola, and sets in the other.

2d. In all places situated between the equator and the polar circles, the track of the shadow, on a horizontal plane, is still an hyperbola; for it may be easily seen that this plane cuts the two opposite cones, united at their summits, which are described by the solar ray that passes over the point of the style; since in all these latitudes the two tropics are intersected by the horizon.

3d. In all places situated under the polar circle, the line described by the shadow on a horizontal plane, when the sun is in the tropic, is a parabolic line: but that described on other days is hyperbolic.

4th. In places situated between the polar circle and the pole, as long as the sun rises and sets, the track described by the shadow of the summit of the style, is an hyperbola: when the sun has attained to such a high latitude that he only touches the horizon, instead of setting, the track is a parabola; and when the sun remains the whole day above the horizon, it is an ellipsis, more or less elongated.

5th. Lastly, it may be easily seen that under the pole the track of the shadow of the summit of the style is always a circle; since the sun, during the whole day, remains at the same altitude.

Corollary.—As the arcs of the signs are nothing else than the track of the shadow of the summit of the style, when the sun in his diurnal motion passes over the parallel belonging to the commencement of each sign, it follows that these arcs are all conic sections, having their axis in the meridian or substylar line. In horizontal dials, constructed for places between the equator and the polar circles, and in all vertical dials, whether south, north, east, or west, constructed for places in the temperate zone, they are hyperbolas. This may be easily perceived, on the first view, in most of the dials in our latitudes.

These observations, which perhaps may be considered by common gnomonists as of little importance, appeared to us worthy the consideration of those more versed in geometry; especially as some of them may not have attended to them. For this reason we resolved to give them a place in this work.

PROBLEM XXXV.

To know the hours on a sun-dial, by the moon shining on it.

This problem will not appear difficult to those who know that the moon's passage by the meridian is every day later by about 48 minutes; that when new, she passes the meridian exactly at the same time as the sun; and when full, 12 hours after.

First, find the moon's age, which is given in every common almanac, where the days and hours of the new and full moon are always marked. Let us suppose then, that at the time when you wish to know the hour, 6 days and a half have elapsed since new moon. Multiply 48 minutes, or $\frac{4}{5}$ of an hour, by $6\frac{1}{2}$, and the product will be $\frac{26}{5}$, or 5 hours 12 minutes, which must be added to the hour

indicated by the dial. If the dial therefore indicates 4 hours, the real time will be 9 hours 12 minutes.

But the hour may be found much more exactly in the following manner. First find at what hour of the day the moon has passed, or will pass the meridian; which may be determined by the help of a common almanac, where the times of the moon's rising and setting are marked; for if the interval between the rising and setting be halved, it will give the time of the moon's passing the meridian nearly.

Let us suppose then, that the moon has passed the meridian at 3^h 30^m in the afternoon; the difference of this passage from that of the sun, were the moon fixed in the heavens, would be 3½ hours later than the sun. Consequently if the moon indicates, on a sun-dial, 7½ in the evening, we may conclude, on the supposition of the moon being motionless, that it is eleven at night. But since, in this interval of 7½ hours, the moon has had a retrograde motion towards the east, which occasions in her passage of the meridian a retardation of 48^m daily; which is at the rate of 2 minutes per hour, we shall have for 7½ hours 15^m, which must be added to the hour indicated by the moon, over and above the quantity by which her passage over the meridian has been later than that of the sun.

If the moon had passed the meridian before the sun, it would be necessary to deduct from the hour indicated by the moon the quantity by which she preceded the sun, and to add to the remainder as many times two minutes as the hours she indicated. But this calculation, however short, may be avoided by means of the following small machine.

This machine consists of two circular plates of brass or wood, or paste-board, pl. 18 fig. 36, one of which AIGH is fixed, and the other *b e f l* moveable. On the fixed plate is described a circle *a i g h*, divided into 24 equal parts, representing the 24 hours of the day; each of which must

be subdivided into halves and quarters. Above this piece is applied the other plate *b e f l*, in such a manner as to be moveable around the centre c, which is common to both: and the circumference of the latter is divided into parts which represent the hours indicated by the moon on a sun-dial. These hours are not equal to those of the sun described on the fixed plate, but must be each 2 minutes larger; since the moon's daily retardation is about 48 minutes, or 12 minutes in 6 hours. Therefore, since the degree of a sign is equal to 4 minutes in time, it is evident that 3 degrees are equivalent to 12 minutes of time. For this reason, having drawn the south-line ACG, set off on each side from the point *b*, to *e* and *l* 93 degrees for 6 hours; and divide each of these spaces into six equal parts, to represent as many hours; then into halves and quarters, as seen in the figure.

To use this instrument, place the index *n b* of the moveable piece at the hour of the moon's passing the meridian on the proposed day; then observe the hour indicated by the moon on a horizontal sun-dial, and opposite to the same hour on the moveable piece, you will find, on the other, the true hour of the day.

PROBLEM XXXVI.

To construct a dial to show the hour by the Moon.

To employ a dial of this kind, it is necessary to know the moon's age, which may be always found either by a common almanac, or by some of the methods we have already pointed out, under the head astronomy.

To describe a lunar dial on any plane whatever, such for example as a horizontal one, first trace out on it a horizontal sun-dial for the given latitude, and draw the two lines 5 7, 3 9 parallel to the equinoctial, pl. 18 fig. 37; the first of which being assumed as the day of full moon; the second will represent that of new moon, where the lunar hours correspond with the solar; and hence the hour-

points marked on those two parallels, by lines proceeding from the centre of the dial A, are common to the sun and the moon.

Then divide the space bounded by the two parallel lines 3 9, 5 7 into 12 equal parts; and through the points of division draw as many parallel lines, which will represent those days of the moon when she successively recedes an hour by her own motion towards the east, and on which she consequently passes the meridian every day an hour later. The first parallel, 4, 10, being the day on which the moon passes the meridian an hour later than the sun, the point B of 11 hours, by the moon, will be the point of noon or 12, according to the sun; as the next 5, 11 represents a day on which the moon passes the meridian 2 hours after the sun, the point C of 10 hours by the moon, will be the point of noon by the sun; and so of the rest.

It is now evident, that if the points 12, B, C, and all the others belonging to noon, which can be found by the same method, be joined by a curved line, this curve will be the lunar meridian. The other lunar hour-lines may be easily traced out also by a similar process.

Because the interval between the moon's conjunction with the sun and her opposition, that is, between the time of new and full moon, or that when she is diametrically opposite to the sun, so that she rises when the sun sets, is about 15 days, all the preceding parallels, except the two first 38, 39, must be effaced; and instead of dividing the interval into 12 equal parts, it must be divided into 15; in order that you may draw, through the points of division, other parallels, which will represent the days of the moon's age; and which therefore must be marked with the proper figures along the meridian line, as seen in the plate; by which means the true hour of the night may be known, when the moon shines, in the following manner.

In the centre of the dial A, fix an axis or pin, so as to form at that centre with the line A 12 an angle equal to the

elevation of the pole above the plane of the dial, which we suppose to be horizontal: this axis, by its shadow on the current day of the moon, will indicate the hour as required.

PROBLEM XXXVII.

To describe the arcs of the signs on a sun-dial.

Of the appendages added to sun-dials, the arcs of the signs may be classed among the most agreeable; for by their means we can know the sun's place in the different signs, and as we may say can fol w his progress through the zodiac. We therefore thought it our duty not to omit, in this work, the method of describing them.

For the sake of brevity, we shall suppose that the plane is horizontal. First describe a dial such as the position of the plane requires, that is, a horizontal one, and fix in it an upright style, terminated by a spherical button, or by a circular plate, having in its centre a hole, of a line or two in diameter; according to the size of the dial. Then proceed as follows:

Let it be required, for example, to trace out the arc corresponding to the commencement of scorpio or pisces. First find, by the table of the sun's altitude, at each hour of the day in the latitude of London, for which we suppose the dial to be constructed, the altitude when he enters these two signs. As this altitude is 26° 43′, make the triangle STE, pl. 19 fig. 38, in which ST is the height of the style, and such that the angle SET shall be equal to 26° 43′: the point E will be the first point of the arc of these two signs.

Then find, in the same table, the sun's altitude at one in the afternoon of the same day, which will be found equal to 25° 30′; and construct the triangle STF, in such a manner that the angle F shall be 25° 30′; then from the bottom of the style S, as a centre, with the radius SF, describe an arc of a circle, intersecting the lines of I and XI hours in

the two points G and H: these will be the points of the arc of those signs on the lines of I and XI.

If the same operation be repeated for all the other hours, you will have as many points, through which if a curved line be drawn, by means of a very flexible ruler, you will obtain the arc of the signs scorpio and pisces.

By employing the like construction, the arcs belonging to the other signs may be obtained.

Another Method.

According to this method, the table of the sun's altitude at the different hours of the day is not requisite. A simple graphic operation is sufficient; but as a figure called the *triangle of the signs* is employed, it is necessary that we should first show how it is constructed.

Draw the line AB, pl. 19 fig. 39, of an indefinite length; and from the point A, as a centre, with any radius AB, describe an arc of a circle; make the arcs BE and B*e* each equal to 11° 30′, which is the sun's declination at the commencement of taurus and virgo, scorpio and pisces, the two former northern, and the two latter southern; and draw the lines AE, A*e*; the former of which will belong to the first two signs, and the latter to the other two.

In like manner make BF and B*f* equal to 20° 12′, and draw AF, and A*f*; the former of which will correspond to the signs gemini and leo, and the latter to sagittarius and aquarius.

Lastly, if BG and B*g* be made equal to 23° 28′; the line AG will correspond to cancer, and A*g* to capricorn.

We shall now suppose that it is required to describe the arcs of the signs on a horizontal dial: having fixed in the proper place, as above directed, an upright style ST, fig. 39 and 40, draw the equinoctial and hour-lines; and on AB raise a perpendicular AD, equal to TP the distance of the summit of the style from the centre of the dial P.

Now, if you are desirous of having marked on the me-

ridian the seven points of division of the arcs of the signs, make AC, fig. 39, equal to RT, the distance of the summit of the style from the equinoctial; and draw the line DC, which will intersect the lines of the signs, in the points 6, 4, 2, C, 1, 3, 5: if these points be transferred in the same order to the meridian, fig. 40, making R 6 equal to C 6, R 4 to C 4, R 2 to C 2, R 1 to C 1, &c, you will have the points through which the sun passes at noon, on the days when he enters into the different signs.

Let it now be required to find the same points on one of the hour-lines, that for example of III and IX. From the bottom of the upright style S let fall on that hour-line PM a perpendicular SV, fig. 40, and continue it till it meets, in the point N, the semicircle described on PM as a diameter: then make AH, fig. 39, equal to PN, and AI equal to PM; and draw HI through the triangle of the signs: this line will be intersected by the seven lines of the signs in seven points, which being transferred, in the same order, to the hour-line proposed, will determine those where it will be met by the shadow of the summit of the style, on the sun's entrance into each of the signs.

If all the points, corresponding to the same sign on the hour-lines, be then joined, by making a curved line to pass through them, it will be the parallel of that sign.

Of the different kinds of Hours.

Every thing hitherto said has related only to the equinoctial and equal hours; such as those by which time is reckoned in England, the day being supposed to begin at midnight, and the hours being counted to the following midnight, to the number of 24, or twice twelve. This is the most common method of computing the hours in Europe. The astronomical hours are almost the same; the only difference is, that the latter are counted, to the number of 24, from the noon of one day to the noon of the day following.

But there are some other kinds of hours, which it is proper we should here explain; because they are sometimes traced out on sun-dials: such are the natural or Jewish hours, the Babylonian, the modern Italian, and those of Nuremberg.

The natural or Jewish hours begin at sun-rise; and there are reckoned to be 12 between that period and sun-set: hence it is evident that they are not of equal length, except on the day of the equinox: at every other time of the year they are unequal. Those of the day, in our hemisphere, are longer from the vernal to the autumnal equinox: those of the night are, on the other hand, longer while the sun is passing through the other half of the zodiac.

The Babylonian hours were of equal length, and began at sun-rise; they were counted, to the number of 24, to sun-rise of the day following.

The modern Italian hours, for the ancient Romans counted nearly as we do from midnight to midnight, are reckoned to the number of 24, from sun-set to sun-set of the day following; so that on the days of the equinox noon takes place at the 18th hour, and then, as the days lengthen, the astronomical noon happens at $17\frac{1}{2}$ hours, then at 17 hours, &c; and vice versa. This singular and inconvenient method has had its defenders, and that even among the French; who have found that with a pencil, and a little astronomical calculation, one may fix the hour of dinner with very little embarrassment.

However, as these hours are still used throughout almost the whole of Italy, we think it our duty to show here the method of describing them, by way of a Gnomonical curiosity.

PROBLEM XXXVIII.

To trace out, on a dial, the Italian hours.

Describe first on the proposed plane, which we here suppose to be a horizontal one, a common horizontal dial,

with the astronomical or European hours: delineate on it also the arcs of the solstitial signs, cancer and capricorn; as well as the equinoctial line, which is the arc of the equinoctial signs.

Then observe that, on the days of the equinox, noon, for a dial constructed at London, takes place at the end of the 18th Italian hour; and on the day of the summer solstice at 17 minutes after the 16th hour. Noon, therefore, or 12 hours, counted according to the astronomical hours, corresponds, on the day of the equinox, to the 18th Italian hour; and on the day of the so tice to 17 minutes after the 16th; consequently the 18th Italian hour, on the day of the summer solstice, will correspond to 17 minutes past 2, counted astronomically. Join therefore, (pl. 20 fig. 41), by a straight line, the point of noon marked on the equinoctial line, and that of 2 hours 17 minutes on the tropic or arc of the sign cancer, and inscribe there 18 hours. Join also by transversal lines 1 hour on the equinoctial and 3^h 17^m on the arc of cancer; then 2^h and 4^h 17^m; &c; and before noon 11^h and 1^h 17^m; 10^h and 12^h 17^m; 9^h and 11^h 17^m; &c: efface then the astronomical hours, which we suppose ought not to appear, and continue the above transversal lines till they meet the parallel of capricorn, inscribing at their extremities the proper numbers; by which means you will have your dial traced out as seen fig. 4, pl. 20.

Remark.—It may be easily seen, by the above example, what calculation will be necessary for a latitude different from that of London, where the length of the day at the summer solstice, is 16 hours 34 minutes, and at the winter solstice only 7 hours 44 minutes. In another latitude, where the longest day is only 14 hours and the shortest 10, noon at the summer solstice will take place at the end of the 17th Italian hour. Noon therefore, or 12 hours, counted astronomically, will on the day of the solstice correspond to the 17th Italian hour; and consequently

the 18th Italian hour, at the same period, will correspond to 1 in the afternoon counted astronomically. To have the hour-line of the 17th Italian hour therefore, nothing will be necessary, but to join the point of 1 in the afternoon, on the arc of cancer, and the point of noon on the equinoctial. And the case will be the same with the other hours.

PROBLEM XXXIX.

To trace out on a dial the lines of the natural or Jewish hours.

We have already said, that the equal hours which can be counted from sun-rise to sun-set, to the number of 12, are called the natural hours, for it is this interval of time which really forms the day.

This kind of hours may be easily traced out on a dial, which we shall here suppose to be horizontal. For this purpose, it will be first necessary to draw the equinoctial, and the two tropics by the preceding methods.

Now it must be observed that as, in the latitude of London, the sun, on the day of the summer solstice, rises at $3^h\,43^m$, and sets at $8^h\,17^m$, the interval between these periods is equal to $17^h\,34^m$; consequently, if we divide this duration into 12 parts, each of these will be about $1\frac{1}{2}$ hour; for this reason, draw lines from the centre of the dial to the points of division on the equinoctial, corresponding to $5\frac{1}{2}$ hours, to 7 hours, to $8\frac{1}{2}$ hours, to 10 hours, to $11\frac{1}{2}$ hours, to 1 hour, and so on; but marking only, on the tropic of cancer, the points of intersection which these hours form with it.

In like manner, as the sun at the winter solstice, in the latitude of London, rises at $8^h\,8^m$, and sets at $3^h\,52^m$, the duration of the day is only 7 hours 44 minutes; which being divided into 12 parts, gives for each about 40 minutes, or $\frac{2}{3}$ of an astronomical hour. Draw therefore the hour-lines corresponding to $8\frac{2}{3}$ hours, to 9^1 hours, to 10

hours, and so on; marking only the points where they intersect the tropic of capricorn; then, if the corresponding points of division, on the two tropics and the equinoctial, be joined by a curved line, the dial will be described, as seen plate 21 fig. 44.

If more exactness be required, it will be necessary to trace out two more parallels of the signs, viz, those of taurus and scorpio, and to find on each, by a similar process, the points corresponding to the natural hours: the natural hour-lines may then be made to pass through 5 points, by which means they will be obtained with much more exactness.

PROBLEM XL.

To find the hour, by means of some of the circumpolar stars.

The hour may be known by a star's passage by the meridian, or even by its altitude; for by means of any Ephemeris, and a short calculation, we can easily determine how much any star precedes or is behind the sun in culminating, or coming to the meridian; and when this is known, together with its declination, the hour may be found by observing its altitude. But as this process would be too complex for the generality of our readers, we shall confine ourselves to a solution of the above problem; to facilitate which, a small instrument, called the nocturnal, has been invented. It is adapted for employing the most brilliant of the two last stars in the little bear, which are called its guards. The construction of it is as follows. Provide a circular piece of wood or metal, pl. 20 fig. 42, and having described on it a circle, divide its circumference into 365 parts, corresponding to the days of the year; which must be afterwards distributed into months, according to the number that each contains.

To this circular piece apply another, moveable around the centre, and divide the circumference of it into 24 equal parts, denoting the 24 hours of the day. At each of these

divisions there must be a small notch on the edge, in order that these parts may be counted in the dark by the touch. One of these notches however must be longer than the rest, for a purpose which will be explained hereafter.

Then affix to the edge of the lower piece a small handle; the middle of which ought to be in a line with the centre of the instrument, passing through the 7th of November; because on that day the above star passes the meridian at the same time as the sun: that is, above the pole at noon, and below it at midnight.

Lastly, adapt to the instrument an index, moveable around a pin in the centre; and let a hole be pierced in the pin, in order to apply the eye to it.

To use this instrument, first make the edge of the longest notch correspond with the day of the month: then apply your eye to the centre, and, turning towards the north, look at the pole star, holding the plane of the instrument in a direction as perpendicular as possible to the visual ray, and the handle of it in the vertical plane; then move the index till the edge of it touches the above star, or the brightest of the guards of the little bear, and count the number of notches between the index and the longest notch; this number will be that of the hours elapsed after midnight.

The instrument might be easily adapted to any other star: nothing would be necessary but to make the small handle of the instrument correspond with the day of the month when the star passes the upper meridian with the sun: in every thing else the construction would be the same.

We shall terminate this part of our work with a sort of gnomonical pleasantry.

PROBLEM XLI.

To tell the hour of the day by means of the left hand.

It may be easily conceived that there can be very little

precision in a method of this kind; and therefore we attach no more value to it than it deserves.

Extend the left hand in a horizontal position, so that the inside of it shall be turned towards the heavens: then take a bit of straw or wood, and place it at right angles, at the joint, between the thumb and the fore-finger: it must be equal in length to the distance from that joint to the end of the fore finger, and must be held upright, as represented in the figure, pl. 20 fig. 43, at A: this piece of stick or straw supplies the place of a style.

Turn the bottom of the thumb towards the sun, the hand being still extended, till the shadow of the muscle which is below the thumb terminate at the line of life, marked C. If the wrist or bottom of the hand be then turned towards the sun, the fingers being kept equally extended, the shadow of the bit of straw or stick will indicate the hour. When the shadow falls at the tip of the fore finger, it denotes 5 in the morning or seven in the evening; at the end of the middle finger, it denotes 6 in the morning and evening; at the end of the next finger, 7 in the morning and 5 in the evening; at the end of the little finger, 8 in the morning and 4 in the afternoon; at the nearest joint of the little finger, 9 in the morning and 3 in the afternoon; at the next joint of the little finger, 10 in the morning and 2 in the afternoon; at the root of the little finger, 11 in the morning and 1 in the afternoon; in the last place, when the shadow falls on that line of the hand marked D, which is called the table line, it will indicate 12 o'clock or noon.

Some curious operations in regard to Gnomonics we have been obliged here to omit; as it would have been necessary to add the demonstrations. We however think it our duty to terminate this article with a list of the principal works on Gnomonics, which may be consulted by those who are desirous of farther information on this subject.

We shall not speak here of the Gnomonics of Clavius, because that mathematician seems to have studied the art of rendering what is simple of itself exceedingly obscure. We shall even confine ourselves to French and English works, as our object is not to give a complete bibliography of the art.

La Gnomonique of M. de la Hire, which appeared in 1683, in duodecimo, is worthy of attention, though a certain kind of obscurity generally prevails throughout the works of that mathematician; it contains the solution of a great many problems relating to the astronomical part of dialling.

Ozanam's work on the same subject is clearer, and better adapted to the capacity of common readers; it still holds a place among other works of the same kind, of a more modern date. The celebrated Picard did not think it beneath him to teach the method of constructing large sun-dials by trigonometrical calculation. This treatise may be found in the 7th volume of the old Memoirs of the Academy.

An academician of Montpellier, published in the Memoirs of the Royal Academy of Sciences, for the year 1707, the analogies employed to determine the hour-angles for all dials, however situated; together with the demonstrations of them.

After that period a great many treatises on gnomonics appeared in France; such as *La Gnomonique de M. Rivard*, Paris 1767, 8vo. A clear and methodical work, which has gone through several editions: that of M. de Parcieux, at the end of his *Trigonometrie Rectiligne et Spherique*, published at Paris in 1741, 4to; a work which ought to be studied by all those who wish to acquire a correct knowledge of this part of the mathematics. The article on gnomonics in the 4th volume of Wolf's Course of the Mathematics is exceedingly clear and concise. We can recommend also to those desirous of delineating sun-dials

with great exactness, *La Gnomonique pratique, ou l'Art de tracer les Cadrans solaires avec beaucoup de précision*, &c, par Dom Bèdos de Celles; a work first printed in 1770, 8vo, and afterwards in 1774 with a great many additions. The author employs chiefly trigonometrical calculation, and enters into minute details respecting every thing that relates to practice; for one may be perfectly well acquainted with the theory, and yet embarrassed in the application of it. Useful tables, calculated for the whole extent of France, will be found in *La Gnomonique mise à la portée de tout le monde*, par Joseph-Blaise Garnier, Marseilles, 1773, 8vo. In other respects, this work is of no great value. In regard to the *Horlogiographie* of Father de la Madelaine, though very common, we can say nothing farther than that it is fit only for country stonemasons, who make it a part of their business to construct sun-dials.

We cannot here help taking notice of the ingenious manner in which the celebrated S'Gravesande, in his Essay on Perspective, printed at Leyden in 1711, considers the general problem of tracing out a sun-dial: he reduces it to a simple problem of perspective, which he solves according to the principles of that branch of optics. This part of his work is remarkable for its elegance, its precision, and its universality. To the above list of works on gnomonics, we shall add in English, *Emerson's Dialling*, published along with his Mathematical principles of Geography; also *Martin's Principles of Dialling*; and, for those who wish to describe dials merely by the rule and compasses, *Leadbetter's Mechanic Dialling*.

APPENDIX.

Containing a general method of describing sun-dials whatever be the declination or inclination of the plane.

When this part of our work was nearly printed off, it occurred to us that our geometrical readers might perhaps find fault with us for omitting to give a geometrical method of describing inclining and declining dials. Finding that the matter destined for this volume would leave us sufficient room, we shall therefore here describe a very simple and ingenious method for that purpose; as by means of a few calculations, the construction of any dial, however complex be the inclination and declination of its plane, will be as easy as that of a common horizontal or vertical dial.

This method is founded on a very ingenious consideration, viz, that any plane whatever is always a horizontal plane to some place of the earth; for a plane being given, it is evident that there is some point of the earth the tangent or horizontal plane of which is parallel to it. It is evident also, that two such parallel planes will show the same hours at the same time. Thus, for example, if we suppose at London a plane inclining and declining in such a manner, as to be parallel to the horizontal plane of Ispahan; then a dial traced out on that plane, as if it were horizontal, will give the hours of Ispahan; so that when the shadow falls on the substyle, we may say that it is noon at Ispahan, &c.

But as the hours of Ispahan are not those wanted at London, it is necessary that we should find out the means of delineating those of London, which will not be attended with much difficulty, when the difference of longitude between these two cities is known. Let us suppose then that it is exactly 45 degrees, or 3 hours: when it is noon at London then, it will be 3 in the afternoon at Ispahan; and

when it is 11 in the forenoon at the former, it will be 2 in the afternoon at the latter, &c. Consequently, on this dial, which we suppose to be horizontal, if we assume the line of 3 o'clock as that of noon, and mark it 12; and if we assume the other hour-lines in the same proportion, we shall have at London the horizontal dial of Ispahan, which will indicate not the hours of Ispahan, but those of London, as required.

We flatter ourselves that we have here explained the principle of this method in a manner sufficiently clear, to make it plain to such of our readers . have a slight knowledge of geometry or astronomy; but to render the application of it more familiar, we shall illustrate it by an example.

Let us suppose then, at London, a plane forming with the horizon an angle of 12 degrees, and declining towards the west 22½ degrees.

The first operation here is, to find the longitude and latitude of that place of the earth where the horizontal plane is parallel to the given plane.

For this purpose, let us conceive an azimuth AI perpendicular to the given plane, pl. 22 fig. 45, and in this azimuth, which we suppose to be traced out on the surface of the earth, let us assume on that side which is towards the upper part of the plane, an arc AH, equal to the inclination of that plane to the horizon: the extremity of this arc, that is the point H, will be that point of the earth where the horizon is parallel to the given plane. This is so easy to be comprehended that it requires no demonstration. Let us next conceive a meridian PH, drawn from the pole P to the point H: it is evident that this will be the meridian of the given plane; and that the angle APH, formed by this meridian and that of London, will give the difference of longitude of the two places. We must therefore determine this triangle, and to find it we have three things given, viz, 1st, AP the complement of the latitude

of London, which is 38° 29′; 2d, AH the distance of London from the place, the horizontal plane of which is parallel to the given plane, and which is 12°; 3d, the angle PAH, comprehended between these two sides, which is equal to the right angle HAL plus PAL, or that which the plane forms with the meridian.

By resolving this spherical triangle, it will be found, that the angle at the pole APH, or that formed by the two meridians, is 5° 59′; which is the difference of longitude between the two places A and H.

The latitude of the place H will be found also by the solution of the same triangle; for it is measured by the complement of the arc PH, of the triangle PAH: according to calculation it is 40° 15′*.

Thus, a plane inclining 12° at London, and declining to the west 22½ degrees, is parallel to the horizontal plane of a place which has 5° 59′ of longitude west from London, and 40° 15′ of latitude. The latter also is the angle which the style ought to form with the substyle; for the angle which the axis of the earth forms with the horizontal plane is always equal to the latitude.

It is here evident, that when it is noon at the place H, it will be 23m 56s after noon at the place A; for 5° 59′ in longitude correspond to 23m 56s in time. Consequently, at the place A, when the shadow of the style falls on the

* Trigonometrical calculation may be avoided by means of a graphic operation exceedingly simple, and which is a consequence of that taught in Prob. XXII. In a circle of a convenient size, pl. 22 fig. 46, assume an arc *pa* equal to PA, fig. 45; make *ah* equal to AH; and from the point *h* let fall a perpendicular *hi*, on the radius *ca*. On *hi* describe a quadrant, or make *hk* equal to the arc which measures the declination of the plane, or equal to the supplement of the angle PAH, draw *kl* perpendicular to *hi*, and from the point *l*, draw *lm* perpendicular to the radius *cp*, and let *lm* be continued till it meet the circle in *n*: the arc *pn* will be equal to PH, and if an arc of a circle be described on *mo*, and if *lx* be drawn perpendicular from the point *l*, so as to meet this arc in *x*, the angle *xml* will be equal to the required angle *p* of the triangle APH.

substyle, which is the meridian of the plane, it will be 23^m 56^s after twelve at noon. To find therefore the hour of noon, it will be necessary to draw, on the west side of the substyle, an hour-line corresponding to 11^h 36^m 4^s, or 11^h 36^m. By the like reasoning, it will be found that 11 in the morning, at the place A, will correspond to 10^h 36^m, at the place H, &c. In the same manner, 1 in the afternoon, at the place A, will correspond to 12^h 36^m, or 36^m after twelve, at the place H: 2 o'clock will correspond to 1^h 36^m; 3 o'clock to 2^h 36^m, and so of the rest.

Thus if we suppose the substyle of the plane, on which the dial ought to be described, to be the meridian, it will be necessary to describe a dial which shall indicate, in the forenoon, 11^h 36^m; 10^h 36^m; 9^h 36^m; 8^h 36^m; &c; and in the afternoon 12^h 36^m; 1^h 36^m; 2^h 36^m; 3^h 36^m; 4^h 36^m; &c.

When these calculations have been made, the dial may be easily constructed. For this purpose, first find, by prob. 3, the substyle, which is the meridian of the plane. We shall suppose that it is PE, fig. 47, and that P is the centre of the dial. Having assumed PB of a convenient length, draw, through the point B, the line ABC, perpendicular to PE: if A be the western side, the line P*d*, which corresponds to 11 hours 36 minutes, or which is distant from the meridian 24 minutes in time, may be found by making use of the following analogy:

As radius,

Is to the cosine of the latitude, which is 40° 15′;

So is the tangent of the hour-angle corresponding to 24^m in time, or the tangent of 6°,

To a fourth term, which will be the tangent of the angle BP*d*.

By this analogy, it will be found equal to 80 parts of which PB contains 1000: if 80 of these parts therefore, taken from a scale, be set off from B towards *d*, and if P*d*

be then drawn, we shall have the hour-line of 11 hours 36 minutes, for the plane of the dial, or of the place H.

The line P*e*, of 10 hours 36 minutes, will be found in like manner, by this analogy:

As radius,

Is to the cosine of 40° 15′;

So is the tangent of the hour-angle corresponding to 10^h 36^m, or the tangent of 21°, to the tangent of the angle BP*e*.

This tangent will be found equal to 293 of the above parts: if this number of parts therefore, taken from the same scale, be laid off from B to *e*, we shall have the hour-line P*e*, corresponding to 10 hours 36 minutes.

The lines of the other hours before noon may be found in the like manner: the two first terms of the analogy are the same, and the third is always the tangent of an angle successively increased by 15°: these tangents therefore will be those of 6°, 21°, 36°, 51°, 66°, the logarithms of which must be added to the cosine of 40° 15′; and if the logarithm of radius be subtracted, the remainders will be the logarithms of the tangents of the hour-lines: these tangents themselves will be for B*d*, B*e*, &c, 80, 293, 554, 942, 1732, 4814, &c, in parts of which the radius or PD contains 1000.

A similar operation must be performed for the hours in the afternoon. As 36^m in time correspond to 9°, the first hour-angle will be 9°; the second, by adding 15°, will be 24°; the third 39°; the fourth 54°; &c. The following proportions then must be employed:

As radius,

Is to the cosine of 40° 15′;

So is the tangent of 9°, or 24°, or 39°, &c.

To a fourth term,

Which will be the tangent of the angle BP*l*, or BP*m*, or BP*n*, &c.

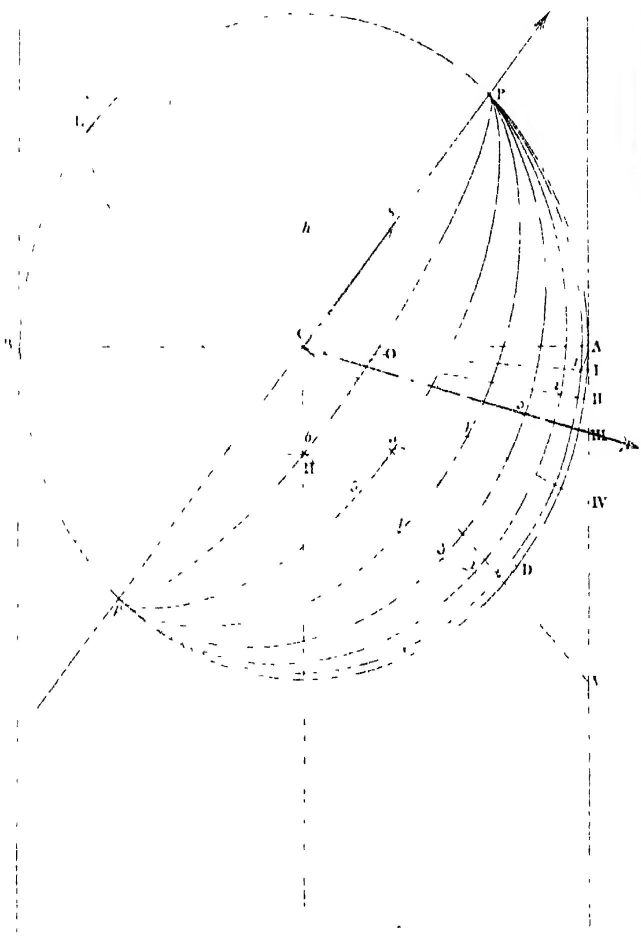
Hence, if the logarithm of the sine of 49° 45′ be suc-

cessively added to the logarithmic tangent, of 9°, 24°, 39°, 54°, &c, and if radius be subtracted from the different sums, we shall have the logarithms of the tangents of the angles which the hour-lines Pl, Pm, Pn, &c, form with the substyle; and these tangents themselves will respectively be 121, 339, 618, 1050, 1988, 7268, parts, of which PB contains 1000. If these numbers therefore, taken from the same scale as before, by means of a pair of compasses, be set off from B to l, from B to m, from B to n, &c, and if the lines Pl, Pm, Pn, Po, &c, be then drawn, the dial will be nearly completed; as nothing will be necessary but to mark the point d with XII, because Pd is the meridian of the place A; and to mark the other hour-points with the numbers which belong to them, as seen in the figure.

To avoid the trouble of tracing out more hour-lines than are necessary, it will be proper first to determine at what hour the sun rises and sets on the given plane, at the time of the longest day; which may be easily done by means of the following consideration.

It may be readily seen that if we suppose two parallel planes, in two different places of the earth, the sun will begin to illuminate both of them at the same moment; and that he will also set to both at the same time. The plane of the dial in question, being parallel to the horizontal plane of a place which has 40° 15′ of north latitude, nothing is necessary but to know at what hour the sun will rise in regard to that plane on the longest day. But it will be found, that in the latitude of 40° 15′ the longest day is 15 hours 24 minutes, or that the sun rises on that day 7 hours 42 minutes before noon, and sets at 42 minutes past 7 in the evening. It will be sufficient then, on the dial in question, to make the first hour-line in the morning that of 4 hours 15 minutes, and the last in the evening 7 hours 30 minutes.

Fig 1



MATHEMATICAL

AND

PHILOSOPHICAL

RECREATIONS.

PART EIGHTH.

Containing some of the most Curious Problems in Navigation.

NAVIGATION may be classed among those arts which do the greatest honour to the human invention; for in no department of science is the ingenuity of man displayed to more advantage than in this art, by which he conducts himself through the wide expanse of the ocean, without any other guide than the heavenly bodies and a compass; by which he subdues the winds, and even employs them to enable him to brave the fury of the ocean, which they excite against him; in short, an art which connects in social intercourse the two worlds; forms the principal source of the industry, commerce, and opulence of nations. Hence one of our poets very justly says,

> Le trident de Neptune est le sceptre du monde.

But, this is not a proper place for entering into a dissertation on the utility of navigation. As mathematicians

therefore, we shall only observe, that navigation may be considered under two points of view. According to the first, it is a science which depends on astronomy and geography: considered in this manner it is called piloting, which is the art of determining the course that ought to be pursued in order to go from one place to another, and of knowing at all times that point of the earth at which a ship has arrived. According to the other, it is an art founded on mechanics and the moving powers of the vessel: considered under this point of view, it is called manœuvring, and teaches how to give to that ponderous mass, which cleaves the billows, the necessary direction by means of the sails and the rudder.

We shall here present the reader with every thing most curious in both these parts of navigation.

PROBLEM I.

Of the curve which a vessel describes on the surface of the sea, when she sails on the same point of the compass.

When a ship is about to set sail, it is necessary to find out the proper course; that is, to determine the direction in which she ought to proceed, in order to arrive, in the shortest time and with the greatest safety, at the place of her destination. When this direction, or the angle it forms with the meridian, has been determined, it is always pursued, unless particular circumstances prevent it. A vessel by thus steering for several days, on the same point of the compass, describes a line which always forms the same angle with the meridians: this is what is called the *loxodromic* line, or oblique course; and there hence results on the surface of the earth a peculiar curve, the nature and properties of which have excited the attention of mathematicians. On these properties the practical rules of navigation have been founded; and, as they are very remarkable, they deserve to be explained.

We presume that the reader is acquainted with the na-

ture of the compass, the different points, &c; and with the elements of navigation; for it is impossible that we should here enter into details merely elementary.

Let us suppose that the sector ACB, pl. 1 fig. 1, represents a portion of the spherical surface of the earth, of which C is the pole, and AB the equator, or only the arc of a parallel comprehended between two meridians, as AC and BC; and that CD, CE, and CF, represent so many meridional arcs, very near to each other.

Let a vessel depart from the point A of the arc AB, the meridian of which is AC, and proceed on a course forming with that meridian the angle CAH, less than a right angle, for example an angle of 60 degrees; the vessel will describe the line AH, by which means she will always change her meridian. When she arrives at H, under the meridian CD, let her continue in the same course, making with the meridian the angle CHI, equal to the former; and so on, describing the lines AH, HI, IK, &c, always making the same angle (60°) with the meridians CA, CH, CI, CK, &c. As her course is continually inclined to the meridian at an angle of 60 degrees, it may be readily seen that the line AHIK, will not be the arc of a great circle on the surface of the sphere, for it is demonstrated in spherics, that if AHK were a circle of this kind, the angle CHI would be greater than CAH, and CIK greater than CHI, and so on. The case would be the same if the curve AHIK were an arc of a lesser circle of the sphere; hence there is reason to conclude, that the curve described by a ship, when she always proceeds on the same course, is a peculiar curve, which constantly approaches the pole.

REMARKS.—I. It is here evident, that when the loxodromic angle vanishes; that is, when the vessel steers directly north or south, the loxodromic line is an arc of the meridian.

But, if the angle be a right angle, and if the vessel be under the equator, she will describe an arc of the equator.

In the last place, if out of the equator, she will describe a parallel.

II. If the loxodromic line AKL, be divided into several parts, so small that they may be considered as straight lines, and if as many parallels or circles of latitude be made to pass through the points of division H, I, K, &c, all these circles will be equal and equally distant from each other; so that, by making meridional arcs to pass through the same points of division, the portions of these meridians, such as DH, MI, NK, &c, will be equal, as well as the corresponding arcs AD, HM IN, &c. This equality however will not be in degrees, but in miles, as may be easily demonstrated; for the triangles ADH, HMI, INK, &c, are evidently similar, because the hypothenuses, AH, HI, IK, &c, being equal in length, the other sides will be respectively equal also. On the other hand, it is evident that if AD, which is part of a great circle, be equal in length, or in miles, to HM, which is part of a lesser circle, the latter must contain a greater number of minutes or degrees than the former.

III. When a very small portion of the loxodromic line, such as AH, has been passed over, always pursuing the same course, on the vessel's arrival at H, if the difference of latitude, or the arc DH, be determined by observation, it will be easy to find the distance sailed AH; since DH is to AH, as the sine of the angle HAD, which is known, is to radius. If the angle CAH, for example, be 60 degrees, and consequently HAD 30 degrees; and if DH be equal to half a degree, or 30 nautical miles, the distance AH will be 60 nautical miles; for the sine of 30 degrees is exactly equal to half the radius.

IV. If the course and distance sailed be known, the difference of latitude may be found in like manner.

V. The loxodromic angle CAH, or HAD, being known, as well as the difference of latitude DH, the value of the arc AD may be found; for DH is to AD as the sine of the

angle HAD is to its cosine. But when the length of the arc of a parallel, or the number of miles it contains, is known, the degrees and minutes it contains may be determined also. In this manner, the difference of longitude produced by the vessel's change of position, while passing over the small loxodromic arc AH, is obtained; and if the same operation be performed in regard to all the other small arcs HM, IN, &c, we shall have the whole difference of longitude, produced by the vessel's passing over any loxodromic arc AK. The difficulty of this operation arises from these arcs being dissimilar, though equal in length. But geometricians have found means to avoid these calculations, by ingenious tables or other operations, the explanation of which does not fall within the plan of this work.

VI. This curved line has one property which is very singular, that it always approaches the pole without ever reaching it. This evidently follows from the nature of it; for if we suppose it to arrive at the pole, it will intersect all the meridians in that point; consequently, since it cuts each meridian under the same angle, it will cut them all at the pole under the same inclination, which is absurd; since they are all inclined in that point to each other. It will therefore approach the pole more and more, making an infinite number of circumvolutions around it, but without ever reaching it. Hence, according to mathematical rigour, a ship which continually pursues the same course, the cardinal points excepted, will always approach the pole, without ever arriving at it.

VII. Though the loxodromic line, when it forms an acute angle with the meridians, must make an infinite number of circumvolutions around the pole before it reaches it, its length is however finite; for it can be demonstrated, that the length of a loxodromic line, such as AKL, is to the length of the arc of the meridian that indicates the difference of latitude, as radius to the cosine, or

sine complement, of the angle which the loxodromic line forms with the meridian; consequently the difference of latitude is to the loxodromic distance sailed, as the cosine of the above angle is to radius.

The above remark is principally intended for geometricians; and exhibits a kind of paradox which must astonish those to whom truths of this kind are not familiar: those, however, who comprehend the preceding demonstrations, can entertain no doubt of it. But, for the sake of farther illustration, let us suppose a loxodromic line inclined to the meridian at an angle of 60 degrees, with its infinite circumvolutions around the pole; if we employ the following proportion, As the cosine of 60 degrees, or the sine of 30°, is to radius; so is 90 degrees difference of latitude to a fourth term, this fourth term will be the absolute length of the loxodromic line. But the sine of 30 degrees is equal to half the radius; and hence it follows that the fourth part of the circle is the half of the above loxodromic line; or this line, notwithstanding the infinite number of its circumvolutions, is exactly equal to a semicircle of the sphere.

PROBLEM II.

How a Vessel may sail against the Wind.

What is here proposed, will no doubt seem a paradox to those unacquainted with the principles of mechanics. Nothing however is more common in navigation, as this is always done when a vessel, according to the nautical term, is beating up on different tacks, or keeping as near to the wind as possible. But when we say a vessel can sail against the wind, we do not mean that she can proceed on a course directly opposite to the point from which the wind blows; it is only by making an acute angle with the rhumb line passing through that point, which is sufficient; for by several tacks she can then advance in a direction contrary to that of the wind.

Let us suppose a vessel, pl. 1 fig. 2, the keel of which is AB, and let one of the sails CD be set in such a manner, as to form with the keel an angle BED of 40 degrees: if the direction of the wind be EF, making with the same keel an angle of 60 degrees, for example, it is evident that the angle DEF will be 20 degrees; consequently the sail will be impelled by a wind falling on it at an angle of 20 degrees. But according to the principles of mechanics, the action of a power falling obliquely on any surface, is exercised in a direction perpendicular to that surface, and therefore if EG be drawn perpendicular to CD, the line EG will be the direction according to which the effort of the wind is exercised on the sail CD, but with a diminished force on account of the obliquity of the stroke.

If the vessel were round, it would proceed in that direction; but as, in consequence of its length, it can move with much greater facility in the direction of its keel EH, than according to any other, it will assume a direction EK, somewhere between EG and EH, but much nearer to the latter than to the former, almost in the ratio of its facility to move according to EH and EG. The angle KEF therefore, which the ship's course forms with the direction of the wind, may be an acute angle. If the angle KEH, for example, be 10 degrees, the angle KEF will be 70} degrees, consequently the vessel will lie almost two points nearer to the wind. But it is shown by experience, that a vessel may be made to go on a course still nearer to the direction of the wind, or to lie closer to it by about one point more; for if the vessel be well constructed, there are 22, of the 32 points comprehended in the compass, which may serve to make her proceed to the same place.

It is indeed true, that the nearer a ship lies to the wind, or to speak in common terms, the sharper the angle of the wind's incidence on the sail, the less will be its force to push the vessel forwards; but this is compensated by the quantity of sail that may be set, for in this case none of

the sails hurt each other, and a vessel can absolutely carry all her sails. What therefore is lost in consequence of the weakness of the force exerted on each, is gained by the quantity of surface exposed to the wind.

It may be easily conceived how advantageous this property of vessels is to navigation; for whatever be the wind, it may be employed to convey a ship to any determinate place, even if it should blow directly from that quarter. For let us suppose, pl. 1 fig. 3, that the direct course is from E to F, and that th wind blows in the direction FS; the vessel must be kept as near the wind as possible, to describe the line EG, making with EF the acute angle FEG; having proceeded some time in the direction EG, the vessel must then tack about, to run down GH; then HI; then IK; and so on; by which means she will always approach nearer to the place of her destination.

PROBLEM III.

Of the force of the Rudder, and the manner in which it acts.

The force, by which the rudder of a ship makes her move in any direction, at pleasure, excites no small degree of astonishment; especially when we consider the weak action of the enormous rudders with which some of the barges that navigate our rivers and canals are furnished. The cause of this phenomenon we shall here endeavour to explain and illustrate.

The rudder of a barge or vessel has no action unless impelled by the water. It is the force resulting from this impulse, which being applied in a direction transversal to the poop, tends to make the vessel turn around a point of its mass, called the spontaneous centre of rotation. The prow of the vessel describes around this point an arc of a circle, in a direction opposite to that described by the poop; hence it follows that the prow of the vessel turns towards that side to which the rudder is turned, consequently opposite to that side towards which the tiller

or lever of the rudder is moved. Hence, when the tiller is moved to the starboard side, the vessel turns towards the larboard, and *vice versa*.

A force, and even a certain degree of intensity, must therefore be applied to the rudder to make the vessel turn; and on this account the construction of the vessel is so contrived, as to increase this force as much as possible; for while the barges which navigate our rivers are in general very broad behind, and screen as we may say the rudder, so that the water flowing along their sides can scarcely touch it, the sterns of vessels intended for sea are made narrow and slender, so that the water flowing along their sides must necessarily strike against the rudder, if in the least moved from the direction of the keel. Let us therefore endeavour to estimate nearly the force which results from this impulse.

A vessel of 900 tons, when fully laden, draws 13 or 14 feet of water, and its rudder is about 2 feet in breadth. Let us now suppose that the vessel moves with the velocity of 2 leagues per hour, which makes 176 yards per minute, or about 9 feet per second; if the rudder be turned in such a manner as to make with the keel continued an angle of 30 degrees, the water flowing along the sides of the vessel will impel the rudder under the same angle, that is 30 degrees. The part of the rudder under water being 14 feet in length and 2 in breadth, presents a surface of 28 square feet, impelled at an angle of 30 degrees, by a body of water flowing with the velocity of 9 feet per second. But the action of such a current, if it impelled a similar surface in a perpendicular direction, would be 2205 pounds, which must be reduced in the ratio of the square of the sine of incidence to that of radius, or in the ratio $\frac{1}{4}$ to 1, since the sine of 30 degrees is $\frac{1}{2}$, radius being 1. The effort therefore of the water will be 551 pounds. Such is the force exercised perpendicularly on the rudder; and to find the quantity of this force that acts in a direction per-

pendicular to the keel, and which makes the vessel turn, nothing is necessary but to multiply the preceding effort by the cosine of the angle of inclination of the rudder to the keel, which in this case is $\sqrt{\frac{3}{4}}$ or 0·866, which will give 477 pounds.

The above computation is made on the old supposition, that the force of the water is diminished in proportion as the square of the sine of the incident angle is less than the square of the radius. But, by more accurate experiments it is found (Dr. Hutton's Math. and Philos. Dictionary, Tab. 3, Resistance), that at an angle of 30 degrees, the absolute force is diminished only in the ratio of 840 to 278; hence then, the whole force 2205 pounds, reduced in this ratio, comes out 730 pounds, for the effective or perpendicular force on the rudder, to turn it or indeed the ship about, supposing the rudder held or fixed firm in that position.

But there is one cause which renders this effort more considerable: the water which flows along the sides of the vessel does not move in a direction parallel to the keel, but nearly parallel to the sides themselves, which terminate in a sort of angle at the stern-post, or piece of timber which supports the hinges of the rudder; so that this water bears more directly on the rudder by an angle of about 30 degrees: hence, in the above case, the angle under which the water impels the rudder will be nearly 60 degrees: we must therefore make this proportion, as the square of radius is to the square of the sine of 60 degrees, or as 1 is to $\frac{3}{4}$; so is 2205 to 1653. The force therefore which acts in a direction perpendicular to the keel, is 1653 pounds. Or, by the table in the dictionary above quoted, as 840 is to 729 (for 60°), so is 2205 to 1913 pounds, the perpendicular force.

This effort will no doubt appear very inconsiderable when compared with the effect it produces, which is to turn a mass of 900 tons; but it must be observed that this

effort is applied at a very great distance from the point of rotation and from the vessel's centre of gravity; for this centre is a little beyond the middle of the vessel towards the prow, as the anterior part swells out, while the posterior tapers towards the lower works in order that the action of the rudder may not be interrupted. On the other hand, it can be shown that what is called the spontaneous centre of rotation, the point round which the vessel turns, is also a little beyond the middle and towards the prow; hence it follows, that the effort applied at the extremity of the keel, towards the stern, acts to move the vessel's centre of gravity, by an arm of a lever 12 or 15 times as long as that by which this centre of gravity, where the weight of the vessel is supposed to be united, exerts its action. And lastly, there is no comparison between the action exercised by this weight when floating in water, and that which it would exert if it were required to raise it only one line. It needs therefore excite no surprise, that the weight of one ton, applied with this advantage, should make the vessel's centre of gravity revolve around its centre of rotation.

If the ship, instead of going at the rate of 2 leagues per hour, sails at the rate of 3, the force applied to the rudder will be to that applied in the former case, in the ratio of 9 to 4; consequently, if the position of the rudder be as above supposed, the actual force will be 3719 pounds, or rather 4304 pounds: if the velocity of the vessel were 4 leagues per hour, this force in the same position of the rudder, would be 4 times as much as at first, or 6612 pounds, or rather 7652 pounds.

Hence it is evident why a vessel, when moving with rapidity, is more sensible to the action of the helm; for when the velocity is double, the action is quadrupled: this action then follows the square or duplicate ratio of the velocity.

PROBLEM IV.

What angle ought the rudder to make in order to turn the vessel with the greatest force?

If the water moves in a direction parallel to the keel when it impels the rudder, it will be found that this angle ought to be 54 degrees 44 minutes; but, as already observed, the water is carried along in an angular manner towards the direction of the keel continued; which renders the problem more difficult. If we suppose this angle to be 15 degrees, which Bouguer considers as near the truth, it will be found that the angle in question ought to be 46 degrees 40 minutes.

Ships do not receive the whole benefit of this force; for the length of the tiller does not permit the helm to form with the keel an angle of more than 30 degrees.

PROBLEM V.

Can a vessel acquire a velocity equal to, or greater than that of the wind?

This can never take place in a direct course, or when the ship sails before the wind; for besides that in this case a part of the sails hurt or intercept the rest, it is evident that if the vessel should by any means acquire a velocity equal to that of the wind, it would no longer receive from it any impluse; its velocity then would begin to slacken in consequence of the resistance of the water, until the wind should make an impression on the sails equal to that resistance, and then the vessel would continue to move in a uniform manner, without any acceleration, with a velocity less than that of the wind.

But, when the course of the vessel is in a direction oblique to that of the wind, this is not the case. Whatever may be its velocity, the sail is then continually receiving an impulse from the wind, which still approaches more to equality, as the course approaches a direction perpen-

dicular to that of the wind: therefore, however fast the vessel advances, it may continually receive from the wind a new impulse to motion, capable of increasing its velocity to a degree superior to that even of the wind itself.

But for this purpose it is necessary that the construction of the vessel should be of such a nature, that, with the same quantity of sail, it can assume a velocity equal to $\frac{4}{11}$ or $\frac{1}{4}$ that of the wind. This is not impossible, if all the canvas which a vessel can spread to the wind, in an oblique course, were exposed in one sail in a direct course. This then being supposed, Bouguer shows, that if the sails be set in such a manner, as to make with the keel an angle of about 15 degrees, and if they receive the wind in a perpendicular direction, the vessel will continually acquire a new acceleration, in the direction of the keel, until her velocity be superior to that of the wind, and that in the ratio of about 4 to 3.

It is indeed true, that, as the masts of vessels are placed at present, it is not possible that the yards can form with the keel an angle less than 40 degrees; but some navigators assert, that by means of a small change this angle might be reduced to 30 degrees. In this case, and supposing that the vessel could acquire in the direct line a velocity equal to $\frac{1}{4}$ that of the wind, the velocity which it would acquire by receiving the wind on the sails at right angles, might extend to 1·034 that of the wind, which is a little more than unity, and therefore somewhat more than the velocity of the wind.

If we suppose the same velocity possible in the direct course, and that the sail forms with the keel an angle of 40 degrees, it will be found that the velocity acquired by the vessel in an oblique course, will be nearly $\frac{1}{8}$ the velocity of the wind.

This at least will be the case, if in this position of the sails, in regard to the wind, they do not hurt or obstruct each other. If all these circumstances therefore be com-

bined, it appears that though it is possible, speaking mathematically, that a vessel can move with the same velocity as the wind, or even with a greater, it will be very difficult to produce this effect in practice.

PROBLEM VI.

Given the direction of the wind, and the course which a vessel must pursue in order to reach a proposed place; what position of the sails will be most advantageous for that purpose?

Let us suppose that the wind blows from the north, and that the ship's course is due east. If the ship, when her head is directed to that point, has her yards parallel to the keel, her progress will be = 0; as she will receive no impulse but in a direction perpendicular to the keel. On the other hand, if the yards be perpendicular to the keel, as the sails will not catch the wind, the vessel in this case again will not move. Thus, from the first position to the latter, the impulse in the direction of the keel, and consequently the velocity, goes on first increasing, and then decreasing. There is some position therefore at which this impulse is strongest, or what is called a *maximum*, and which will make the vessel move with the greatest velocity. The question is to determine it.

Geometricians have solved the problem, and have found, that to determine this angle, that between the wind and the proposed course must be divided in such a manner, that the tangent of the apparent angle, which the wind forms with the yard, shall be double to that which the yard forms with the course, or with the keel. In this case therefore, the sail at first must be placed in such a situation, as to make with the keel an angle of 35 degrees 16 minutes, and consequently with the wind an angle of 54 degrees 44 minutes.

We say the sail at first must be set in this manner; for as soon as the vessel has acquired a greater velocity, this

angle will cease to be the most favourable, and will become less so, the more the velocity is accelerated, as must be the case, till the impulse of the wind be in equilibrio with the resistance which the vessel suffers from the water; but in proportion as the velocity is accelerated, the wind strikes the sail more obliquely, and loses its force: for this reason, the sail must be disposed in such a manner, as to form with the keel an angle always more acute, and this angle may be reduced to 30 degrees and less; so that the wind shall make with the sail an angle of 60 degrees and more.

We have here considered the question independently of lee-way; but if this be taken into account, supposing it for example in the present case to be one point, it will be necessary to make the vessel's head lie a point nearer to the wind: the angle then which the wind forms with the course will be from 78 to 79 degrees; and it will be found that on the outset, the angle formed by the wind and the sail ought to be 48° 45′; and that of the yard with the keel 29° 45′, which must gradually be reduced to 24 or 25 degrees. By then steering WNW ¾ W, the vessel will really proceed East with the greatest velocity possible, or nearly so; and as in the neighbourhood of those points which give a *maximum*, the progressive increase is insensible, this greatest velocity will always be nearly obtained, even when the above angles are not very exact.

PROBLEM VII.

In what manner must a vessel at sea be directed, so as to proceed from any given place to another by the shortest course possible?

As the loxodromic line, which navigators generally follow at sea, is not the shortest way from one place to another, it is natural to ask whether there be not some means by which the shortest course can be pursued; for it is evident, *cæteris paribus,* that the way being shorter, the voyage would be sooner ended.

As this is no doubt possible, we shall first show how it may be done, and then examine with what advantage it is attended.

Every one knows that the shortest way from one place to another, on the surface of the earth, is the arc of a great circle drawn from the one to the other. Nothing then is necessary but to keep the vessel continually on the arc of a great circle, or at least to deviate very little from it.

Let us suppose then, that a vessel is bound from London to the island of Trinidad. It will be found by trigonometrical calculation, that the arc of a great circle drawn from London to Trinidad, makes at London with the meridian, an angle of 69° 44′, and at Trinidad of 37° 30′; while that of the loxodromic line with the meridian is at London 50° 40′. The angle formed by the course with the meridian, at the time of departure, ought therefore to be 69° 44′.

But to keep the vessel in this great circle, it will be necessary to change the angle every day; and strictly speaking every hour and every moment, otherwise the vessel will describe small loxodromic lines, and not the arc of a great circle. The following method, which, if not perfectly exact, approaches very near the truth, may be employed to effect this change.

As the angle at Trinidad is 37° 30′, it may be easily seen, that from the time of the vessel's departure, till that of her arrival at the place of destination, the angle of the course must be gradually diminished, from 69° 44′ to 37° 30′. Let us divide the difference, which is 32° 14′, into 10 equal portions, which will each be 3° 13′. Every time then that the difference of longitude is one tenth of the whole, or about 5° 37′, that is when the vessel has made about 111 leagues of departure towards the west, it will be necessary to keep 3° 13′ more to the south. By these means the vessel will be kept nearly on the arc of a great circle, passing through London and Trinidad.

These angles might be more exactly determined by

means of trigonometry; that is by drawing a meridian at about every 4 degrees of longitude, and successively solving the spherical triangles thence resulting; but we confess that we never had the courage to attempt a calculation so useless. For, if we examine what advantage would arise from this operation, it will be found of very little importance. The distance from Plymouth to Trinidad, measured on a great circle drawn from the one to the other, is about 1212 leagues; and if the loxodromic line drawn from the one to the other be measured, it will be found to be about 1254. It is therefore not worth while to seek for the shortest course to save about 40 leagues; especially as in sea voyages, the principal object is not to pursue the shortest route, but to take advantage of the wind whatever it may be, in order to complete the voyage.

PROBLEM VIII.

What is the most advantageous form of construction for the prow of a vessel, in order that she may sail better, or be easier steered?

If one only of these objects were to be attained, that for example of cleaving the water with the greatest facility, the problem might be easily solved. The sharper a vessel is at the prow, the easier she can cut the water, and consequently will be better calculated for moving with rapidity.

But an object still more important than velocity, is that of being easily worked: without this property, a vessel, like a refractory horse, would render useless the whole art of the navigator. But it is shown both by experience and reason, that a vessel, to be manageable, must be narrow towards the stern in the part immersed, in order that the water which runs along her sides may strike the rudder with more facility. She will also be managed with more

ease, the farther the centre of gravity is from the stern; and for this reason the most obtuse and the widest part of the vessel must be towards the head. This is actually the case in regard to all vessels destined for voyages.

Nature, in regard to this point, seems to have provided man with a model in the form of fishes; for it may be readily seen that the thickest part of the fish is towards the head, which in general is even pretty obtuse. Like our ships, they have much more need of being able to turn and direct themselves with ease, than to move with rapidity. The best vessel perhaps would be that constructed according to the exact dimensions of a migrating fish, such as the salmon; which seems to enjoy, in a greater degree than any other, the two properties of moving quick and directing itself with ease.

M. Camus, a gentleman of Lorraine, gives an account, in his Mechanics, of several experiments, from which he endeavours to show, that the model of a vessel will move faster with the thick end foremost, than when cleaving the waves with the other, which is sharper: he even assigns reasons for this idea, but they are certainly ill founded. These experiments are in absolute contradiction to sound theory; and if ships have that form, it is not that they may move faster, but in consequence of the necessity which has been found, of sacrificing the advantage of velocity to that of being easily manœuvred.

M. Montucla here, rather injudiciously, opposes theory to experiment, and censures Camus improperly, whose experiments and reasonings have been confirmed by the more accurate and extensive ones made, in the years 1793-1798, by the English Society for the improvement of Naval Architecture, and may be seen at large in the Report of their Committee, printed in the year 1800.

PROBLEM IX.

What is the most expeditious method of coming up with a vessel which is chased, and which is to the leeward?

When a vessel is descried at sea, and you are desirous of coming up with her, you would be much mistaken if you directed the head of your own vessel towards the one you are pursuing; for unless the chace were proceeding on the same course exactly, you would either be obliged to change your direction every moment, or you would lose the advantage of the wind by falling to the leeward.

If a body *a*, pl. 1 fig. 4, moves in the line *a b c d*, and if it be proposed that another body A should come up with it, the body A ought not to be impelled in the direction A*a*; for in a few moments *a* will have advanced on the line in which it moves, and will have reached the point *b*, for example. Hence if we suppose that the body A always changes its course, directing itself towards the one it pursues, it will describe a curve such as ABCDE, and will at length reach the body *a* by going faster, but not by the shortest way. If it does not change its direction every moment, it will arrive at a point in the line *a a*, which the body *a* has already left, and will pass it, unless it set out to pursue it along the line *a d*, which would still make it lose time.

To cause the body A therefore to come up with *a*, in the least time possible, A must be directed to a point in the line *a e*, fig. 5, so situated, that AE and *a e* shall be to each other in the ratio of their respective velocities. But these lines will be in this ratio, if the body A, at every moment in its course, is that which it pursues similarly situated, in a direction parallel to the direction A*a*, that is, A*a* being directed to the south, if the body *a*, when it reaches *b*, is to the south of the body A when it arrives at B; for it is evident that the lines AE, *a e*, will then be proportional to

the velocities of the two bodies, and they will arrive at the same time at [illegible] e.

Navigators are sensible of this, both from practice and reason; for if a vessel at A espies another at *a*, the course of the latter *a e*, may be ascertained nearly without much difficulty, and the ship in chase, instead of directing her head towards *a*, will follow a course such as A B, inclined from *a*, and at the same time the bearing of the vessel in the direction A*a* will be taken by means of the compass; when A has proceeded some time, and reached B, for example, while *a* has reached *b*, the bearing of the vessel *a* in the direction B*b* will be again taken: if it be still the same, it is a sign that A is gaining ground, for A*a* and B*b* are parallel. If the chace falls a little behind, it shows that she may be pursued in a line making with the direction of her course, a less acute angle; but if she has got a-head, a line more inclined must be pursued to reach her; and if the line be as much inclined as possible, and approaches to parallelism, there is reason to conclude that the chace is a better sailer, and that all hope of reaching her must be given up.

It is here supposed that the chasing vessel has the advantage, or is to windward; for if she be to leeward, the manœuvring must be different, unless she has a great advantage in being able to lie near the wind. But this is not the proper place for enlarging on these manœuvres of the most ingenious of all arts.

PROBLEM X.

On determining the Longitude at Sea.

The determination of the longitude at sea, has afforded no less exercise to mathematicians, than the perpetual motion, the quadrature of the circle, and the duplication of the cube, but with more reason; for no great advantage would be derived from a solution of the two latter, whereas that

of the former would be attended with the greatest benefit to navigation. Navigators might at all times, when the heavens are visible, determine the place at which they have arrived, by observing the longitude and latitude; while in the present state of navigation the longitude can be estimated only in a very vague manner; and nothing is more common, in long voyages from east to west, or the contrary, than for navigators to err a hundred leagues and more in their longitude. The British parliament therefore, many years ago, offered a reward of 20,000l, to the person who should point out a certain method, practicable for common navigators, of determining the longitude at sea.

The problem of the longitude consists in determining the difference between the time reckoned in a vessel at sea, and that reckoned at any determinate place, such as the port whence the vessel sailed, and of which the longitude is known. But the time may be ascertained on board a vessel without much difficulty, provided the sun can be observed at noon, and also the latitude; for, by means of the instruments now employed at sea, the point of noon can be determined within about 2 minutes. By knowing the latitude in which the vessel is, and the sun's declination, the hour can be determined also by the setting of the sun. The operations relating to this subject may be seen in all good works on navigation.

But the difficulty is to find what the hour is, at the same time, in the port from which the vessel sailed. There are however two methods of accomplishing this object, which mathematicians have endeavoured to render certain and practicable: one of them depends on mechanics, the other is purely astronomical.

If the instruments constructed for measuring time, preserved at sea the same regularity of motion as at land, it would be easy on board ship to find on every occasion the hour at a determinate point. For this purpose, a navigator

on leaving the Lizard, for example, would set his time-keeper to the exact hour at that place, and if the time-keeper were always regularly wound up, it would continue to indicate the hour at the Lizard. When it should be required then to determine the ship's longitude, nothing would be necessary but to observe exactly the time of noon, and then to examine the hour indicated by the time-piece: the difference between these would be the difference of longitude. Thus, after the end of a fortnight, if the time-keeper indicated 2 hours 10 minutes, when it was noon on board the ship, there would be reason to conclude that the difference in time, between the Lizard and the place at which the ship had arrived, was 2 hours 10 minutes, which are equal to 32 degrees 30 minutes west from the Lizard: then by means of the latitude, found by observation, it would be easy to determine exactly the ship's place on the earth.

But as a pendulum clock cannot be used, and as the best watches get entirely deranged at sea, it becomes necessary to discover some method of measuring time not subject to this inconvenience; or to improve the instruments already employed so far as to remove it entirely.

Various inventions, supposed to be less subject to the irregularities occasioned by the rolling of a ship at sea, have been proposed for this purpose. It is said, in the preceding editions of this work, that nothing is necessary but to take a good clock of the common construction, to change its large spring for eight others of less force, which together shall exert the same action, to wind them up in successive order, that is one every twenty-four hours, and to substitute for the pendulum a spiral spring with a scapement *à rochet;* in the last place, to preserve this instrument, or several of them, in one or two boxes, deposited in some part of the vessel less sensible of its motion, taking care to keep the air within these boxes at a uniform temperature, which might be easily done by a thermometer.

By these means, says the author, you will have an instrument which will indicate the hour at sea exactly. If an apparatus so simple were sufficient to solve this problem, it would not have occupied so long the talents of mechanicians and astronomers.

Others have had recourse to sand-glasses: the description of one of these, invented by the abbé Soumille, may be seen in the *Mémoires adressés à l'Academie Royale des Sciences, par des Sçavans étrangers*, vol. I. It is ingenious, but we do not know whether it was ever tried, and whether it was attended with success.

Many years of research however gave birth, in England, to the invention of a marine time-keeper, which has the advantage of preserving its uniformity of motion at sea. For this invention we are indebted to Mr. Harrison, who proposed it about the year 1737. Though at that time it did not appear to possess the required regularity, the Board of Longitude conferred a reward on the author, to encourage him to improve his work; and at length after twenty years employed in this labour, and in making various experiments, he again presented it in 1758 to the Board, who gave orders for trying it in a voyage from England to Jamaica. This trial was made with every precaution and formality necessary to ascertain the result; and towards the end of the year 1761, it appeared that Mr. Harrison's time-keeper gave the longitude of Jamaica nearly within 5 seconds in time. On the return of the vessel dispatched for this purpose, the error, notwithstanding the violent storms experienced during the voyage, was only 1 minute 54 seconds in time, or about 33 English miles; and a reward was adjudged to the author in consequence of his having constructed a machine which in such a passage did not err much more than 30 miles.

Mr. Harrison therefore received 5,000l. sterling as part of the reward of 20,000l. offered by the British parliament,

which was to be paid to him after a new experiment, and on his making known the mechanism of his time-keeper, and teaching artists how to construct others of the like kind. This second trial was made in 1765, in a voyage from Portsmouth to Barbadoes, and the result of it having confirmed the success of the former, Mr. Harrison received 5000l. more. He was to be paid the remainder when he had taught a certain number of artists to construct such machines, for the use of navigators. A full account of this interesting discovery, and a description of the mechanism invented by Mr. Harrison, may be seen in various pamphlets, and other works, published on the subject. Navigation however is indebted to England for a certain method of knowing at sea the time at the port of departure; which is an inestimable advantage, and will certainly be the means of preserving many navigators from shipwreck.

Mr. Harrison's invention having been long kept a secret, the French watchmakers, who had already made many attempts to solve the problem, redoubled their efforts to discover it, or to find out some means of the same kind. In order to encourage them, the Academy of Sciences at length proposed, in the years 1767 and 1773, a prize for the construction of a time-keeper similar to that of Mr. Harrison. This prize was gained by M. le Roy; son of the celebrated Julian le Roy, who showed that he had long before that time discovered the principle of the compensation balance, necessary for constructing his time-keeper. It was partly for the purpose of trying it that the Marquis de Courtanveaux caused to be built and fitted out, at his own expence, the Aurora frigate, in which he made a voyage to the Texel in the year 1767. During this voyage, M. le Roy's time-keeper always went with the greatest regularity, notwithstanding the violent agitation which the vessel continually experienced, in a sea where a heavy swell generally prevails; and therefore, though the merit

of the discovery must be allowed to Mr. Harrison and to England, we may say that France had nearly fallen upon it at the same time.

We must here observe that there is another French artist, who has followed so closely the steps of Mr. Harrison, that he disputes with M. le Roy the honour of having made the first time-keeper in France: the artist here alluded to is M. Berthoud, whose time-keepers, tried during the long voyage of M. de Fleurieu, seem also to have answered all the required conditions.

We have already mentioned another method of considering the problem of the longitude, which is merely astronomical. It is therefore necessary that we should make known what astronomers have done in this respect.

When Galileo discovered the satellites of Jupiter, the eclipses of which are so frequent, he conceived an idea of employing them in the solution of the problem respecting the longitude. It is indeed evident, that if the theory of the satellites of Jupiter be brought to sufficient perfection, to determine for any given place, such as London, the moment when they will be eclipsed, and if an eclipse of one of these small planets be observed at sea, together with the exact time when it is seen, nothing will be necessary but to compare that time with the hour and minute at which it has been previously announced for the meridian of London: the difference of time will give the difference of longitude. Thus for example, if an eclipse of the first satellite has been observed at 10^h 20^m in the evening; and if it is found, by consulting the *Nautical Almanac*, that the eclipse is announced for Greenwich observatory at 11^h 55^m in the evening; it is evident that the difference 1^h 35^m, is the difference of time, as reckoned at Greenwich and on board the vessel; which makes 23° 45′ difference in longitude.

Several obstacles however prevent this method from being much employed; for, in the first place, these eclipses

do not happen often enough, as there is only one of the first satellite every 42 hours; and besides, they are not visible during several months, when Jupiter is too near the sun, &c. 2dly. To observe them, telescopes of a certain length are necessary, and it is well known that the rolling of a ship renders it very difficult to observe Jupiter, or any celestial body whatever, with a telescope of considerable length.

Attempts have indeed been made to remedy this inconvenience. Mr. Irwin, an Irish gentleman, proposed in the year 1760, his marine chair; that is to say, a chair suspended in a vessel in such a manner, that a person seated in it can observe, with tolerable ease, the satellites of Jupiter, especially with an achromatic telescope, which will produce the same effect as a much longer one constructed in the usual manner. A trial of it was made by order of the Lords of the Admiralty, and according to the accounts published at that time, it succeeded pretty well; but it would appear that after Mr. Harrison proposed his time-keeper, Mr. Irwin's marine chair was laid aside.

It has been known for more than a century, that if the theory of the moon were brought to sufficient perfection, the problem of the longitude at sea would be solved; for the moment of the moon's appulse to some of the zodiacal stars of the first or second magnitude, might be calculated for any determinate place. Besides, the motion of the moon is so rapid, that her change of position, in a short time, is very sensible. On this account, astronomers, for several years past, have employed themselves with great assiduity, to improve the theory of the moon; and they have indeed so far succeeded, that the errors in calculating the moon's place, do not exceed 2 or 3 minutes in the most unfavourable parts of her orbit; whereas formerly, they amounted to several degrees. The British parliament thought it necessary, by voting a sum of money to the widow and heirs of the late Tobias Meyer, of Gottin-

gen, to reward the successful efforts of that indefatigable and able astronomer, to whom we are indebted for the best tables of the moon ever published. They received therefore a present of 2500l. sterling, and as Euler also had laboured with the greatest success in improving the theory of the moon, the parliament voted him the sum of 500l. Such examples of justice and generosity towards those who have exerted themselves in promoting the general good of mankind, do nations the utmost honour.

Another necessary step was, to render the calculation of these observations sufficiently easy for practice, if not to all seamen, at least to the more enlightened part of them. The Abbé de la Caille is among those who exerted themselves with the greatest success in the accomplishment of this object. He gave formulæ and operations for performing these calculations, in which a ruler and a pair of compasses only are employed, and which require but a moderate knowledge of geometry and astronomy. They may be seen in the edition which he published of Bouguer's *Traité de Navigation*, as well as in the *Connoissance des Temps*, for the years 1765 and 1766. A Nautical Almanac, which contains the moon's appulse to various fixed stars, calculated for the meridian of Greenwich, as well as the instructions and formulæ necessary for employing the observations of the moon in determining the longitude, has been published for several years past at London, under the direction of Dr. Maskelyne, astronomer-royal.

Some time ago a new instrument, for observing the distances of the moon from the fixed stars, was proposed. This instrument, to which the inventor, M. Charnieres, an officer in the French navy, gave the name of *Megametre*, was employed by him to make observations, during a voyage from Europe to America, and in 1768 he published the result of them, which seems to prove, that the instrument may be useful at sea. We do not find however that it

ever met with a favourable reception from navigators; nor do we know the reason.

PROBLEM XI.

If a vessel should be able to reach either of the poles, what method ought the commander to pursue, in order to steer in the direction of a determinate Meridian?

The difficulty which this problem seems, on the first view, to present, arises from this circumstance, that if a vessel were at either of the poles to whichever side she might turn, her head would be directed towards the south or north. Every line drawn from that point, to any point whatever in the horizon, is a meridian; and consequently at the pole there is neither east nor west. But, if there is neither east nor west, how would she steer, or how would it be possible, all the meridians being similar, to find that in the direction of which it would be necessary to proceed, in order to reach the proposed place?

This however is not all: if a vessel should reach one of the poles, it is probable that the compass would become useless, or as the sailors say *run entirely mad;* and there are only two ways of navigating a vessel, either by the magnetic needle, or by observing the stars, or rather by both these methods combined.

Such is the problem, which the astronomer who accompanied the Hon. Capt. Phipps, afterwards Lord Mulgrave, sent out to attempt a passage through the northern ocean, would have had to solve, had the expedition succeeded. If the progress of the vessel had not been stopped by the ice, he would have proceeded to the 90th degree of latitude, in order to arrive by the shortest passage at the strait which separates Asia from America—a strait, the existence of which is now confirmed by the expeditions of the Russians, and by the researches of Captain Cook, and which lies in about the 176th degree of longitude. I proposed

this problem to myself, in consequence of a new attempt which was about to be undertaken in France, by M. de Bouguainville. I have heard that it was proposed to a celebrated astronomer, a member of the Royal Academy of Sciences: I do not know what answer he returned: but my solution is as follows:

Had I been the navigator intrusted with the expedition, that I might not be taken by surprise, I should have provided myself with two or three good time-keepers, all exactly set to the time at the port of departure, which I suppose to be Brest.

Let us now suppose that the sea was found open, and that I had arrived at the north pole. I shall suppose also that my compass had become entirely useless; but that I had the sun on the horizon, which is the case in summer, and therefore such an expedition ought never to be undertaken but at that period, during which the sun is visible in those regions for several months. It is evident that by consulting my time-keepers, the moment when they indicated noon would be that when the sun was on the meridian of Brest; consequently had I been desirous of returning thither, nothing would have been necessary but to turn the ship's head towards the sun, and to steer on that course, in such a manner, as to have the sun at the end of an hour 15 degrees to the starboard; at the end of two hours 30 degrees, &c. It may be readily conceived that by these means, though destitute of a compass, I should have kept my vessel pretty exactly on the line of the determinate meridian.

Now, if the meridian, on which it was necessary I should steer, had been distant from that of the place of departure 176°, as seems to be the case with that of the strait which separates Asia from America, it may be easily seen that I should have had nothing to do, but to direct the ship's head within about 4 degrees of the point diametrically opposite to the sun, when the time-keepers indicated noon; or to-

wards the sun itself when they indicated 16 minutes after midnight, and then to keep on this course by the method above described; changing every hour the angle formed by the ship's course with the azimuth passing through the sun. If we suppose the mouth of the strait in question to be, in regard to Brest, in the longitude already mentioned, it is evident that I should not have failed to enter it.

But it is to be observed, that this expedient would be necessary only when very near to the pole: at the distance of ten degrees from it, other means of directing the ship's course might be employed. We shall not however enlarge farther on this subject; for it would be of very little use to point out these means, since the latest voyages seem to prove that the arctic pole, at the most favourable seasons, that is to say during the summer of our hemisphere, is surrounded by a covering of ice ten degrees at least in diameter, and which even extends farther towards Asia and America; or, in all probability, adheres to these two continents, except perhaps during some excessively hot summers. In short, I am fully persuaded that the idea of traversing the frozen ocean, in order to proceed to the seas of China and Japan, is a mere chimera; and that if a vessel should even be able to get thither, by steering close along the shores of Asia or America, to the strait above mentioned, the voyage would be attended with so many dangers, and require circumstances so favourable, that it would be madness to attempt it. What indeed would become of a ship if, retarded by any of the accidents so common in those seas, she should be obliged to winter, nearly a whole year, in any port of the almost uninhabited northern coast of Asia? What assistance could she expect from the Samoiedes, or any other of these nations, still more barbarous? If the crew remained there, how could they secure themselves from the intense cold of these climates? If they quitted their vessel, to take up their lodging in a close hut, after carrying thither their provisions, would not the

vessel be exposed to the danger of being plundered or burnt? Such an enterprise would require, that the commercial nation which undertook it, should have a port belonging to it in some advantageous situation, that ships obliged to winter in those cold regions might have a convenient place of shelter. But what appearance is there that Russia, the sole mistress of these countries, will ever consent to such a measure; especially as the Russian government so long concealed the information it had obtained in regard to the strait above mentioned?

MATHEMATICAL

AND

PHILOSOPHICAL

RECREATIONS.

PART NINTH.

Some curious particulars in regard to Architecture.

ARCHITECTURE may be considered under two points of view. According to the first it is an art, the object of which is to unite utility and grace; to give to an edifice that form fittest for the purpose to which it is destined, and at the same time the most agreeable by its proportions; to strike the beholder by magnitude or extent, and to please by the harmony of the different parts and their relation to each other: the more an architect succeeds in uniting all these requisites, the more he will be entitled to rank among the eminent men who have distinguished themselves in this art.

But it is not under this point of view that we here consider it; we shall confine ourselves to the geometrical and mechanical part of architecture, as it presents us with several curious and useful questions, which we shall lay before the reader.

PROBLEM I.

To cut a Tree into a Beam capable of the greatest possible resistance.

This problem belongs properly to mechanics; but on account of its use in architecture, we thought it might be proper to give it a place here, and to discuss it both geometrically and philosophically. We shall first examine it under the former point of view.

Galileo, who first undertook to apply geometry to the resistance of solids, has determined on a very ingenious train of reasoning, that when a body is placed horizontally, and fixed by one of its extremities, as is the case with a quadrangular beam projecting from a wall, if a weight be suspended from the other extremity, in order to break it, the resistance which it opposes is in the compound ratio of the horizontal dimension and the square of the vertical dimension. But this would be more correctly true, if the matter of the body were of a homogeneous and inflexible texture.

It has been shown also, that if a beam is supported at both extremities, and if a weight, tending to break it, be suspended from the middle, the resistance it opposes, is in the ratio of the product of the breadth and square of the depth, divided by half the length.

To solve therefore the proposed problem, we must cut from the trunk of the tree a beam of such dimensions, that the product of the square of the one by the other shall be the greatest possible.

Let AB then, pl. 1 fig. 1, be the diameter of the circle, which is the section of the trunk; the question is, to inscribe in this circle a rectangle, as AEBF, of such a nature, that the square of one of its sides AF, multiplied by the other side AE, shall give the greatest product. But it can be proved that, for this purpose, we must first take, in the

diameter AB, the part AD equal to a third of it, and raise the perpendicular DE, till it meet the circumference in E: if BE and EA be then drawn, and also AF and FB parallel to them, we shall have the rectangle AEBF, of such a nature, that the product of the square of AF by BF, will be greater than that given by any other rectangle inscribed in the same circle. If a beam of these dimensions, cut from the proposed trunk, be placed in such a manner, that its greatest breadth AF shall be perpendicular to the horizon, it will present more resistance than any other that could be cut from the same trunk; and even than a square beam cut from it, though the latter would contain more matter.

REMARK.—Such would be the solution of this problem, if the suppositions from which Galileo deduced his principles, in regard to the resistance of solids, were altogether correct. He indeed supposes that the matter of the body to be broken is perfectly homogeneous, or composed of parallel fibres, equally distributed around the axis, and presenting an equal resistance to rupture; but this is not entirely the case with a beam cut from the trunk of a tree which has been squared.

By examining the manner in which vegetation takes place, it has been found, that the ligneous coats of a tree, formed by its annual growth, are almost concentric; and that they are like so many hollow cylinders, thrust into each other, and united by a kind of medullary substance, which presents little resistance: it is therefore these ligneous cylinders chiefly, and almost wholly, which oppose resistance to the force that tends to break them.

But, what takes place when the trunk of a tree is squared, in order that it may be converted into a beam? It is evident, and it will be rendered more sensible by inspecting fig. 2, that all the ligneous cylinders, greater than the circle inscribed in the square, which is the section of the beam, are cut off on the sides; and therefore the

whole resistance almost arises from the cylindric trunk inscribed in the solid part of the beam. The portions of the cylindric coats which are towards the angles, add indeed a little strength to that cylinder, for they cannot fail of opposing some resistance to the breaking force; but it is much less than if the ligneous cylinder were entire. In the state in which they are they oppose only a moderate effort to flection, and even to rupture. For this reason, there is no comparison between the strength of a joist made of a small tree, and that of another which has been sawn, or cut with several other from the same beam or block. The latter is generally weak and so liable to break, that joists, and other timber of this kind, ought to be carefully rejected from all wooden work which has to support any considerable weight.

We shall here add, that these ligneous and concentric cylinders are not all of equal strength. The coats nearest the centre, being the oldest, are also the hardest; while, according to theory, the absolute resistance is supposed to be uniform throughout.

It needs therefore excite no surprise, that experience should not entirely confirm, and even that it should sometimes oppose the result of theory. Hence we are under considerable obligations to Duhamel and Buffon, for having subjected the resistance of timber to experiments; as it is of great importance in Architecture to know the strength of the beams employed, in order that larger and more timber than is necessary may not be used.

But notwithstanding what has been said, it is very probable that the beam capable of the greatest resistance, which can be cut from the trunk of a tree, is not the square beam; for the following experiments made by Duhamel seem to prove, the size being the same, that the beam which has more depth in proportion than breadth, when the depth is placed vertically, presents so much more resistance; and even without deviating very much from

the law proposed by Galileo, viz, the compound ratio of the square of the vertical dimension and that of the breadth.

Duhamel indeed caused to be broken twenty square bars of the same volume, to determine what form of dressing would render them capable of the greatest resistance. They all had 100 square lines of base, and four of each sort were employed of the different dimensions, to compose the same area.

The first four, which were 10 lines in every direction, sustained a weight of 131 pounds.

Four others which were 12 lines in one direction and $8\frac{1}{3}$ in another, sustained each 154 pounds. The above law would give 157 pounds.

The next four, which were 14 lines in height and $7\frac{1}{7}$ in breadth, supported each 164 pounds. Calculation would give 183 pounds.

Four more, which were 16 lines in height and $6\frac{1}{7}$ in breadth, sustained each 180 pounds. According to calculation they ought to have supported 209 pounds.

The last four, which were 18 lines in height and $5\frac{1}{2}$ in breadth, sustained each 243 pounds. Calculation would have given only 233 pounds. It is very singlar that in this case calculation should give less than experience; while in the other cases the result was contrary.

Buffon began experiments on a larger scale, in regard to the resistance of timber, an account of which may be seen in the Memoirs of the Academy of Sciences for the year 1741. It is to be regretted that he did not pursue this subject, on which no one could have thrown more light. It appears to result from these experiments, that the resistance increases less than in the square of the vertical dimension, and decreases in a ratio somewhat greater than the inverse of the length.

In short, the result of the whole is, that to solve the proposed problem, it would be necessary to have physical

data of which we are not yet in possession; that the beam capable of the greatest resistance, that can be cut from the trunk of a tree, is not a square beam; and that in general many researches are to be made respecting the lightening of carpenter's work, which often contains forests of timber in a great part useless.

PROBLEM II.

Of the most perfect form of an arch. Properties of the catenarian curve, and their application to the solution of this problem.

The most perfect arch, no doubt, would be that, the voussoirs of which being exceedingly thin, and even smooth on the sides in contact, should maintain themselves in complete equilibrium. It may easily be perceived that, in consequence of this form, very light materials might be employed; and we shall show also that its push or thrust on the piers would be much less than that of any other arch of the same height, constructed on the same piers.

This property and this advantage are found in a curve well known to geometricians under the name of the catenarian, and called by the French *la Chainette*. This name has been given to it because it represents the curve assumed by a chain ACB, pl. 1 fig. 3, composed of an indefinite number of infinitely small and perfectly equal links, or by a rope perfectly uniform and exceedingly flexible, when suspended freely by its two extremities.

The determination of this curve was one of those problems which Leibnitz and Bernoulli proposed towards the end of the 17th century, in order to show the superiority of their calculation over the common analysis; which indeed is hardly sufficient to solve a problem of this nature. But we must here confine ourselves to a few of the properties of the curve in question.

If the curve ABC, fig. 3 and 4 pl. 1, be disposed in such a manner, that its summit shall be uppermost; and if a

multitude of globes be so arranged, that their centre shall be in the circumference of this curve, they will all remain motionless and in equilibrium: much more will this equilibrium subsist, if, instead of balls, we substitute thin voussoirs, the joints of which will pass through the points in contact, as they will touch each other in a surface far more extensive than the points in which we suppose the balls to touch each other.

Now to describe a curve of this kind is attended with no difficulty; for let us suppose that the space AB, comprehended between the two piers A and B of fig. 5, is to be covered with an arch, and that the elevation of this arch is to be SC. Trace out on a wall a horizontal line *ab*, fig. 6, equal to AB; then from the middle of *ab* draw *c* perpendicular to it, and equal to SC; and having fixed to the points, *a* and *b*, the two ends of a very flexible rope or chain, formed of small links perfectly equal and very moveable, so that when suspended freely it shall pass through the point *c*, mark out on the wall a sufficient number of the points or eyes of these links, without deranging them: the curve described through these points will be the one required; and nothing will be easier than to trace out the plan of it on the wall as represented by ACB fig. 5.

Then trace out at an equal distance, both without and within ACB, two curves, which will represent the extrados and intrados of the arch to be constructed. Divide the curve AC into any number of equal parts at pleasure; and through these points of division draw lines perpendicular to the curve, which may be done mechanically with sufficient exactness for practice: these perpendiculars will divide the arch into voussoirs; and you will thus have a plan of the arch described on the wall. From this plan it will be easy to construct the pannel or model boards for cutting the stones according to the proper form. If these operations are accurately performed, were the line AB a

hundred feet, and the height SC still more, the voussoirs of this arch would maintain themselves in equilibrium, however small the part in contact might be: for, mathematically speaking, they ought to maintain themselves in equilibrium even if the surfaces in contact were highly polished and slippery: consequently the equilibrium will subsist much more when cut in the usual manner.

Now to find the force with which an arch of this kind pushes against its piers, or tends to overturn them, draw a tangent to the point *a* the commencement of the curve, fig. 6, which may be done mechanically by assuming two points very near the curve, and drawing through these points a line which will meet in *t*, the axis *sc* continued*. This tangent being given, it can be demonstrated in mechanics that the whole weight of the semi-arch *ac*, is to the weight or force with which it pushes the pier in a horizontal direction, as *st* is to *sa*. On the other hand, we must add to the weight of the pier, the force with which the semi-arch presses upon it perpendicularly; that is to say, the absolute weight of the semi-arch: in this manner the thickness of the pier may be found, by the following arithmetical operation, which we shall here substitute for a geometrical construction, as the latter might appear too complex to the generality of our readers.

We shall suppose the span AB to be 60 feet, pl. 1 fig. 5 and 6, and consequently AS will be 30 feet; we suppose SC to be 30 feet also, in order that we may compare the push or thrust of this arch with that of a semi-circular one. Let the length AC be 45 feet 1 inch 8 lines†, and the breadth of the arch 1 foot; for, on account of the reasons

* This tangent may be drawn geometrically in the following manner: make use of this proportion, as 2*sc* is to *ac* + *sc*, so is *ac* — *sc* to a fourth term, which we shall call *cu*; if you then say, as *cu* is to *ac*, so is *as* to *st*, the point *t* will be that where the tangent to the point *a* will meet the axis.

† It is found by calculation that this ought to be the length.

above mentioned, it may be constructed with safety in this light manner. If the height of the pier then be 40 feet, required the thickness it ought to have in order to overcome the thrust of the arch.

It will be found, on this supposition, that the tangent of the point *a*, the commencement of the catenarian curve or arch, will meet its axis *sc* produced, in a point *t* so situated, that *st* will be $71\frac{7}{10}$ feet. If *sa* be then divided by *st*, we shall have the number $\frac{300}{717}$, which must be reserved, and which we shall call N.

Now take a fourth proportional to the height of the pier, the length AC of the semi-arch, and to its thickness, and let the half of this fourth proportional, which in this case is $\frac{9}{16}$, be called D.

Then multiply AC by the thickness 1, and the product by double the reserved number N, which will give $37\frac{3}{7}$; to this number add the square of D, and extract the square root of the sum, which will be $6\frac{1}{4}$: if the above number D be taken from this root, we shall have 5 feet 7 inches, for the breadth of the pier*. The pier being constructed of materials homogeneous to the arch, it is certain that it will resist the force with which the latter tends to throw it down; for, to simplify the calculation, we have made a supposition which is not altogether exact, but which increases in some measure the breadth of the pier. This observation we think necessary, that we may not be accused of committing a wilful error.

If this breadth be compared with that necessary to support a circular arch forming a complete semicircle, the latter will be found to be much greater, for it ought to be near 8 feet.

The push of an arch constructed on a circular foundation, such as the arch of a dome, being only about one half

* This determination of the breadth or thickness of the pier, if not mathematically correct, may at least be considered as sufficiently near for practical purposes.

of that exerted on its piers by a vault arch of the same thickness, it thence follows that, on the above supposition, the side of such a dome would require only 33½ inches in thickness. But it can be demonstrated, even by the figure of the catenarian curve, that the arch may be but about a foot in thickness. Hence we may see how ill founded was the objection made to the architect of the church of Saint Genevieve, of its being impossible to construct, on the base he employed, the dome which he projected; for he could have done it even if we suppose his construction to be such as the author of the objection traced out to him, according to the precepts of Fontana, or rather according to the mode which that architect followed in the construction of his domes. What then would have been the case, had the architect alluded to, instead of first constructing a cylinder of 36 feet, which it however appears was never his design, made his arch rise immediately in a catenarian curve, above the circular cornice, which crowned his pendentives, or above a socle of small height? It is evident that the push of this arch would have been much less; and it would not be surprising if it should be found by calculation that his piers would have been capable of sustaining the arch raised above them, even supposing them insulated, and not allowing them any support from the re-entering angles of the church, which might have been made to rest against them.

We shall conclude with observing, that if it were required to find, by principles similar to those which gave rise to the discovery of the catenarian curve, the most advantageous form for a dome, the problem would be exceedingly difficult; for if we suppose this arch divided into small sectors, it will be evident that the weight of the voussoirs is not equal, and that their relation depends even on the form given to the arch. What has been here said, ought therefore to be considered only as an approximation of the most advantageous figure which the arch, in that case, ought to have.

PROBLEM III.

How to construct a hemispherical arch, or what the French Architects call an arch en cul-de-four, which shall have no thrust on the piers.

The dispute carried on, some years ago, with a considerable degree of warmth, respecting the possibility of executing the cupola of the new church of Saint Genevieve, gave me an opportunity of examining whether, even on the supposition that the supporters would be necessarily too weak to resist the thrust of an arch 63 feet in diameter, there might not be found some resources to render the construction of the cupola possible. I soon found that it was possible, by means of a very simple artifice, to construct an hemispherical arch, or an arch in the form of a semi-spheroid, which should have no sort of thrust on its piers, or on the cylindric tower by which it is supported. This will be readily conceived from the following reasoning and illustration.

It is evident that a hemispherical arch would exert no thrust on its support, if the first row were of one piece. But though this is impossible, the deficiency may be supplied, and such an arrangement may be made, that not only the first row, but that several of those above it, shall be disposed in such a manner, that their voussoirs can have no movement capable of disjoining them, as we shall here show. The hemispherical arch will then exert no kind of thrust on its supporters; so that it may not only be sustained by the lightest cylindric pier, but even by simple columns, which would furnish the means of rearing a work very remarkable on account of its construction. Let us see then, how the voussoirs of any row can be connected in such a manner as to have no motion tending to make them recede from the centre. There are several methods of accomplishing this object.

1st. Let A and B, pl. 2 fig. 7 n° 1, be two contiguous voussoirs, which we shall suppose to be 3 feet in length, and a foot and a half in breadth. Cut out on the contiguous sides two cavities in the form of a dove-tail, 4 inches in depth, with an aperture of the same extent at *ab*; 5 or 6 inches in length, and as much in breadth at *cd*. This cavity will serve to receive a double key of cast iron, as appears in the same figure n°. 2; or even of common forged iron, which will be still more secure, as the latter is not so brittle as the former. These two voussoirs will thus be connected together in such a manner, that they cannot be separated without breaking the ve-tail at its re-entering angle; but as each of its dimensions in this place will be 4 inches, it may be easily seen that an immense force would be required to produce that effect; for we are taught, by well-known experiments on the strength of iron, that it requires a force of 4500 pounds to break a bar of forged iron an inch square by the arm of a lever of six inches: consequently 288000 would be necessary to break a bar of 16 square inches, like that in question. Hence there is reason to conclude that these voussoirs will be connected together by a force of 288000 pounds, and as they will never experience an effort to disjoin them nearly so great, as might easily be proved by calculation, it follows that they may be considered as one piece.

They might even be still farther strengthened in a very considerable degree: for the height of these dove-tails might be made double, and a cavity might be cut in the middle of the bed of the upper voussoir, fit to receive it entirely: the dove-tail could not then be broken without breaking the upper voussoir also. But it may be easily seen that, to produce this effect, an immense force would be required.

2d. But as some persons may condemn the use of iron in works of this kind, we shall propose another method, not attended with the same inconvenience, if it really be

one*; and in which nothing is employed but stone combined with stone.

Let A and B, fig. 8, be two contiguous voussoirs of the first row, and C the inverted voussoir of the upper next row which ought to cover the joining. Each of the two former voussoirs being divided into two, cut out in the middle of each half a hemispherical cavity, half a foot in diameter; then take with great exactness the distance of the centres of the cavities *a* and *c*, which are in two contiguous voussoirs, and by these means cut out two similar cavities in the lower bed of the voussoir, which is to be placed in connection on the preceding. Then fill the cavities *a* and *c* with two globes of very hard marble, and place the upper voussoir in such a manner that these two globes shall fit exactly into the cavities of its lower bed. If this operation be dexterously performed throughout the whole range of the first, second, and third rows, it may be easily perceived, that all these voussoirs will form together one solid body, the parts of which cannot be separated; for the two voussoirs A and B cannot be disunited without breaking either the balls of marble which connect them with the upper voussoir, or breaking the upper voussoir through the middle. But even if we suppose this effect, which could not be produced without a force almost inconceivable, or at least far superior to the action of the arch, the two halves of the broken voussoir being themselves sustained in a similar manner by the superior voussoirs, no tendency to separate from each other could thence result: the three rows therefore of the arch would form

* All architects, indeed, are not so nice in their choice of materials; but it appears to us that the frequent use of iron for strengthening buildings is subject to much inconvenience and danger. We at least wish that public monuments were constructed without it for if they can support themselves without iron it is needless: if iron is essential to strength, it will certainly be consumed in the course of time by rust, and the edifice will then tumble to pieces, or be greatly injured. The use of iron then in this case is attended with bad consequences.

only one piece, and there would be no thrust. It will be sufficient if the base of this arch have such a thickness as to prevent it from being crushed by its absolute weight; and a very moderate thickness, if the materials be good, will answer this purpose.

We think we have proved therefore, by these two methods, that a hemispherical arch might be constructed without any thrust on its supporters; consequently, if we even suppose that the Architect of Saint Genevieve had adopted the form of Fontana's domes, and had begun by raising on his pendentives a tower of about 36 feet in height, to be crowned by a hemispherical dome, it would not have been impossible to give it a solid construction.

PROBLEM IV.

In what manner the thrust of arches may be considerably diminished.

Architects, in our opinion, have not considered with sufficient attention the resources afforded by mechanics, for diminishing, on many occasions, the thrust of arches. We shall therefore present the reader with some observations on that subject.

When the manner in which an arch tends to overturn its piers is analyzed, it appears that the arch necessarily divides itself somewhere in its flanks, and that the upper part acts in the form of a wedge or a lever on the remainder of the arch, and on the pier, which are supposed to form one body. This consideration then suggests, that to diminish the thrust of the arch, or increase the stability of the pier, the commencement of the flanks ought to be loaded; and that the thickness of the voussoirs near the key ought to be considerably lessened: in short, to make the arch, instead of having a uniform thickness throughout its whole extent, to be very thick at its origin, and at the key to be no thicker than what is necessary to resist the pressure of the flanks. It may be easily perceived, that

as by this method a part of the force which acts to overturn it, is thrown upon that which resists being overturned, the latter will gain a great advantage over the former.

It is to arches in the form of a dome, in particular, that this consideration is applicable; and not only might this method be employed, but also heterogeneity of materials. For this purpose, let us suppose ourselves in the place of the architect of Saint Genevieve, and that it is necessary to construct his dome by first raising a round tower 36 feet in height, to be afterwards crowned by an arch, which we shall suppose to be hemispherical, though he was allowed to make it a little more elevated than that form, in order that it might appear hemispherical when seen at a moderate distance. It is found that giving to this arch the uniform thickness of a foot and a half, the tower ought to be 4½ feet in thickness at the utmost, which added to some necessary enlargement at the foundation, for the sake of solidity, exceeds the breadth of the basis which might be given to it in a part of its circumference. But, according to the above considerations, what would prevent this tower, and the first rows, even as far as towards the middle of the flanks of the arch, from being constructed of materials much more ponderous than the rest of the arch? For we are acquainted with some stones, such as hard and coarse marble, which weigh 230 pounds the cubic foot; while the Saint Leu, in the neighbourhood of Paris, weighs only 132, and brick much less. Instead of giving to the arch the uniform thickness of a foot and a half, why might it not be made 3 feet at the spring, and only 8 inches towards the summit? But by making the following suppositions, namely that the tower and the first rows of the arch, as far as the middle of the flanks, are of the hard stone in the neighbourhood of Paris, which weighs 170 pounds the cubic foot, and the rest of brick which weighs only 130; and that the arch at its spring, as far as the middle, is 2½ feet in thickness, and only 8 inches towards the summit;

we have found that the tower in question ought to be only 1 foot 8½ inches in thickness, to be in equilibrium with the thrust of the arch. If this tower therefore were made 3 feet in thickness, it is evident to the most timid architect, that it would be more than sufficient to counteract every effect of lateral pressure; and it would be still more so were it made 3½ feet in thickness to a certain height, such as that of 9 feet, for example, and then 3 feet or 2 feet 9 inches to the commencement of the arch; as a pier is strengthened by throwing to its lower part a portion of its thickness, instead of making it equally thick throughout; since the point on which it ought to turn, in order to be thrown down, is removed farther back.

But this is enough on a subject which we have introduced here occasionally.

PROBLEM V.

Two persons, who are neighbours, have each a small piece of ground, on which they intend to build; but, in order to gain as much room as possible, they agree to construct a stair common to both houses, and of such a nature, that the inhabitants shall have nothing in common except the entrance and the vestibule. What method must the architect pursue to carry this plan into execution?

The stairs here proposed may be constructed in the following manner, of which there are some examples.

Let fig. 9 n°. 1, pl. 2, be a plan of the stairs, the form of which is of such a nature as to ascend, without being too steep, from the lower to the first story in one revolution, or somewhat less. In a common vestibule A, the entrance to which is through a common door P, construct on the right at B the commencement of the ramp intended for the house on the right; and make it circulate from right to left, as far as a landing place, which must be constructed above the landing place B: the stairs may then be continued in the same manner to a second or third story.

The commencement of the other stair-case must be on the side diametrically opposite at c; and must circulate in the same direction, in order to arrive, after one revolution, at a landing place forming the entrance to the first story of the house on the left; so that if the inside railing of these stair-cases be open, as it may be easily made, those who ascend or descend by one of them, can see those who are on the other, without having any communication but by the common vestibule A and the door of entrance. A section of this double stair-case is seen fig. 9 n°. 2.

At the castle of Chambord there is a stair-case nearly of this form, which serves for the whole building. For, as this edifice consists of four grand vestibules, or immense saloons, placed opposite to each other, in the form of a Greek cross, and into which all the apartments open, Serlio, the architect, constructed the stair-case in the centre of this cross, and, by means of a double ramp, those who enter from the south vestibule on the ground floor, and who front the stair-case before them, arrive after one revolution at the southern vestibule or saloon of the first story, and *vice versa.*

But though the form of this stair-case is very ingenious, it has some great defects, which might have been easily avoided. 1st. The entrance of the stair-case, instead of being directly opposite to the middle of each saloon, is a little on one side. 2d. There is no landing-place before the door which forms the entrance into this story. 3d. The interior railing, which might have been light, and almost entirely open, has only a very small number of apertures.

If the ground would admit, the same artifice might be employed to construct a stair-case with four ramps, all separate from each other, in order to ascend to four different apartments. The plan of a stair of this kind, which is said to have been constructed at Chambord, may be seen in Palladio. That of Serlio, on account of the four gal-

leries to be entered, would no doubt have been much more beautiful, had it been built on the same plan; but we can assert that the stair of Chambord has only two ramps as above described.

REMARK.—Some stairs are distinguished by another peculiarity, namely, the boldness of their construction. Such are those stairs in the form of a screw, the helix of which forms a spiral entirely suspended, so that there remains in the middle a vacuity of greater or less extent. This bold construction depends on the manner in which the steps are cut, and then being fixed by one end in the wall, which on one side supplies the place of a rail. A full account of the mechanism of them may be seen in most works on architecture.

PROBLEM VI.

To construct a floor with joists, the length of which is little more than the half of that necessary to reach from the one wall to the other.

Let the square ABCD, pl. 2 fig. 10, be the frame of the floor, which is to be covered with joists a little more in length than the half of one of the sides AB. On the sides of this square assume the parts AG, BI, CL, and DE equal to the given length of the joists, which must be arranged as seen in the figure; that is, first place EF, and introduce below it GH, with its end H resting on IK; and let K, the end of IK, rest upon LM, the end of which M must be made to rest upon the first joist EF. It may be easily proved, that in this position these joists will mutually support each other without falling.

It is almost needless to observe that the end of each joist must be cut in such a manner, as to enter a notch made for it in the joist on which it rests, and into which it ought to be well fitted. However, as a notch cut into a joist must lessen its strength, it would perhaps be better to

make the end of each joist rest on an iron stirrup of a sufficient size, and affixed to them in a secure manner.

But it is not necessary that the joists should be a little longer than half the breadth of the frame to be covered: a floor may be constructed with pieces of wood much shorter, if they be cut and arranged in a proper manner.

Let us suppose, for example, that an area of 12 feet square is to be covered, and that the pieces of wood, intended to support the floor, are only 2 feet in length. Cut the extremities of one of these pieces of wood in an oblique form, or into a bevel, as represented by the section ACD or BEF, fig. 11; and in the middle of the same piece, form on each side a notch, for receiving the end of another piece cut in like manner. If the same operation be performed on all the rest, they may then be arranged as seen in the figure; a bare view of which will give a better idea of the artifice here employed, than a long description. The oblong spaces, which remain along the walls, may be filled up with pieces of wood half the length of the former. The scaffolding may then be removed with great safety, for these pieces of wood will form a solid floor, and will mutually support each other, provided none of them is destroyed: for it is to be observed, that the breaking of one would make the whole fall to pieces.

Dr. Wallis, at the end of the third volume of his works, gives a great variety of these combinations, and he says that this invention was employed in some parts of England. But on account of the reasons already mentioned, it is to be considered rather as ingenious than useful, and fit only to be adopted when there is a great scarcity of timber, and for floors which have very little weight to support.

Remark.—Instead of pieces of wood, if we suppose stones to be cut in the same manner, it is evident that they would form a flat arch; but in this case, to avoid the danger of breaking, it would be necessary that they should be at most 2 feet in length, and of a suitable width and

thickness. An arch of this kind is generally called the flat arch of M. Abeille; because it was proposed by that engineer, to the Academy of Sciences, in 1699. It is attended with this advantage, that its whole thrust is exerted on the four walls, which serve to support it; whereas a flat arch, constructed according to the usual method, exerts its thrust or push only against two. But this advantage is more than overbalanced by the danger of the whole tumbling to pieces, if one stone only should be deficient. Frezier has treated on this subject at some length, in his work on cutting stones; and has shown how to vary the compartments of the intrados, or lower part, as well as of the extrados, or upper part, which might be formed with these arches. But we must here repeat, that these things are more curious than useful, or rather that this construction is very dangerous.

PROBLEM VII.

Of suspended Arches, called by the French Trompes dans l'Angle.

One of the boldest works in masonry, is that kind of arch called, by the French, *Trompe dans l'Angle**. Let us suppose a conical arch, as SAFBS, pl. 3 fig. 12, raised on the plane of a triangle ASB; if from the middle of the base there be drawn two lines EC and ED, which in general are parallel to the respective sides SD and SC; and if upon these be raised two planes DEF and CEF, perpendicular to the base; these two planes will cut off, towards the summit S, a part of the arch, as FDSCF, the half of which CFDC will be suspended, or project beyond the foundation. This truncated part of the conical arch FDSCF, is what is called a *trompe dans l'Angle;* because in general it is constructed in a re-entering angle to support some projecting part of

* These arches are called *trompes*, because they have a resemblance to the mouth of a trumpet.

an edifice. For this purpose, on the curvilinear planes CF and DF, there are raised walls which, though suspended, have sufficient strength, provided the voussoirs be exactly cut; are long enough to be inserted in the half which is not suspended; and provided also that this part is properly loaded.

Works of this kind are common; but the most singular is one at Lyons; which supports a considerable part of a house, situated on the stone-bridge. One cannot see, without some uneasiness, the corner of this edifice, which is three or four stories in height, project several yards over the river. It is said to be the work of Desargues, a gentleman of the Lyonnese, and an able geometrician, who lived in the time of Descartes. In that case, this work must have stood about 150 years; which seems to prove that this kind of construction has a real and greater solidity than is commonly supposed.

REMARK.—If the suspended arch be a right arch, that is to say a portion of a right cone ASBF; and if the section planes FED and FEC be respectively parallel to SC and SD, the curves FC and FD, as is well known, will be parabolas, having their summit in D, and CE or DE for their axis.

We must here take notice of a geometrical curiosity, which is, that the conical surface FCSDF, though a curve, and terminated in part by curved lines, is equal to a rectilineal figure; for if DG be drawn parallel to the axis SE, it can be demonstrated, that the conical surface in question, is equal to one and a third of the rectangle of SB or SF by EG.

PROBLEM VIII.

A gentleman has a quadrangular irregular piece of ground, as ABCD, *in which he is desirous of planting a quincunx, in such a manner, that all the rows of trees, whether transversal or diagonal, shall be right lines. How must he proceed to carry this plan into execution?*

We shall suppose this quadrilateral to be so irregular, that the opposite sides AB and DC, pl. 3 fig. 13, meet in a point F, and the sides AD and CB in another point E. Continue these sides, two and two, to the points of meeting, E and F, which must be joined by a straight line EF; then through the point D draw a line parallel to EF; continue BC and BA till they meet that parallel, in H and G, and divide GD and DH into the same number of equal parts, which we shall suppose to be four: if through the points of division in GD, as many straight lines be drawn to the point F; and if straight lines be drawn, in like manner, through the points of division in DH to the point E, these lines will intersect the sides of the quadrilateral, and each other, in points, which will be those where the trees must be planted, in order to solve the problem.

For the demonstration we might refer to prob. 24 of Optics, where we have shown how a quadrilateral, such as ABCD, may be the perspective representation of a parallelogram. We shall however here repeat it.

Through the points D and H draw the lines D *a* and H *b*, inclined to GH at an angle of 45 degrees from right to left; and through the points G and D two other lines, G *b* and D *c*, inclined also 45 degrees to GH, but in a contrary direction: these four lines will necessarily cut each other at right angles, and form a rectangle *a b c* D, of which, according to the rules of perspective, the quadrilateral ABCD would be the representation, to an eye situated in the point I, which divides EF into two equal parts, and is at a distance from the plane of the picture equal to IE or IF.

Let us suppose then that the oblong *a b c* D is divided into similar oblongs, by four lines parallel to its sides: these lines, if continued till they meet GD and DH, will divide them into the same number of equal parts; and as DC and GAB are the perspective representations of D *c* and G *a b*, the lines proceeding from the equal divisions of GD, and ending at the point F, will be the perspective representa-

tions of lines parallel to *a b* or D *c*. The case will be the same with the lines parallel to the two sides D *a* and *c b*. The small quadrilaterals then formed by these lines cutting each other in the quadrilateral A B C D, will be the perspective pictures of the oblongs into which *a b c* D is divided. But all the points which are in a straight line in the object, will be in a straight line in the picture; consequently, as the rows of trees planted at the angles of the divisions of the oblong *a b c* D necessarily form straight lines, both transversely and diagonally, their places in the quadrilateral A B C D, which are the pictures of these angles in the oblong, will also form straight lines in the same direction; for, in perspective representations, the pictures of straight lines are always straight lines.

If the opposite sides *a b* and *c* D, of the given quadrilateral, be very unequal, they must not be divided into the same number of parts; for in that case they would be too unequal, since in a plantation of this kind the quadrilaterals ought to be nearly perfect squares. For example, if one side *a b* be 100 yards, and the other 40, by dividing each of them into 20, the divisions on one side would be 5, and on the other 2 yards, which would form figures too oblong. On this supposition, it would be much better to divide the first into 16 and the second into 6, which would give divisions almost square, namely of 6¼ yards in one direction, and 6⅔ in the other, but in this case there would be no diagonal row of trees, either in the oblong *a b c* D, or in the proposed quadrilateral A B C D. In short, by dividing one of the lines G D or D H into 16 parts, and the other into 6, all the rows of trees in the irregular figure will be straight lines.

To have a real quincunx *, it will be sufficient, after this operation, to draw, in each small quadrilateral of the plant-

* A real quincunx is that where there is a tree in the middle of each square; for the word *quincunx* means five trees in a square, which cannot be arranged otherwise.

ation, two diagonals, and to plant a tree in the point where they intersect each other: all these new trees will form straight lines also.

PROBLEM IX.

To construct the frame of a roof, which, without tie-beams, shall have no lateral thrust on the walls on which it rests.

We have seen at Paris, in a garden of the *faubourg Saint-Honoré*, a small building, in the form of a tent, the walls of which were only a few inches in thickness, and which were covered by a roof without tie-beams; the whole being lined in the inside, it had the real appearance of a tent. It was used as a summer apartment in the day-time, and formed a retreat truly delightful.

What surprized those who had any knowledge of architecture, was, how the roof of this small edifice could be constructed without tie-beams: for however light it might be, the walls were so thin, that any common roof must have overturned them. The artifice, said to have been the invention of M. Arnoult, superintendant of the theatres *des Menus-Plaisirs*, was as follows:

Two rafters, CD and ED, pl. 3 fig. 14, resting on the two beams AB and *ab*, were strongly joined together at the summit D. From the angles, which these two rafters formed at C and E, proceeded two other pieces of timber, which were well united to the beams at F and G, to the rafters at H and I, and also to each other at K, by means of a double notch. For the sake of greater security, the pieces CD and FH, and ED and GI, were bound together by two cross pieces at L and M. It is evident that these four inclined pieces can have no tendency to separate, or to exert any lateral thrust on the walls upon which the beams AB and *ab* are placed: for they cannot separate without rendering the angle D more obtuse. For this purpose, it would be necessary that the angle K should become more obtuse also; but the junctions at I and H oppose any movement

of this kind; consequently this frame work will rest on the beams A a and a b, without separating in any manner, and will exert no lateral pressure against the walls.

It is hence evident that this artifice might be of great use in architecture, especially when it is required to cover an extensive building, the walls of which are thin, and to avoid the disagreeable effect produced by tie-beams, when not concealed from the sight.

PROBLEM X.

On measuring arches en cul-de-four, surhaussé, and surbaissé.

The appellation of *cul-de-four* is applied to vaults on a plan commonly circular, a section of which through the axis is an ellipsis, or as the French architects call it an *anse de panier*. They differ from hemispherical arches in this, that in the latter the height of the summit above the plane of the base is equal to the radius of that base; while in the former this height is greater or less: if greater the arch is called *cul-de-four surhaussé*, if less, it is called *cul-de-four surbaissé*. Both these arches are represented pl. 4 fig. 15 and 16. The first is an arch *en cul-de-four surhaussé*; the second arch *en cul-de-four surbaissé*. In the language of geometry, the one is an elongated semi-spheroid, or an arch formed by the circumvolution of a semi-ellipse around its greater semi-axis; the other a semi-spheroid formed by the circumvolution of the same semi-ellipsis about its less semi-axis.

Books of architecture contain, in general, rules so false for measuring the superficial content of these arches, that we think it necessary to give here methods more correct. Bullet and Savot, for example, say that nothing is necessary but to multiply the circumference of the base by the height; as if the arch to be measured were hemispherical. This is an egregious error, and it is surprizing that those authors did not observe that if this rule were correct, the

superficial content of some arches *en cul-de-four surbaissé*, would be less than the circle covered by them, which is absurd.

For let us suppose, by way of example, an arch of a foot in height, on a circle of 7 feet diameter; the area of this circle, according to the approximation of Archimedes, will be equal to 38½ square feet; but if the circumference, 22 feet, be multiplied by one foot in height, we should have only 22 square feet; which is not two-thirds of the base. In this case, the builder would be cheated of more than two-thirds of what he ought to receive. We shall therefore give rules for measuring such arches, sufficiently correct to be employed in the common purposes of architecture.

I. *For arches en cul-de-four surhaussé, or the Oblong Spheroid.*

The radius of the base and the height of the arch being given, first make this proportion: As the height is to the radius of the base, so is the latter to a fourth term, the third of which must be added to two-thirds of the radius of the base.

Then find the circumference corresponding to a radius equal to that sum, and multiply this circumference by the height: the product will be the superficial content or curve surface nearly.

Example.—Let the height be 10 feet, and the radius of the base 8. Then say as 10 is to 8, so is 8 to $6\frac{2}{5}$, the third of which is $2\frac{2}{15}$: two-thirds of 8 are $5\frac{1}{3}$, which added to $2\frac{2}{15}$, make $7\frac{7}{15}$ feet, or 7 feet 5 inches 7 lines.

But the circumference corresponding to $7\frac{7}{15}$ feet radius, or $14\frac{14}{15}$ feet diameter, is $46\frac{14}{15}$ feet, which multiplied by 10 feet, the height of the arch, gives for product $469\frac{1}{3}$ square feet, or 52 yards $1\frac{1}{3}$ foot.

By Bullet's rule, the superficial content would have been 55 yards 7 feet; the difference of which in excess is 3

yards 5 feet, or about a 14th of the whole; and this in an arch which does not deviate much from a hemisphere: if it deviated more, the error would be considerably greater.

II. *For arches en cul-de-four surbaissé, or the Oblate Spheroid.*

The rule for these arches is nearly the same as the preceding. Find a third proportional to the height and the radius of the base; and add two-thirds of it to the radius of the base; then find the circumference corresponding to the sum as a radius, and multiply it by the height: the product will be the superficial content nearly of the given arch.

Let the radius of the base of an arch *en cul-de-four surbaissé* be 10 feet, and the height be 8. As 8 is to 10, so is 10 to $12\frac{1}{2}$, two-thirds of which are $8\frac{1}{3}$; on the other hand, the third of 10 is $3\frac{1}{3}$, which added to the former, gives 11^{2} feet.

But the circumference corresponding to $11\frac{2}{3}$ feet radius, or $23\frac{1}{3}$ diameter, is $73\frac{1}{3}$, which multiplied by the height, that is 8 feet, gives for product $586\frac{2}{3}$ feet, or 65 yards $1\frac{2}{3}$ foot = the superficial content of the arch.

According to Bullet's rule the superficial content would have been 55 yards 7 feet; which makes an error in defect of 9 yards $3\frac{2}{3}$ feet, or above a 6th part of the whole surface.

Remark.—It would be easy to give, for those who are geometricians, rules still more exact; as it is well known that the dimensions of prolate spheroids depend on the measurement of a truncated elliptical or circular segment; and that of the surface of an oblate spheroid, on the measurement of an hyperbolical space; consequently the former may be determined by means of a table of sines and circular arcs, and the other by employing a table of logarithms.

In regard to the method above given, it is deduced from

the same principles; but by considering a segment of a circle or hyperbola of a moderate extent, as a parabolic area, which when this segment forms but a small part of the space to be measured, is liable only to a very small error: in many cases this consideration supplies practical rules exceedingly convenient.

Some architects may perhaps ask: Of what advantage is it to be able to ascertain with precision the superficial content of these domes, as a few yards more or less can be of little importance? But it may be said in reply, that for the same reason, accurate measurement in general is of little utility. To such persons it is of no consequence that Archimedes has demonstrated that the surface of a hemisphere is equal to that of a cylinder of the same base and height; or, to speak according to their own terms, that the surface of a hemispherical arch is equal to the product of the circumference of the base by the height. If they employ, in regard to the arches in question, rules so erroneous, it is because they consider them as exact, and because they have been taught them by people so ignorant of geometry, as not to be able to give them better ones.

PROBLEM XI.

To measure Gothic or Cloister arches, and arches d'arête, or Groin Arches.

It frequently happens that on a square, an oblong, or polygonal space or edifice, an arch vault is raised, consisting of several *berceaux* or vaults, which commencing at the sides of the base, unite in a common point as a summit, and form in the inside as many re-entering angles or groins, as there are angles in the figure which serves as a base. These arches are called *arcs de cloitre*, cloister arches. A representation of them is seen fig. 17 pl. 4.

But if the space or edifice, a square for example, be covered with two *berceaux* or vaults, (fig. 18), which seem to penetrate each other, and which form two ridges or re-

entering angles, intersecting each other at the summit of the vault, such an arch is called an arch *d'arête*, or a groin arch. The most remarkable properties of these arches are as follow.

1st. The superficial content of every circular cloister arch, on any base, whether square or polygonal, is exactly double that of the base, in the same manner as the superficial content of a hemispherical arch, or arch *en cul-de-four* or *en plein ceintre*, is double that of the circular base.

It may indeed be said, that a hemispherical arch is only a cloister arch on a polygon of an infinite number of sides.

When the superficial content therefore of such an arch is to be measured, nothing will be necessary but to double the surface of the base, provided the *berceaux* be *en plein ceintre*, or a complete semi-circle; for if they are greater or less, they will have to the base, the same ratio that an arch *en cul-de-four surhaussé*, or *surbaissé*, has to the circle of its base.

2d. A cloister arch, and a groin arch on a square, form together the two complete *berceaux* or vaults, raised upon that square.

This may be readily seen in fig. 19. Therefore if from two *berceaux* or vaults, the cloister arch be deducted, there will remain the groin arch, which in this case gives a simple method for measuring groin arches; for if the superficial content of the cloister arch be subtracted from the superficial content of the two vaults, the remainder will be that of the groin.

If the base, for example, be 14 feet in both directions, the circumference of the semi-circle of each will be 22 feet, and the superficial content will be 22 by 14 or 308 square feet; consequently the superficial content of both the *berceaux* will be 616 square feet. But the interior surface of the gothic arch is twice the base, or twice 196, that is 392; and if this number be subtracted from 616, we shall

have 224 square feet, for the superficial content of the groin arch.

3d. If the solid content of the interior of such an arch be required. Multiply the base by two-thirds of the height. This is evident from the reason already given in regard to the superficial content; for arches of this kind, both in regard to their solidity and superficial content, are to a prism of the same base and height, in the same ratio as the hemisphere to the circumscribed cylinder.

4th. The solidity of the space contained by a groin arch on a square or oblong plane, is $\frac{2}{7}$ of the solid having the same base and height, supposing the approximate ratio of the diameter of the circle to the circumference, to be as 7 to 22. This may be easily demonstrated also, by observing, that the interior solid of such an arch, is equal to the sum of the two vaults or demi-cylinders, minus once the solidity of the cloister arch, which is twice comprehended in this double, and consequently ought to be deducted.

PROBLEM XII.

How to construct a wooden bridge of 100 feet and more in length, and of one arch, with pieces of timber, none of which shall be more than a few feet in length.

We shall here suppose, that the pieces of timber intended for a bridge of this kind, are 12 or 14 inches square, and only about 12 feet in length: or that particular circumstances have prevented rows of piles from being sunk in the bed of the river, to support the beams employed in constructing the work. In what manner must the architect proceed to build the bridge, notwithstanding these difficulties?

The execution of this plan is not impossible: for it might be accomplished in the following manner. First trace out, on a large wall, a plan of the projected bridge, by describing two concentric arches at such a distance from each other, as the length of the pieces of timber to be em-

ployed will admit; which we shall suppose, for example, to be 100 feet, giving them the form of an arc of 90 degrees from one pier to another: then divide this arc into a certain number of equal parts, in such a manner, that the arc of each shall not exceed 5 or 6 feet.

On the supposition here made, of the distance of 100 feet between the two piers, an arc of 90 degrees which covers it would be 110 feet in length, and its radius would be 70 feet. Divide then this arc into 22 equal parts, of 5 feet each, and with the above pieces of timber, joined together, form a kind of voussoirs, 8 or 10 feet in height, 5 feet in breadth at the intrados, and 5 feet 8 inches 6 lines at the extrados; for such are the proportions of these arcs, according to the above dimensions. Fig. 20 represents one of these voussoirs, which, as it is evident, consists of four principal pieces of strong timber, at least 10 inches square, which meet two and two at the centre of their respective arcs; of three principal cross bands at each face, as AC, BD, EF, *ac*, *bd*, *ef*, which must be exceedingly strong, and therefore ought to be 12 or 14 inches in height, and 10 inches in breadth; and, lastly, of several lateral bands, between the two faces, to bind them together in different directions, and to prevent them from giving way. A voussoir of this kind may be about 6 feet in length, that is between the two faces AEFB and *aefb*.

An arch must then be formed of these voussoirs, exactly in the same manner as if they were stone, and when they are all arranged in their proper places, the different pieces may be bound together according to the rules of art, either with pins or braces. Several arches or ribs of this kind must be formed, close to each other, according to the intended breadth of the bridge; and the pieces may be bound together in the same manner as the first, so as to render the whole firm and secure. By these means we shall have a wooden bridge of one arch, which it would be very difficult to construct in any other manner.

It now remains to be examined whether these voussoirs will have sufficient strength to resist the pressure which they will exert on each other. The following calculation will show that there can be no doubt of it.

It appears from the experiments of Muschenbroeck *, and the theory of the resistance of bodies, that a piece of oak 12 inches square, and 5 feet in length, can sustain in an upright position, without breaking, 264 thousand pounds; hence it follows that a cross band, as AC or EF, 5 feet in length and 12 inches by 10, can support 220000; but for the greater certainty we shall reduce this weight to 150000: therefore, as we have six bands of this length, a few inches more or less, in each of these voussoirs, it is evident that the effort which one of these voussoirs is capable of sustaining, will be at least 900 thousand pounds. Let us now examine what is the real effort to be resisted.

We have found, by calculating, the absolute weight of such a voussoir, and even supposing it to be considerably increased, that it will weigh at most between 7 and 8 thousand pounds or 7500. The weight then resting on one of the piers, most loaded, having 10 voussoirs to support, will be charged only with the weight of 75000 pounds; a weight however which, on account of the position of the voussoirs, will exert a pressure of 115000 pounds; but we shall suppose it to be even 120000. There is reason therefore to conclude, from this calculation, that such a bridge would not only have strength to support itself, but also to bear, without any danger of breaking, the most ponderous burthens: it even appears that it would not be necessary to make the pieces of timber so strong.

If the expence of such a bridge be compared with that attending the common method, it will perhaps be found to be much less; for one of these voussoirs would contain no more than 140 or 150 square feet of timber, which at

* Essais de Physique, vol. i. chap. ii.

the rate of 25s. per foot, would be only 15£; so that the 22 voussoirs, of one course or rib, would cost 330£; consequently, if we suppose the breadth of the bridge to consist of four courses or ribs, the whole would amount only to 1320£. It must indeed be allowed that to complete such a bridge, other expences would be required; but the object here proposed, was to show the possibility of constructing it, and not to calculate the expense.

The idea of such a bridge first occurred to me in consequence of a dangerous passage I met with in the province of Cusco, in Peru; where I was obliged to cross a torrent, that flows between two rocks, about 125 feet distant from each other, and more than 150 feet in height. The inhabitants of the country have constructed there a *Travita**, where I was in danger of perishing. When I arrived at the next village, I began to reflect on the best means of constructing in this place a wooden bridge, and I contrived the above expedient. I proposed my plan to the Corregidor, Don Jayme Alonzo y Cuniga, a very intelligent man, who, being fond of the French, received me with great politeness. He approved of my idea, and agreed that, at the expence of a thousand piasters, a bridge of 12 feet in breadth, which all Peru would come to see through curiosity, might be constructed in that place. But as I set out three days after, I do not know whether this project, with which this worthy man seemed highly pleased, was ever carried into execution.

It may here be remarked, that it would be easy to

* This is an Indian bridge, the very idea of which is enough to make one shudder. A man is placed in a large basket, fastened by a pulley to a rope which is extended from the one side of a torrent to the other. The basket and rope are both constructed of those creeping plants, which the inhabitants of America employ in almost all their works. As soon as the man has got into the machine, it is drawn over to the opposite side, by means of a rope fastened to the pulley. If the rope, used for dragging over the machine, should break, the man must remain suspended for some hours, until means have been found to relieve him from his painful situation.

arrange the voussoirs of a bridge of this kind, in such a manner that, in case of necessity, any one of them might be taken out, in order to substitute another in its stead; which would afford the means of making all the necessary repairs.

PROBLEM XIII.

Is it possible to construct a Plat-band, or Frame, which shall have no lateral thrust?

It would be of great advantage to be able to execute a work of this kind, for one of the obstacles which architects experience, when they employ columns, arises from the thrust of their architraves, which requires that the lateral columns should be strengthened by different means. This embarrassment they are particularly liable to, when they make detached porches to project before an edifice, like that of Sainte-Genevieve: the two frames, that of the face and the side, exert such a push on the angular column or columns, that it is very difficult to secure them; and it is even sometimes necessary to renounce them, if stones cannot be found sufficiently large to make architraves of one piece, from column to column, at least in the spaces nearest the angles.

These difficulties would be obviated, if frames could be made without any thrust. This we do not think impossible; and we propose the problem to architects in the hope that some of them will be able to solve it.

PROBLEM XIV.

Is it a perfection, in the Church of St. Peter at Rome, that those who see it, for the first time, do not think it so large as it really is; and that it appears of its real magnitude after they have gone over it?

Though we announced, in the beginning of this work, that we meant to exclude from it whatever was mere matter of taste; as the above question is connected with

physical and metaphysical reasons, we are of opinion that it may be admitted.

The impression which the church of St. Peter at Rome makes, on the first view, has been boasted of as a perfection. Every person, as far as we have heard or read, who enters this edifice, for the first time, conceives the extent of it to be far less than it is generally accounted to be by public report. To have a just idea of its grandeur, one must have seen, and in some measure studied, every part of it.

Before we venture to say any thing decisive on this subject, it may perhaps be of some use to examine the causes of this first impression. In our opinion, it arises from two sources.

The first is the small number of principal parts into which this immense edifice is divided, for, from the entrance to the middle, which constitutes the dome, there are only three lateral arcades. But, though dividing a large mass into many small parts tends, in general, to diminish its effect, there is still a medium to be observed; and it appears to us that Michael Angelo kept too far below it.

The second cause of the impression which we here examine, is the excessive size of the figures and ornaments, which serve as appendages to the principal parts. We can indeed judge of the size of objects beyond our reach, only by comparing them with neighbouring objects, the dimensions of which are familiar to us. But if these objects, the dimensions of which are known, or are nearly given by nature, accompany others to which they have a ratio that approaches too near to equality, it must necessarily follow that the latter, in the imagination of the spectator, will lose a part of their magnitude. Such is the case with the church of St. Peter at Rome: the figures placed in niches, which decorate the spaces between the pillars of the arcades; those between the pilasters and

those which ornament the tympana of the lateral groupes, are truly gigantic; but they are human figures; they are besides, for the most part, raised very high; consequently they appear less, and make the principal parts which they accompany to appear less also.

By some people, this illusion is considered to be a master-piece of the art and genius of the celebrated architect, the principal author of this monument. Shall we be permitted to differ from them? For what is the object which the constructors of this immense edifice had in view; and which will be the m of all those who raise edifices that exceed the usual measures? Doubtless to excite astonishment and admiration. We are convinced that Michael Angelo would have been much mortified, had he heard a stranger, just arrived at Rome, and entering St. Peter's for the first time, say publicly: "This is the church respecting the immensity of which we have heard so much: it is a large building; but not so large as generally reported."

In our opinion, it would display much more ingenuity to construct an edifice which, though of a moderate size, should immediately excite in the mind the idea of considerable extent; than to construct an immense one which, on the first view, should appear of a moderate size. We do not think that on this subject there can be any difference of opinion. Whatever then may be the perfection, which it must be allowed the church of St. Peter possesses, so far as harmony of proportion, beauty and magnificence of architecture, are concerned, we are of opinion that Michael Angelo missed his aim in regard to the object in question; and it is probable that he would have approached much nearer to it, had he employed less gigantic appendages. If the children, for example, which support the *bénitiers** had been of less size; if the figures which accompany the

* Benitiers are vases for holding holy-water.

archivaults of his lateral arcades, as well as those which decorate the niches between the pilasters, had been on a scale not so enormous, a comparision of the one with the other would have made the principal parts appear much greater. Those who turn their eyes from these gigantic objects, and direct them towards a man near the middle, or at the extremity of the church, experience this effect: it is then, by comparing their own size with that of the principal parts of the edifice in the neighbourhood, that they begin to form an idea of its extent, and are struck with astonishment; but this second impression is the effect of a sort of reasoning, and the sensation, when produced in this manner, has not the same energy, as when it is the effect of a first view.

While we are on this subject, we shall take the liberty of offering a few observations on the means of enlarging, as we may say, any space by the help of the imagination. In our opinion, nothing contributes more to produce this effect, than insulated columns, that is to say columns not regularly connected; for, when coupled or grouped, they always produce this effect more or less, though it would doubtless be much better to employ them single. The result is, that every time the spectator changes his position, different openings occur, and a variety of aspects which astonish and deceive the imagination.

But when columns are employed, they ought to be large; for in the same degree as they have a majestic appearance when constructed on a grand scale, they are, in our opinion, mean and diminutive when small, and particularly when supported on pedestals. The court of the Louvre, though in other respects beautiful, would have a much more striking effect, if the columns, instead of being mounted on meagre pedestals, rose from the ground supported merely by a socle, like those in some of the vestibules of that palace. One might almost say, and there is some reason to think, that pedestals were invented to

render fit for use columns collected at hazard, and which have not the requisite dimensions.

If Michael Angelo then, instead of forming his lateral spaces of immense arcades supported by pillars, decorated with pilasters, had employed groupes of columns; if instead of placing only three rows of lateral arcades, between the entrance and the part of the dome, he had placed a greater number, which this arrangement would have allowed him to do, and if the figures employed amidst this decoration had not far exceeded the natural size, we entertain no doubt that the spectator would have been struck with astonishment on the first view, and that the edifice would have appeared much larger.

But it is to be observed, at the same time, that the knowledge which we now possess, in regard to the resistance of materials, and the philosophy or mechanical part of architecture, was not known at the time when Michael Angelo lived. It is probable that he durst not venture to load columns, even when grouped, with a weight so considerable as that which he had to raise upon these pillars. But it is proved, by late experiments in regard to stones, that there is no weight that an insulated column, six feet in diameter, made of very hard stone, well chosen and prepared, is not capable of supporting. Our ancient churches, called improperly *gothic*, are a proof of it; for there are some of them, the whole mass of which rests on pillars scarcely six feet in diameter, and often less: they therefore in general convey an idea of extent, which the Greek architecture, employed in the same places, does not excite.

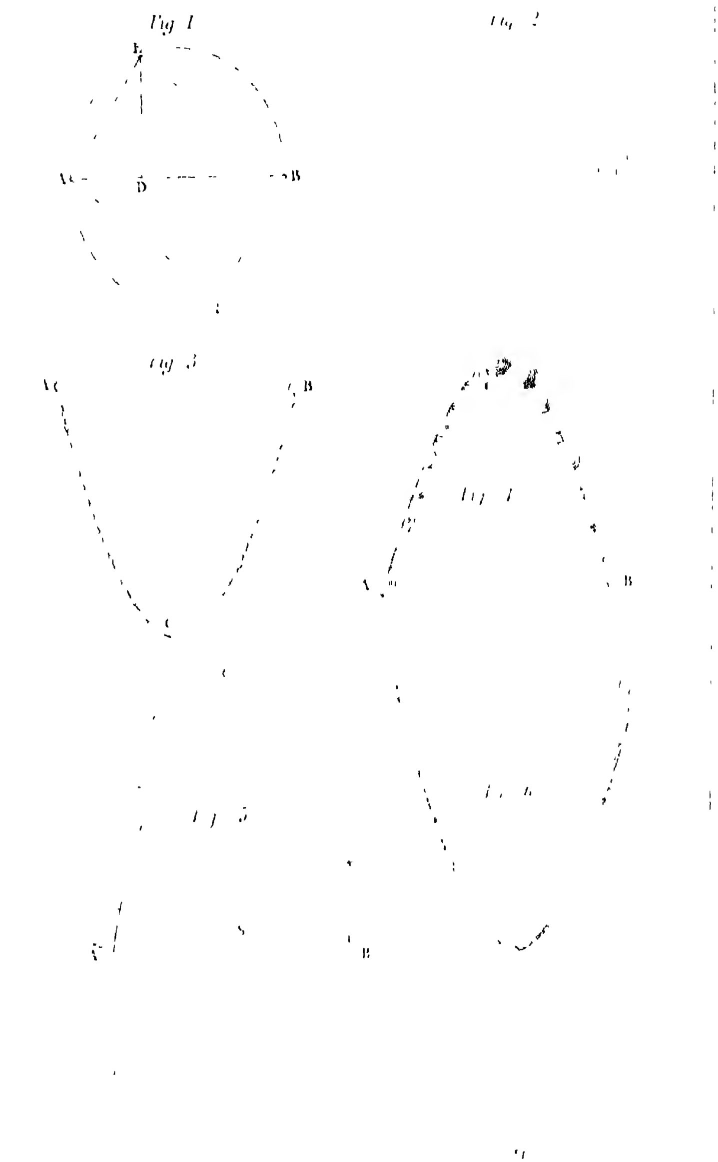


Fig. 1
E
A
D
B
Fig. 2
Fig. 3
A
B
C
Fig. 4
A
B
Fig. 5
Fig. 6
B

MATHEMATICAL

AND

PHILOSOPHICAL

RECREATIONS.

PART TENTH.

Containing the most curious and amusing operations in regard to Pyrotechny.

WHY it has been usual to consider pyrotechny as a branch of the mathematics, we do not know. The least reflection will readily show, that it is an art by no means mathematical, though dimensions, proportions, &c, are employed in it. There are a great number of other arts which have a much better claim to be included among these sciences.

However, as we might be blamed for omitting an art which affords a considerable field for amusement, and as it is connected, at least, with natural philosophy, we shall make it the subject of one of the divisions of this work. But as we do not intend to give a complete treatise of pyrotechny, we shall confine ourselves to those parts which are most common and most curious: we shall also avoid every thing that relates to the fatal art of destroying men. We can see no amusement in the motion of a bullet,

which carries off files of soldiers, nor in the action of a bomb or shell that sets fire to a town. The preceding editors and continuators of Ozanam, seem to have possessed a very military spirit, if they considered all these things as harmless recreation. For our part, having imbibed other principles in that happy country, Pennsylvania, we shudder even at the idea of introducing such atrocities under the form of amusement.

Pyrotechny, as we consider it in this work, is the art of managing fire, and of making, by means of gunpowder and other inflammable substances, various compositions, agreeable to the eye, both by their form and their splendour. Of this kind are rockets, serpents, sheaves of fire, fixed or revolving suns, and other pieces employed in decorations and fire-works.

Gunpowder being the most common ingredient in pyrotechny, we shall begin with an account of its composition.

ARTICLE I.

Of Gunpowder.

Gunpowder is a composition of sulphur, saltpetre, and pounded charcoal: these three ingredients mixed together, in the proper quantities, form a substance exceedingly inflammable, and of such a nature, that the discovery of it could be owing only to chance. A single spark is sufficient to inflame, in an instant, the largest mass of this composition. The expansion, suddenly communicated either to the air, lodged in the interstices of the grains of which it consists, or to the nitrous acid which is one of the elements of the saltpetre, produces an effort which nothing can resist; and the most ponderous masses are driven before it with inconceivable velocity. We must however observe that this invention, to which the epithet of *diabolical* is frequently applied, is not so destructive to the human race as it might at first appear: battles seem to have been attended with less slaughter since gunpowder began to be used;

and, as is remarked by the celebrated Marshal Saxe, the noise and smoke produced by fire arms, during a battle, are more terrible than the execution they make. We must however except cannon when well directed: but let us return to our subject, and give an account of the process for making gunpowder.

Sulphur is found ready formed, and almost in its last degree of purity, in volcanic productions. It is found also, and much more frequently, in the state of sulphuric acid; that is to say combined with oxygen: it is in this state that it is found in argil, gypsum, &c. It may be extracted likewise from vegetable substances and animal matters.

To purify sulphur, melt it in an iron pan; by which means the earthy and metallic parts will be precipitated; and then pour it into a copper-kettle, where it will form another deposit of the foreign matters, with which it is mixed. After keeping it in fusion some time, pour it into cylindric wooden moulds, in order that it may be formed into sticks.

Saltpetre, or, as it is called in the modern chemistry, nitrate of potash, exists in a natural state, but in small quantities. It is found sometimes at the surface of the ground, as in India, and sometimes on the surface of calcareous walls, the roofs of cellars, under the arches of bridges, &c.

To extract the saltpetre from the lime of walls, or other earths impregnated with it, the earths are put into casks, placed on timbers, and water is poured over them to the height of about three inches. When the water has remained in that state five or six hours, it is suffered to run off by apertures made in the bottom of the casks, from which it falls into a gutter that conveys it to a common reservoir sunk in the earth. When the sediment has been deposited, the clear liquor is drawn off into a proper vessel, in order to be evaporated.

When the liquor is in a state of ebullition, in proportion as it evaporates, there is precipitated calcareous earth, and then muriate of soda. To know when it is sufficiently evaporated, put a drop of it on a piece of cold iron, and if it becomes fixed, and assumes a white solid globular form, it is time to slacken the fire. The liquor must then be left at rest for twenty-four hours, after which it is run off, and set to crystallize.

It is needless to describe charcoal, as it is every where known. We shall only observe, that the charcoal found by experience to be fittest for the composition of gunpowder, is that made from the alder, willow, or black dog-wood.

To make gunpowder, mix together 6 parts of pounded nitre, well purified, 1 part of pounded sulphur, exceedingly pure, and 1 part of pounded charcoal, adding a quantity of water sufficient to reduce them to a soft paste. Put the whole into a wooden or copper mortar, and with a pestle of the same materials, to prevent inflammation, pound these ingredients for 24 hours, to mix them thoroughly; taking care to keep them always moderately moist. When they are well incorporated, pour the mass upon a sieve pierced with small holes of the size which you intend to give to the grains of the powder. If it be then pressed, shaking the sieve, it will pass through in grains, which must be dried in the sun or over a stove without fire. When dry, it ought to be put into vessels capable of preserving it from moisture.

Every one knows that, in consequence of the great consumption of gunpowder, certain machines, called *powder mills*, have been invented. These machines consist of a beam turned by means of a water wheel, and furnished with a great number of projecting arms, which raise up and let fall in succession a series of pestles or stampers, below which are placed copper vessels or mortars containing the matter to be pounded and incorporated. These

mills, however, are exceedingly disagreeable neighbours; for notwithstanding the precautions taken, there are few of them which do not some time or other blow up. On this account they ought always to be erected at a distance from towns or dwellings.

As the enlarged state of chemistry has introduced some improvements in the art of making gunpowder, we shall here, in addition to what has been above said, give the following account of the process employed for this purpose in some of the English manufactories.

Gunpowder is made of three ingredients: salt-petre, charcoal, and brimstone; which are combined in the following proportions: for each 100 parts of gunpowder, saltpetre 75 parts, charcoal 15, and sulphur 10.

The saltpetre is either that imported principally from the East Indies, or that which has been extracted from damaged gunpowder. It is refined by solution, filtration, evaporation, and crystallization; after which it is fused; taking care not to use too much heat, that there may be no danger of decomposing the nitre.

The sulphur used, is that which is imported from Sicily, and is refined by melting and skimming, the most impure is refined by sublimation.

The charcoal formerly used in this manufacture, was made by charring wood in the usual manner. This mode is called charring in pits. The wood is cut into pieces of about three feet in length; it is then piled on the ground, in a circular form, three, four, or five cords of wood making what is called a pit, and then covered with straw, fern, &c. kept down by earth or sand, and vent-holes are made, as may be necessary, in order to give it air. As this method is uncertain and defective, the charcoal now used in the manufacturing of gunpowder, is made in the following manner. The wood to be charred is first cut into pieces of about nine inches in length, and put into an iron cylin-

der placed horizontally. The [illegible] of the cylinder is then closely stopped, at the other end there are pipes connected with casks. Fire being made under the cylinder, the pyro-ligneous acid, attended with a large portion of hydrogen gas, comes over. The gas escapes, and the acid liquor is collected in the casks. The fire is kept up till no more gas or liquor comes over, and the carbon remains in the cylinder.

The several ingredients, being thus prepared, are ready for manufacturing. They are first round separately to a fine powder; they are then mixed together in the proper proportions; and the composition in this state is sent to the gunpowder mill, which consists of two stones placed vertically, and running on a bed-stone. On this bed-stone the composition is spread out, and moistened with as small a quantity of water as will reduce it to a proper body, but not to a paste: after the stone runners have made the proper revolutions over it, it may then be taken off.

A powder mill is a slight wooden building, with a boarded roof. Only about 40 or 50 lb. of composition is worked here at a time, as explosions may happen by the runners and bed-stone coming into contact, and even from other causes. These mills are worked either by water or by horses.

The composition, when taken from the mill, is sent to the corning house, to be corned or grained. Here it is first formed into a hard and firm mass, it is then broken into small lumps, and afterwards grained, by these lumps being put into sieves, in each of which is a flat circular piece of lignum vitæ. The sieves are made of parchment skins, having round holes punched through them. Several of these sieves are fixed in a frame, which by proper machinery has such a motion given to it, as to make the lignum vitæ runner in each sieve go round with great velocity, so as to break the lumps of powder, and by forcing

it [illegible] to form it into grains of several sizes. These are then separated from the dust by sieves and reels made for that purpose.

The grains are next hardened, and the rougher edges are taken off by shaking them a sufficient time in a close reel, moved in a circular direction with a proper velocity.

The powder for guns, mortars, and small arms, is generally made at one time, and always of the same composition. The only difference is in the size of the grains, which are separated by sieves of different fineness.

The gunpowder thus corned, dusted and reeled, which is called glazing, as it gives it a small degree of gloss, is then sent to the stove and dried; care being taken not to raise the heat so much as to decompose the sulphur. The heat is regulated by a thermometer placed in the door of the stoves, if dried in a gloom-stove *.

A gunpowder stove dries the powder either by steam or by the heat from an iron gloom, the powder being spread out on cases, placed on proper supports around the room.

If gunpowder is injured by damp in a small degree, it may be recovered by again drying it in a stove; but if the ingredients are decomposed, the nitre must be extracted, and the gunpowder re-manufactured.

There are several methods of proving and trying the goodness and strength of gunpowder. The following is one by which a tolerably good idea may be formed of its purity, and also some conclusion as to its strength.

Lay two or three small heaps, about a dram or two of

* This kind of stove consists of a large cast-iron vessel, projecting into one side of a room, and heated from the outside, till it absolutely glows. From the construction it is hardly possible that fire can be thrown from the gloom, as it is called; but stoves heated by steam passing through steam-tight tubes, or otherwise, ought certainly to be preferred, for the most cautious workman may stumble, and if he has a case of powder in his hand, some of it may be thrown upon the gloom; and it is not improbable that some of the accidents which have happened to powder mills may have been occasioned in this manner.

the powder, on separate piec[illegible]; fire one of them by a red hot [illegible]ds rapidly, with a good report, [illegible]om white specks, and without burn[illegible] if sparks fly off and set fire to the adj[illegible]od-ness of the ingredients and prope[illegible] the powder may be safely inferred; but if otherwise, it is either badly made, or the ingredients are impure.

The editor of this English edition of the Recreations, has been fortunate enough to succeed in constructing the most convenient and most accurate eprouvette that has perhaps ever been contrived, for accurately determining the comparative strength of gunpowder. It consists of a small cannon, or gun, suspended freely, like a pendulum, with the axis of the gun horizontal. This being charged with the proper charge of powder, and then fired, the gun swings, or recoils backward, and the instrument itself shows the extent of the first or greatest vibration, which indicates the strength to the utmost nicety.

Having thus given an account of almost every thing necessary to be known in regard to the process for making gunpowder, we shall now say a few words respecting the physical causes of its inflammation and exploding.

Gunpowder being composed of the above ingredients, when a spark, struck from a piece of flint and steel, falls on this mixture, it sets fire to a certain portion of the charcoal, and the inflamed charcoal causes the nitre with which it is mixed or in contact to detonate, and also the sulphur, the combustibility of which is well known. Portions of the charcoal contiguous to the former take fire in like manner, and produce the same effect in regard to the surrounding mass: thus the first portion inflamed, inflames a hundred others, these hundred communicate the inflammation to ten thousand; the ten thousand to a million, and so on. It may be easily conceived that an inflammation, the progress of which is so rapid, cannot fail to extend it-

se[illegible] ort time, from the one extremity [illegible] t mass.

W[illegible] of this inflammation, that granulated powder inflames with much more rapidity than that which is not granulated. The latter only puffs away slowly, while the other takes fire almost instantaneously; and of the granulated kinds of gunpowder, that in round grains, like the Swiss powder, inflames sooner than that in oblong irregular grains, like the French. The reason of this is, that the former leaves to the flame of the grains, first inflamed, larger and freer interstices, which produce the inflammation with more rapidity.

In regard to the expansion of inflamed gunpowder, is it occasioned by the air interposed between its grains, or by the aqueous fluid which enters into the composition of the nitre? We doubt much whether it be the air, as its expansibility does not seem sufficient to explain the phenomenon; but we know that water when converted into vapour by the contact of heat, occupies a space 14000 times greater than its original bulk, and that its force is very considerable.

In the foregoing account however Montucla seems to have missed the true cause of the expansive force of fired gunpowder, the discovery of which is chiefly due to the English philosophers, and particularly to the learned and ingenious Mr Robins. This author apprehends that the force of fired gunpowder consists in the action of a permanently elastic fluid, suddenly disengaged from the powder by the combustion, similar in some respects to common atmospheric air, at least as to elasticity. He showed, by satisfactory experiments, that a fluid of this kind is actually disengaged by firing the powder; and that it is *permanently* elastic, or retains its elasticity when cold, the force of which he measured in this state. He also measured the force of it when inflamed, by a most ingenious method, and found its strength in that state to be about a thousand times

the strength or elasticity of com[illegible]his however is not its utmost deg[illegible]und to increase in its force when [illegible] than those employed by Mr. Robins; so [illegible] that, by more accurate and effectual ex[illegible] have found its force rise as high as 1600 or 1800 times the force of atmospheric air in its usual state. Much beyond this it is not probable it can go, nor indeed possible, if there be any truth in the common and allowed physical principles of mechanics. With an elastic fluid of a given force, we infallibly know, or compute the effects it can produce, in impelling a given body; and on the other hand, from the effects or velocities with which given bodies are impelled by an elastic fluid, we as certainly know the force or strength of that fluid. And these effects we have found perfectly to accord with the forces above mentioned. If any gentleman therefore thinks he has discovered that fired gunpowder is 50 or 60 times as strong, as above stated, he must have been deceived by mistaking or misapplying his own experiments; and we apprehend it would not be difficult, if this were the proper place, to show, that this has actually been the case.

Mr. Robins's discovery and opinion have also been corroborated by others, among the best chemists and philosophers. Lavoisier was of opinion that the force of fired gunpowder depends, in a great measure, on the expansive force of uncombined caloric, supposed to be let loose, in a great abundance, during the combustion or deflagration of the powder. And Bouillon Lagrange, in his Course of Chemistry, says, when gunpowder takes fire, there is a disengagement of azotic gas, which expands in an astonishing manner, when set at liberty; and we are even still ignorant of the extent of the dilatation occasioned by the heat arising from the combustion. A decomposition of water also takes place, and hydrogen gas is disengaged with elasticity; and by this decomposition of water there is

for[illegible]d even sulphurated hydrogen gas, [illegible] the smell emitted by burnt powder.

R[illegible]. It is ridiculous therefore to believe in the existence of *white gunpowder*; that is, a kind of powder which impels a ball without any noise; for there can be no force without sudden expansion, nor sudden expansion without a concussion of the air, which produces sound.

II. It was childish to give precepts, as in the preceding editions of this work, for making red, blue, green, &c, gunpowder; as they could answer no good purpose.

We shall now proceed to our principal object, the construction of the most common and curious pieces of fire-works.

ARTICLE II

Construction of the Cartridges of Rockets.

A rocket is a cartridge or case made of stiff paper, which being filled in part with gunpowder, saltpetre, and charcoal, rises of itself into the air, when fire is applied to it.

There are three kinds of rockets: small ones, the calibre of which does not exceed a pound bullet; that is to say, the orifice of them is equal to the diameter of a leaden bullet which weighs only a pound: for the calibres, or orifices of the moulds or models used in making rockets, are measured by the diameters of leaden bullets. Middle sized rockets, equal to the size of a ball of from one to three pounds. And large rockets, equal to a ball of from three to a hundred pounds.

To give the cartridges the same length and thickness, in order that any number of rockets may be prepared of the same size and force, they are put into a hollow cylinder of strong wood, called a mould. This mould is sometimes

of metal; but at any rate it ou[illegible]y hard wood.

This mould must not be con[illegible]piece of wood, called the former or roller, a[illegible]olled the thick paper employed to make the [illegible] the calibre of the mould be divided into 8 equal p[illegible], the diameter of the roller must be equal to 5 of these parts. See fig. 1 pl. 1, where A is the mould, and B the roller. The vacuity between the roller and the interior surface of the mould, that is to say $\frac{3}{8}$ of the calibre of the mould, will be exactly filled by the cartridge.

As rockets are made of different sizes, moulds of different lengths and diameters must be provided. The calibre of a cannon is nothing else than the diameter of its mouth; and we here apply the same term to the diameter of the aperture of the mould.

The size of the mould is measured by its calibre; but the length of the moulds for different rockets, does not always bear the same proportion to the calibre, the length being diminished as the calibre is increased. The length of the mould for small rockets ought to be six times the calibre, but for rockets of the mean and larger size, it will be sufficient if the length of the mould be five times or even four times the calibre of the moulds.

At the end of this section we shall give two tables, one of which contains the calibres of moulds below a pound bullet; and the other the calibres from a pound to a hundred pounds bullet.

For making the cartridges, large stiff paper is employed. This paper is wrapped round the roller B, fig. 1 pl. 1, and then cemented by means of common paste. The thickness of the paper when rolled up in this manner, ought to be about one eighth and a half of the calibre of the mould, according to the proportion given to the diameter of the roller. But if the diameter of the roller be made equal to

¼ th[illegible] the thickness of the cartridge must [illegible]f of that calibre.

W[illegible] formed, the roller B is drawn out, by t[illegible] until it is distant from the edge of the [illegible]ength of its diameter. A piece of cord is then m[illegible]e to pass twice round the cartridge at the extremity of the roller. And into the vacuity left in the cartridge, another roller is introduced, so as to leave some space between the two. One end of the pack-thread must be fastened to something fixed, and the other to a stick conveyed between the legs, and placed in such a manner, as to be behind the person who choaks the cartridge. The cord is then to be stretched by retiring backwards, and the cartridge must be pinched until there remains only an aperture capable of admitting the piercer DE. The cord employed for pinching it is then removed, and its place is supplied by a piece of pack-thread, which must be drawn very tight, passing it several times around the cartridge, after which it is secured by means of running knots made one above the other.

Besides the roller B, a rod C, pl. 1 fig. 1, is used, which being employed to load the cartridge, must be somewhat smaller than the roller, in order that it may be easily introduced into the cartridge. The rod C is pierced lengthwise, to a sufficient depth to receive the piercer DE, which must enter into the mould A, and unite with it exactly at its lower part. The piercer, which decreases in size, is introduced into the cartridge through the part where it has been choaked, and serves to preserve a cavity within it. Its length, besides the nipple or button, must be equal to about two-thirds that of the mould. Lastly, if the thickness of the base be a fourth part of the calibre of the mould, the point must be made equal to a sixth of the calibre.

It is evident that there must be at least three rods, such

as c, pierced in proportion to the [illegible] of the piercer, in order that the powder [illegible] in by means of a mallet, may be uniformly [illegible]hout the whole length of the rocket. It m[illegible] perceived also, that these rods ought to be [illegible] very hard wood, to resist the strokes of the mallet.

In loading rockets, it is more convenient not to employ a piercer. When loaded on a nipple, without a piercer, by means of one massy rod, they are pierced with a bit and a piercer fitted into the end of a bit-brace. Care however must be taken to make this hole su[illegible]ed to the proportion assigned for the diminution of the piercer. That is to say, the extremity of the hole at the choaked part of the cartridge, ought to be about a fourth of the calibre of the mould; and the extremity of the hole which is in the inside for about two thirds of the length of the rocket, ought to be a sixth of the calibre. This hole must pass directly through the middle of the rocket. In short, experience and ingenuity will suggest what is most convenient, and in what manner the method of loading rockets, which we shall here explain, may be varied.

After the cartridge is placed in the mould, pour gradually into it the prepared composition; taking care to pour only two spoonfuls at a time, and to ram it immediately down with the rod c, striking it in a perpendicular direction with a mallet of a proper size, and giving an equal number of strokes, for example, 3 or 4, each time that a new quantity of the composition is poured in.

When the cartridge is about half filled, separate with a bodkin the half of the folds of the paper which remains, and having turned them back on the composition, press them down with the rod and a few strokes of the mallet, in order to compress the paper on the composition.

Then pierce three or four holes in the folded paper, by means of a piercer, which must be made to penetrate to

the composition of the rocket, as seen at a, fig. 2 pl. 1. These holes serve to form a communication between the body of the rocket and the vacuity at the extremity of the cartridge, or that part which has been left empty.

In [illegible] this vacuity is filled with granulated powder, which serves to let them off: they are then covered with paper, and pinched in the same manner as at the other extremity. But in other rockets, the pot containing stars, serpents, and running rockets, is adapted to it, as will be shown hereafter.

It may be sufficient however to make, with a bit or piercer, only one hole, which must be neither too large nor too small, such as a fourth part of the diameter of the rocket, to set fire to the powder, taking care that this hole be as straight as possible, and exactly in the middle of the composition. A little of the composition of the rocket must be put into these holes, that the fire may not fail to be communicated to it.

It now remains to affix the rocket to its rod, which is done in the following manner. When the rocket has been constructed as above described, make fast to it a rod of light wood, such as fir or willow, broad and flat at the end next the rocket, and decreasing towards the other. It must be as straight and free from knots as possible, and ought to be dressed, if necessary, with a plane. Its length and weight must be proportioned to the rocket; that is to say, it ought to be six, seven, or eight feet long, so as to remain in equilibrium with it, when suspended on the finger, within an inch or an inch and a half of the neck. Before it is fired, place it with the neck downwards, and let it rest on two nails, in a direction perpendicular to the horizon. To make it ascend straighter and to a greater height, adapt to its summit A a pointed cap or top, as c, made of common paper, which will serve to facilitate its passage through the air.

These rockets in general are made in a more complex manner, several other things being added to them to render them more agreeable, such for example as a petard, which is a box of tin-plate, filled with fine gunpowder, placed on the summit. The petard is [illegible] on the composition, at the end where it has been filled; and the remaining paper of the cartridge is folded down over it to keep it firm. The petard produces its effect when the rocket is in the air and the composition is consumed.

Stars, golden rain, serpents, saucissons, and several other amusing things, the composition of which we shall explain hereafter, are also added to them. This is done by adjusting to the head of the rocket, an empty pot or cartridge, much larger than the rocket, in order that it may contain serpents, stars, and various other appendages, to render it more beautiful.

Rockets may be made to rise into the air without rods. For this purpose four wings must be attached to them in the form of a cross, and similar to those seen on arrows or darts, as represented at A plate 1 fig. 3. In length, these wings must be equal to two-thirds that of the rocket; their breadth towards the bottom should be half their length, and their thickness ought to be equal to that of a card. But this method of making rockets ascend is less certain, and more inconvenient than that where a rod is used; and for this reason it is rarely employed.

We shall now show the method of finding the diameters or calibre of rockets, according to their weight; but we must first observe that a pound rocket, is that just capable of admitting a leaden bullet of a pound weight, and so of the rest. The calibre for the different sizes may be found by the two following tables, one of which is calculated for rockets of a pound weight and below; and the other for those from a pound weight to 50 pounds.

I. Table of the calibre of moulds of a pound weight and

Ounces.	Lines.	Drams.	Lines.
16	19½	14	7¾
12	17	12	7
8	15	10	6⅓
7	14¾	8	6¼,
6	14¼	6	5⅔
5	13	4	4½
4	12⅓	2	3¾
3	11½		
2	9⅙		
1	6½		

The use of this table will be understood merely by inspection; for it is evident that a rocket of 12 ounces ought to be 17 lines in diameter; one of 8 ounces, 15 lines; one of 10 drams, 6⅓ lines; and so of the rest.

On the other hand, if the diameter of the rocket be given, it will be easy to find the weight of the ball corresponding to that calibre. For example, if the diameter be 13 lines, it will be immediately seen, by looking for that number in the column of lines, that it corresponds to a ball of 5 ounces.

II. Table of the calibre of moulds from 1 to 50 pounds ball.

Pounds.	Calibre.	Pound.	Calibre.	Pounds.	Calibre.	Pounds.	Calibre.
1	100	14	241	27	300	40	341
2	126	15	247	28	304	41	344
3	144	16	252	29	307	42	347
4	158	17	257	30	310	43	350
5	171	18	262	31	314	44	353
6	181	19	267	32	317	45	355
7	191	20	271	33	320	46	358
8	200	21	275	34	323	47	361
9	208	22	280	35	326	48	363
10	215	23	284	36	330	49	366
11	222	24	288	37	333	50	368
12	228	25	292	38	336		
13	235	26	296	39	339		

The use of the second table is as follows: If the weight of the ball be given, which we shall suppose to be 24 pounds, seek for that number in the column of pounds, and opposite to it, in the column of calibres, will be found the number 288. Then say, as 100 is to 19½, so is 288 to a fourth term, which will be the number of lines of the calibre required; or multiply the number found, that is 288, by 19½, and from the product 5616, cut off the two last figures: the required calibre therefore will be 56·16 lines, or 4 inches 8 lines.

On the other hand, the calibre being given in lines, the weight of the ball may be found with equal ease: if the calibre, for example, be 28 lines, say as 19½ is to 28, so is 100 to a fourth term, which will be 143·5, or nearly 144. But in the above table, opposite to 144, in the second column, will be found the number 3 in the first; which shows that a rocket, the diameter or calibre of which is 28 lines, is a rocket of a 3 pounds ball.

ARTICLE III.

Composition of the Powder for Rockets, and the manner of filling them.

The composition of the powder for rockets must be different, according to the different sizes; as that proper for small rockets, would be too strong for large ones. This is a fact respecting which almost all the makers of fire-works are agreed. The quantities of the ingredients, which experience has shown to be the best, are as follow:

For rockets capable of containing 1 or 2 ounces of composition.

To one pound of gunpowder, add two ounces of soft charcoal; or to one pound of gunpowder, a pound of the coarse powder used for cannon; or to 9 ounces of gunpowder, 2 ounces of charcoal; or to a pound of gunpowder, an ounce and a half of saltpetre, and as much charcoal.

For rockets of two or three ounces.

To 4 ounces of gunpowder, add an ounce of charcoal; or to 9 ounces of powder, add 2 ounces of saltpetre.

For a rocket of four ounces.

To 4 pounds of gunpowder, add a pound of saltpetre, and 4 ounces of charcoal: you may add also, if you choose, half an ounce of sulphur; or to one pound two ounces and a half of gunpowder, add 4 ounces of saltpetre, and 2 ounces of charcoal; or to a pound of powder, add 4 ounces of saltpetre, and one ounce of charcoal; or to 17 ounces of gunpowder, add 4 ounces of saltpetre, and the same quantity of charcoal; or to 3 ounces and a half of gunpowder, add 10 ounces of saltpetre, and 3 ounces and a half of charcoal. But the composition will be strongest, if to 10 ounces of gunpowder, you add 3 ounces and a half of saltpetre, and 3 ounces of charcoal.

For rockets of five or six ounces.

To 2 pounds 5 ounces of gunpowder, add half a pound of saltpetre, 2 ounces of sulphur, 6 ounces of charcoal, and 2 ounces of iron filings.

For rockets of seven or eight ounces.

To 17 ounces of gunpowder, add 4 ounces of saltpetre, and 3 ounces of sulphur.

For rockets of from eight ten ounces.

To 2 pounds and 5 ounces of gunpowder, add 8 ounces of saltpetre, 2 ounces of sulphur, 7 ounces of charcoal, and 8 ounces of iron filings.

For rockets of from ten to twelve ounces.

To 17 ounces of gunpowder, add 4 ounces of saltpetre, 3 ounces and a half of sulphur, and one ounce of charcoal.

For rockets of from fourteen to fifteen ounces.

To 2 pounds 4 ounces of gunpowder, add 9 ounces of saltpetre, 3 ounces of sulphur, 5 ounces of charcoal, and 8 ounces of iron filings.

For rockets of one pound.

To one pound of gunpowder, add an ounce of sulphur, and 3 ounces of charcoal.

For a rocket of two pounds.

To one pound four ounces of gunpowder, add 2 ounces of saltpetre, 1 ounce of sulphur, 3 ounces of charcoal, and 2 ounces of iron filings.

For a rocket of three pounds.

To 30 ounces of saltpetre, add 7 ounces and a half of sulphur, and 11 ounces of charcoal.

For rockets of four, five, six, or seven pounds.

To 31 pounds of saltpetre, add 4 pounds and a half of sulphur, and 10 pounds of charcoal.

For rockets of eight, nine, or ten pounds.

To 8 pounds of saltpetre, add 20 ounces of sulphur, and 44 ounces of charcoal.

We shall here observe, that these ingredients must be each pounded separately, and sifted; they are then to be weighed and mixed together for the purpose of loading the cartridges, which ought to be kept ready in the moulds. The cartridges must be made of strong paper, doubled, and cemented by means of strong paste, made of fine flour and very pure water.

Of Matches.

Before we proceed farther, it will be proper to describe the composition of the matches necessary for letting them off. Take linen, hemp or cotton thread, and double it eight or ten times, if intended for large rockets; or only four or five times, if to be employed for stars. When the match has been thus made as large as necessary, dip it in pure water, and press it between your hands, to free it from the moisture. Mix some gunpowder with a little water, to reduce it to a sort of paste, and immerse the match in it; turning and twisting it, till it has imbibed a sufficient quantity of the powder; then sprinkle over it a little dry powder, or strew some pulverised dry powder upon a smooth board, and roll the match over it. By these means you will have an excellent match: which if dried in the sun, or on a rope in the shade, will be fit for use.

ARTICLE IV.

On the cause which makes rockets ascend into the air.

As this cause is nearly the same as that which produces

recoil in fire-arms, it is necessary we should first explain the latter.

When the powder is suddenly inflamed in the chamber, or at the bottom of the barrel, it necessarily exercises an action two ways at the same time; that is to say, against the breech of the piece, and against the bullet or wadding, which is placed above it. Besides this, it acts also against the sides of the chamber which it occupies; and as they oppose a resistance almost insurmountable, the whole effort of the elastic fluid, produced by the inflammation, is exerted in the two directions above mentioned. But the resistance opposed by the bullet, being much less than that opposed by the mass of the barrel or cannon, the bullet is forced out with great velocity. It is impossible, however, that the body of the piece itself should not experience a movement backwards; for if a spring is suddenly let loose, between two moveable obstacles, it will impel them both, and communicate to them velocities in the inverse ratio of their masses: the piece therefore must acquire a velocity backwards nearly in the inverse ratio of its mass to that of the bullet. We make use of the term nearly, because there are various circumstances which give to this ratio certain modifications; but it is always true that the body of the piece is driven backwards, and that if it weighs with its carriage, a thousand times more than the bullet, it acquires a velocity, which is a thousand times less, and which is soon annihilated by the friction of the wheels against the ground, &c.

The cause of the ascent of a rocket is nearly the same. At the moment when the powder begins to inflame, its expansion produces a torrent of elastic fluid, which acts in every direction; that is, against the air which opposes its escape from the cartridge, and against the upper part of the rocket; but the resistance of the air is more considerable than the weight of the rocket, on account of the extreme rapidity with which the elastic fluid issues through

the neck of the rocket to throw itself downwards, and therefore the rocket ascends by the excess of the one of these forces over the other.

This however would not be the case, unless the rocket were pierced to a certain depth. A sufficient quantity of elastic fluid would not be produced; for the composition would inflame only in circular coats of a diameter equal to that of the rocket; and experience shows that this is not sufficient. Recourse then is had to the very ingenious idea of piercing the rocket with a conical hole, which makes the composition burn in conical strata, which have much greater surface, and therefore produce a much greater quantity of inflamed matter and fluid. This expedient was certainly not the work of a moment.

ARTICLE V.

Brilliant fire and Chinese fire.

As iron-filings, when thrown into the fire, inflame and emit a strong light, this property, discovered no doubt by chance, gave rise to the idea of rendering the fire of rockets much more brilliant, than when gunpowder, or the substances of which it is composed, are alone employed. Nothing is necessary but to take iron-filings, very clean and free from rust, and to mix them with the composition of the rocket. It must however be observed, that rockets of this kind will not keep longer than a week; because the moisture contracted by the saltpetre rusts the iron-filings, and destroys the effect they are intended to produce.

But the Chinese have long been in possession of a method of rendering this fire much more brilliant and variegated in its colours; and we are indebted to father d'Incarville, a jesuit, for having made it known. It consists in the use of a very simple ingredient; namely, cast iron reduced to a powder more or less fine: the Chinese give it a name, which is equivalent to that of *iron sand*.

To prepare this sand, take an old iron pot, and having broken it to pieces on an anvil, pulverise the fragments till the grains are not larger than radish seed: then sift them through six graduated sieves, to separate the different sizes, and preserve these six different kinds in a very dry place, to secure them from rust, which would render this sand absolutely unfit for the proposed end. We must here remark, that the grains which pass through the closest sieve, are called sand of the first order; those which pass through the next in size, sand of the second order; and so on.

This sand, when it inflames, emits a light exceedingly vivid. It is very surprising to see fragments of this matter no bigger than a poppy seed, form all of a sudden luminous flowers of stars, 12 and 15 lines in diameter. These flowers are also of different forms, according to that of the inflamed grain, and even of different colours, according to the matters with which the grains are mixed. But rockets into which this composition enters, cannot be long preserved, as those which contain the finest sand will not keep longer than eight days, and those which contain the coarsest, fifteen. The following tables exhibit the proportions of the different ingredients for rockets of from 12 to 36 pounds.

For red Chinese fire.

Calibres. Pounds.	Saltpetre. Pounds.	Sulphur. Ounces.	Charcoal. Ounces.	Sand of the 1st order. oz.	dr
12 to 15	1	3	4	7	
18 to 21	1	3	5	7	8
24 to 36	1	4	6	8	

For white Chinese fire.

Calibres. Pounds.	Saltpetre. Pounds.	Bruised Gunpowder. Ounces	Charcoal. oz.	dr.	Sand of the 3d order. oz.	dr.
12 to 15	1	12	7	8	11	
18 to 21	1	11	8		11	8
24 to 36	1	11	8	8	12	

When these materials have been weighed, the saltpetre and charcoal must be three times sifted through a hair sieve, in order that they may be well mixed; the iron sand is then to be moistened with good brandy, to make the sulphur adhere, and they must be thoroughly incorporated. The sand thus sulphured must be spread over the mixture of saltpetre and charcoal, and the whole must be mixed together by spreading it over a table with a spatula.

ARTICLE VI.

Of the Furniture of Rockets.

The upper part of rockets is generally furnished with some composition, which taking fire when it has reached to its greatest height, emits a considerable blaze, or produces a loud report, and very often both these together. Of this kind are saucissons, marroons, stars, showers of fire, &c.

To make room for this artifice, the rocket is crowned with a part of a greater diameter, called the pot, as seen fig. 5 pl. 1. The method of making this pot, and connecting it with the body of the rocket, is as follows.

The mould for forming the pot, though of one piece, must consist of two cylindric parts of different diameters. That on which the pot is rolled up, must be three diameters of the rocket in length, and its diameter must be three fourths that of the rocket; the length of the other ought

to be equal to two of these diameters, and its diameter to ⅞ that of the rocket.

Having rolled the thick paper intended for making the pot, and which ought to be of the same kind as that used for the rocket, twice round the cylinder, a portion of it must be pinched in that part of the cylinder which has the least diameter; this part must be pared in such a manner, as to leave only what is necessary for making the pot fast to the top of the rocket, and the ligature must be covered with paper.

To charge such a pot, attached to a rocket; having pierced three or four holes in the double paper which covers the vacuity of the rocket, pour over it a small quantity of the composition with which the rocket is filled, and by shaking it, make a part enter these holes; then arrange in the pot the composition with which it is to be charged, taking care not to introduce into it a quantity heavier than the body of the rocket.

The whole must then be secured by means of a few small balls of paper, to keep every thing in its place, and the pot must be covered with paper cemented to its edges: if a pointed summit or cap be then added to it, the rocket will be ready for use.

We shall now give an account of the different artifices with which such rockets are loaded.

§ I. *Of Serpents.*

Serpents are small flying rockets, without rods, which instead of rising in a perpendicular direction, mount obliquely, and descend in a zig-zag form without ascending to a great height. The composition of them is nearly the same as that of rockets; and therefore nothing more is necessary than to determine the proportion and construction of the cartridge, which is as follows.

The length AC pl. 1, fig. 7, of the cartridge may be about 4 inches; it must be rolled round a stick somewhat

larger than the barrel of a goose quill, and after being choaked at one of its ends, fill it with the composition a little beyond its middle, as to B; and then pinch it so as to leave a small aperture. The remainder BC, must be filled with grained powder, which will occasion a report when it bursts. Lastly, choak the cartridge entirely towards the extremity C; and at the other extremity A place a train of moist powder, to which if fire be applied, it will be communicated to the composition in the part AB, and cause the whole to rise in the air. The serpent, as it falls, will then make several small turns in a zig-zag direction, till the fire is communicated to the grained powder in the part BC; on which the serpent will burst with a loud report before it falls to the ground.

If the serpent be not choaked towards the middle, instead of moving in a zig-zag direction, it will ascend and descend with an undulating motion, and then burst as before.

The cartridges of serpents are generally made of playing cards. These cards are rolled round a rod of iron or hard wood, a little larger, as already said, than the barrel of a goose quill. To confine the card, a piece of strong paper is cemented over it.

The length of the mould must be proportioned to that of the cards employed, and the piercer of the nipple must be three or four lines in length. These serpents are loaded with bruised powder, mixed only with a very small quantity of charcoal. To introduce the composition into the cartridge, a quill, cut into the form of a spoon, may be employed: it must be rammed down by means of a small rod, to which a few strokes are given with a small mallet.

When the serpent is half loaded, instead of pinching it in that part, you may introduce into it a vetch seed, and place granulated powder above it to fill up the remainder. Above this powder place a small pellet of chewed paper, and then choak the other end of the cartridge. If you are

desirous of making larger serpents, cement two playing cards together; and, that they may be managed with more ease, moisten them a little with water. The match consists of a paste made of bruised powder, and a small quantity of water.

§ II. *Marroons.*

Marroons are small cubical boxes, filled with a composition proper for making them burst, and may be constructed with great ease.

Cut a piece of pasteboard, according to the method taught in geometry to form the cube, as seen fig. 8 pl. 1; join these squares at the edges, leaving only one to be cemented, and fill the cavity of the cube with grained powder; then cement strong paper in various directions over this body, and wrap round it two rows of pack-thread, dipped in strong glue: then make a hole in one of the corners, and introduce into it a match.

If you are desirous to have luminous marroons, that is to say marroons which, before they burst in the air, emit a brilliant light, cover them with a paste the composition of which will be given hereafter for stars: and roll them in pulverised gunpowder, to serve as a match or communication.

§ III. *Saucissons.*

Marroons and saucissons differ from each other only in their form. The cartridges of the latter are round, and must be only four times their exterior diameter in length. They are chooked at one end in the same manner as a rocket; and a pellet of paper is driven into the aperture which has been left, in order to fill it up. They are then charged with grained powder, above which is placed a ball of paper gently pressed down, to prevent the powder from being bruised; the second end of the saucisson being afterwards chooked, the edges are pared on both sides, and

the whole is covered with several turns of pack-thread, dipped in strong glue, and then left to dry.

When you are desirous of charging them, pierce a hole in one of the ends; and apply a match, in the same manner as to marroons.

§ IV. *Stars.*

Stars are small globes of a composition which emits a brilliant light, that may be compared to the light of the stars in the heavens. These balls are not larger than a nutmeg or musket bullet, and when put into the rockets must be wrapped up in tow, prepared for that purpose. The composition of these stars is as follows.

To a pound of fine gunpowder well pulverised, add four pounds of saltpetre, and two pounds of sulphur. When these ingredients are thoroughly incorporated, take about the size of a nutmeg of this mixture, and having wrapt it up in a piece of linen-rag, or of paper, form it into a ball; then tie it closely round with a pack-thread, and pierce a hole through the middle of it, sufficiently large to receive a piece of prepared tow, which will serve as a match. This star, when lighted, will exhibit a most beautiful appearance; because the fire as it issues from the two ends of the hole in the middle, will extend to a great distance, and make it appear much larger.

If you are desirous to employ a moist composition in the form of a paste, instead of a dry one, it will not be necessary to wrap up the star in any thing but prepared tow; because, when made of such paste, it can retain its spherical figure. There will be no need also of piercing a hole in it, to receive the match; because, when newly made, and consequently moist, it may be rolled in pulverised gunpowder, which will adhere to it. This powder, when kindled, will serve as a match, and inflame the composition of the star, which in falling will form itself into tears.

Another method of making rockets with stars.

Mix three ounces of saltpetre, with one ounce of sulphur, and two drams of pulverised gunpowder; or mix four ounces of sulphur, with the same quantity of saltpetre, and eight ounces of pulverised gunpowder. When these materials have been well sifted, besprinkle them with brandy, in which a little gum has been dissolved, and then make up the star in the following manner.

Take a rocket mould, eight or nine lines in diameter, and introduce into it a nipple, the piercer of which is of a uniform size throughout, and equal in length to the height of the mould. Put into this mould a cartridge, and by means of a pierced rod load it with one of the preceding compositions; when loaded, take it from the mould, without removing the nipple, the piercer of which passes through the composition, and then cut the cartridge quite round into pieces of the thickness of three or four lines. The cartridge being thus cut, draw out the piercer gently, and the pieces, which resemble the men employed for playing at drafts, pierced through the middle, will be stars, which must be filed on a match thread, which, if you choose, may be covered with tow.

To give more brilliancy to stars of this kind, a cartridge thicker than the above dimensions, and thinner than that of a flying-rocket of the same size, may be employed; but, before it is cut into pieces, five or six holes must be pierced in the circumference of each piece to be cut. When the cartridge is cut, and the pieces have been filed, cement over the composition small bits of card, each having a hole in the middle, so that these holes may correspond to the place where the composition is pierced.

Remarks.—1. There are several other methods of making stars, which it would be too tedious to describe. We shall therefore only show how to make *étoiles à pet*,

or stars which give a report as loud as that of a pistol or musket.

Make small saucissons, as taught in the third section; only, it will not be necessary to cover them with pack-thread: it will be sufficient if they are pierced at one end, in order that you may tie to it a star constructed according to the first method, the composition of which is dry; for if the composition be in the form of a paste, there will be no need to tie it. Nothing will be necessary in that case; but to leave a little more of the paper hollow at the end of the saucisson which has been pierced, for the purpose of introducing the composition; and to place in the vacuity, towards the neck of the saucisson, some grained powder, which will communicate fire to the saucisson when the composition is consumed.

II. As there are some stars which in the end become petards, others may be made, which shall conclude with becoming serpents. But this may be so easily conceived and carried into execution, that it would be losing time to enlarge further on the subject. We shall only observe, that these stars are not in use, because it is difficult for a rocket to carry them to a considerable height in the air: they diminish the effect of the rocket or saucisson, and much time is required to make them.

§ V. *Shower of Fire.*

To form a shower of fire, mould small paper cartridges on an iron rod, two lines and a half in diameter, and make them two inches and a half in length. They must not be choaked, as it will be sufficient to twist the end of the cartridge, and having put the rod into it to beat it, in order to make it assume its form. When the cartridges are filled, which is done by immersing them in the composition, fold down the other end, and then apply a match. This furniture will fill the air with an undulating fire. The

following are some compositions proper for stars of this kind.

Chinese fire. Pulverised gunpowder one pound, sulphur 2 ounces, iron sand of the first order 5 ounces.

Ancient fire. Pulverised gunpowder one pound, charcoal 2 ounces.

Brilliant fire. Pulverised gunpowder one pound, iron filings 4 ounces.

The Chinese fire is certainly the most beautiful.

§ VI. *Of Spark*

Sparks differ from stars only in their size and duration; for they are made smaller than stars; and are consumed sooner. They are made in the following manner.

Having put into an earthen vessel an ounce of pulverised gunpowder, two ounces of pulverised saltpetre, one ounce of liquid saltpetre, and four ounces of camphor reduced to a sort of farina, pour over this mixture some gum-water, or brandy in which gum-adraganth or gum-arabic has been dissolved, till the composition acquire the consistence of thick soup. Then take some lint which has been boiled in brandy, or in vinegar, or even in saltpetre, and then dried and unravelled, and throw into the mixture such a quantity of it as is sufficient to absorb it entirely, taking care to stir it well.

Form this matter into small balls or globes of the size of a pea; and having dried them in the sun or the shade, besprinkle them with pulverised gunpowder, in order that they may more readily catch fire.

Another Method of making Sparks.

Take the saw-dust of any kind of wood that burns readily, such as fir, elder-tree, poplar, laurel, &c, and boil it in water in which saltpetre has been dissolved. When the water has boiled some time, take it from the

fire, and pour it off in such a manner that the saw-dust may remain in the vessel. Then place the saw-dust on a table, and while moist besprinkle it with sulphur, sifted through a very fine sieve: you may add to it also a little bruised gunpowder. Lastly, when the saw-dust has been well mixed, leave it to dry, and make it into sparks as above described.

§ VII. *Of Golden Rain.*

There are some flying-rockets which, as they fall, make small undulations in the air like hair half frizzled. These are called *fusées chevelues*, bearded rockets; they finish with a kind of shower of fire, which is called golden rain. The method of constructing them is as follows.

Fill the barrels of some goose quills with the composition of flying-rockets, and place upon the mouth of each a little moist gunpowder, both to keep in the composition, and to serve as a match. If a flying-rocket be then loaded with these quills, they will produce, at the end, a very agreeable shower of fire, which on account of its beauty has been called golden rain.

ARTICLE VII.

Of some rockets different in their effect from common rockets.

Several very amusing and ingenious works are made by means of simple rockets, of which it is necessary that we should here give the reader some idea.

§ I. *Of Courantins, or Rockets which fly along a rope.*

A common rocket, which however ought not to be very large, may be made to run along an extended rope. For this purpose, affix to the rocket an empty cartridge, and introduce into it the rope which is to carry it; placing the head of the rocket towards that side to which you intend it to move: if you then set fire to the rocket, adjusted in

this manner, it will run along the rope without stopping, till the matter it contains is entirely exhausted.

If you are desirous that the rocket should move in a retrograde direction; first fill one half of it with the composition, and cover it with a small round piece of wood, to serve as a partition between it and that put into the other half; then make a hole below this partition, so as to correspond with a small canal filled with bruised powder, and terminating at the other end of the rocket: by these means the fire, when it ceases in the first half of the rocket, will be communicated through the hole into the small canal, which will convey it to the other end; and this end being then kindled, the rocket will move backwards, and return to the place from which it set out.

Two rockets of equal size, bound together by means of a piece of strong pack-thread, and disposed in such a manner that the head of the one shall be opposite to the neck of the other, that when the fire has consumed the composition in the one, it may be communicated to that in the other, and oblige both of them to move in a retrograde direction, may also be adjusted to the rope by means of a piece of hollow reed. But to prevent the fire of the former from being communicated to the second too soon, they ought to be covered with oil-cloth, or to be wrapped up in paper.

Remark.—Rockets of this kind are generally employed for setting fire to various other pieces when large fire-works are exhibited; and to render them more agreeable, they are made in the form of different animals, such as serpents, dragons, &c; on which account they are called *flying dragons*. These dragons are very amusing, especially when filled with various compositions, such as golden rain, long hair, &c. They might be made to discharge serpents from their mouths, which would produce a very pleasing effect, and give them a greater resemblance to a dragon.

§ II. *Rockets which fly along a rope, and turn round at the same time.*

Nothing is easier than to give to a rocket of this kind a rotary motion around the rope along which it advances; it will be sufficient for this purpose, to tie to it another rocket, placed in a transversal direction. But the aperture of the latter, instead of being at the bottom, ought to be in the side, near one of the ends. If both rockets be fired at the same time, the latter will make the other revolve around the rope, while it advances along it.

§ III. *Of rockets which burn in the water.*

Though fire and water are two things of a very opposite nature, the rockets above described, when set on fire, will burn and produce their effect even in the water; but as they are then below the water, the pleasure of seeing them is lost; for this reason, when it is required to cause rockets to burn as they float on the water, it will be necessary to make some change in the proportions of the moulds, and the materials of which they are composed.

In regard to the mould, it may be eight or nine inches in length, and an inch in diameter: the former, on which the cartridge is rolled up, may be nine lines in thickness, and the rod for loading the cartridge must as usual be somewhat less. For loading the cartridge, there is no need of a piercer with a nipple.

The composition may be made in two ways; for if it be required that the rocket, while burning on the water, should appear as bright as a candle, it must be composed of three materials mixed together, viz, 3 ounces of pulverised and sifted gunpowder, one pound of saltpetre, and 8 ounces of sulphur. But if you are desirous that it should appear on the water with a beautiful tail, the composition must consist of 8 ounces of gunpowder pulverised and sifted, one pound of saltpetre, 8 ounces of pounded and sifted sulphur, and 2 ounces of charcoal.

When the composition has been prepared according to these proportions, and the rocket has been filled in the manner above described, apply a saucisson to the end of it; and having covered the rocket with wax, black pitch, rosin, or any other substance capable of preventing the paper from being spoilt in the water, attach to it a small rod of white willow, about two feet in length, that the rocket may conveniently float.

If it be required that these rockets should plunge down, and again rise up; a certain quantity of pulverised gunpowder, without any mixture, must be introduced into them, at certain distances, such for example, as two, three, or four lines, according to the size of the cartridge.

Remarks.—I. Small rockets of this kind may be made, without changing the mould or composition, in several different ways, which, for the sake of brevity, we are obliged to omit. Such of our readers as are desirous of further information on this subject, may consult those authors who have written expressly on pyrotechny, some of whom we shall mention at the end of the 12th section.

II. It is possible also to make a rocket which, after it has burnt some time on the water, shall throw out sparks and stars; and these after they catch fire shall ascend into the air. This may be done by dividing the rocket into two parts, by means of a round piece of wood, having a hole in the middle. The upper part must be filled with the usual composition of rockets, and the lower with stars, which must be mixed with grained and pulverised gunpowder, &c.

III. A rocket which takes fire in the water, and, after burning there half the time of its duration, mounts into the air with great velocity, may be constructed in the following manner.

Take a flying rocket, furnished with its rod, and by means of a little glue attach it to a water rocket, but only at the middle A, pl. 1 fig. 9, in such a manner, that the

latter shall have its neck uppermost, and the other its neck downward. Adjust to their extremity B a small tube, to communicate the fire from the one to the other, and cover both with a coating of pitch, wax, &c, that they may not be damaged by the water.

Then attach to the flying rocket, after it has been thus cemented to the aquatic one, a rod of the kind described in the 2d article, as seen in the figure at D; and from F suspend a piece of pack-thread, to support a musket bullet E, made fast to the rod by means of a needle or bit of iron wire. When these arrangements have been made, set fire to the part C after the rocket is in the water; and when the composition is consumed to B, the fire will be communicated through the small tube to the other rocket: the latter will then rise and leave the other, which will not be able to follow it on account of the weight adhering to it.

§ IV. *By means of rockets, to represent several figures in the air.*

If several small rockets be placed upon a large one, their rods being fixed around the large cartridge, which is usually attached to the head of the rocket, to contain what it is destined to carry up into the air; and if these small rockets be set on fire while the large one is ascending, they will represent, in a very agreeable manner, a tree, the trunk of which will be the large rocket, and the branches the small ones.

If these small rockets take fire when the large one is half burned in the air, they will represent a comet; and when the large one is entirely inverted, so that its head begins to point downwards, in order to fall, they will represent a kind of fiery fountain.

If the barrels of several quills, filled with the composition of flying rockets, as above described, be placed on a large rocket; when these quills catch fire, they will represent, to an eye placed below them, a beautiful shower

of fire, or of half frizzled hair if the eye be placed on one side.

If several serpents be attached to the rocket with a piece of pack-thread, by the ends that do not catch fire; and if the pack-thread be suffered to hang down two or three inches, between every two, this arrangement will produce a variety of agreeable and amusing figures.

§ V. *A rocket which ascends in the form of a screw.*

A straight rod, as experience shows, makes a rocket ascend perpendicularly, and in a straight line: it may be compared to the rudder of a ship, or the tail of a bird, the effect of which is to make the vessel or bird turn towards that side to which it is inclined: if a bent rod therefore be attached to a rocket, its first effect will be to make the rocket incline towards that side to which it is bent; but its centre of gravity bringing it afterwards into a vertical situation, the result of these two opposite efforts will be that the rocket will ascend in a zig-zag or spiral form. In this case indeed, as it displaces a greater volume of air, and describes a longer line, it will not ascend so high, as if it had been impelled in a straight direction; but, on account of the singularity of this motion, it will produce an agreeable effect.

ARTICLE VIII.

Of Globes and Fire Balls.

We have hitherto spoken only of rockets, and the different kinds of works which can be constructed by their means. But there are a great many other fireworks, the most remarkable of which we shall here describe. Among these are globes and fire balls; some of which are intended to produce their effect in water; others by rolling or leaping on the ground: and some, which are called *bombs*, do the same in the air.

§ I. *Globes which burn on the water.*

These globes, or fire balls, are made in three different forms; spherical, spheroidal, or cylindrical; but we shall here confine ourselves to the spherical.

To make a spherical fire ball, construct a hollow wooden globe of any size at pleasure, and very round both within and without, so that its thickness AC or BD, pl. 1, fig. 10, may be equal to about the ninth part of the diameter AB. Insert in the upper part of it a right concave cylinder EFGH, the breadth of which EF may be equal to the fifth part of the diameter AB; and having an aperture, LM or ON, equal to the thickness AC or BD, that is, to the ninth part of the diameter AB. It is through this aperture that fire is communicated to the globe, when it has been filled with the proper composition, through the lower aperture IK. A petard of metal, loaded with good grained powder, is to be introduced also through the lower aperture, and to be placed horizontally, as seen in the figure.

When this is done, close up the aperture IK, which is nearly equal to the thickness FF or GH, of the cylinder EFGH, by means of a wooden tompion dipped in warm pitch; and melt over it such a quantity of lead that its weight may cause the globe to sink in water, till nothing remain above it but the part GH; which will be the case if the weight of the lead, with that of the globe and the composition be equal the weight of an equal volume of water. If the globe be then placed in the water, the lead by its gravity will make the aperture IK tend directly downwards, and keep in a perpendicular direction the cylinder EFGH, to which fire must have been previously applied.

To ascertain whether the lead, which has been added to the globe, renders its weight equal to that of an equal volume of water; rub the globe over with pitch or grease, and make a trial, by placing it in the water.

The composition with which the globe must be loaded,

is as follows: to a pound of grained powder, add 32 pounds of saltpetre reduced to fine flour, 8 pounds of sulphur, 1 ounce scrapings of ivory, and 8 pounds of saw-dust previously boiled in a solution of saltpetre, and dried in the shade or in the sun.

Or, to 2 pounds of bruised gunpowder, add 12 pounds of saltpetre, 6 pounds of sulphur, 4 pounds of iron filings, and 1 pound of Greek pitch.

It is not necessary that this composition should be beaten so fine as that intended for rockets: it requires neither to be pulverised nor sifted; it is sufficie if it be well mixed and incorporated. But to prevent it from becoming too dry, it will be proper to besprinkle it with a little oil, or any other liquid susceptible of inflammation.

§ II. *Of Globes which leap or roll on the ground.*

I. Having constructed a wooden globe A, pl. 1, fig. 11, with a cylinder C, similar to that above described, and having loaded it with the same composition, introduce into it four petards, or even more, loaded with good grained gunpowder to their orifices, as AB; which must be well stopped with paper or tow. If a globe, prepared in this manner, be fired by means of a match at C, it will leap about, as it burns, on a smooth horizontal plane, according as the petards are set on fire.

Instead of placing these petards in the inside, they may be affixed to the exterior surface of the globe; which they will make to roll and leap as they catch fire. They may be applied in any manner to the surface of the globe, as seen in the figure.

II. A similar globe may be made to roll about on a horizontal plane, with a very rapid motion. Construct two equal hemispheres of pasteboard, and adjust in one of them, as AB, fig. 12, three common rockets C, D, E, filled and pierced like flying rockets which have no petard; these rockets must not exceed the interior breadth of the

hemisphere, and ought to be arranged in such a manner, that the head of the one shall correspond to the tail of the other.

The rockets being thus arranged, join the two hemispheres, by cementing them together with strong paper; in such a manner, that they shall not separate, while the globe is moving and turning, at the same time that the rockets produce their effect. To set fire to the first, make a hole in the globe opposite to the tail of it, and introduce into it a match. This match will communicate fire to the first rocket; which, when consumed, will set fire to the second by means of another match, and so on to the rest; so that the globe, if placed on a smooth horizontal plane, will be kept in continual motion.

It is here to be observed, that a few more holes must be made in the globe, otherwise it will burst.

The two hemispheres of pasteboard may be prepared in the following manner: construct a very round globe of solid wood, and cover it with melted wax; then cement over it several bands of coarse paper, about two inches in breadth, giving it several coats of this kind, to the thickness of about two lines. Or, what will be still easier and better, having dissolved, in glue water, some of the pulp employed by the paper-makers, cover with it the surface of the globe; then dry it gradually at a slow fire, and cut it through in the middle; by which means you will have two strong hemispheres. The wooden globe may be easily separated from the pasteboard by means of heat; for if the whole be applied to a strong fire the wax will dissolve, so that the globe may be drawn out. Instead of melted wax, soap may be employed.

§ III. *Of Aerial Globes, called Bombs.*

These globes are called aerials, because they are thrown into the air from a mortar, which is a short thick piece of artillery of a large calibre.

Though these globes are of wood, and have a suitable thickness, namely, equal to the twelfth part of their diameters, if too much powder be put into the mortar, they will not be able to resist its force; the charge of powder therefore must be proportioned to the globe to be ejected. The usual quantity is an ounce of powder for a globe of four pounds weight; two ounces for one of eight, and so on.

As the chamber of the mortar may be too large to contain the exact quantity of powder sufficient for the fire ball, which ought to be placed immediately above the powder, in order that it may be expelled and set on fire at the same time, another mortar may be constructed of wood, or of pasteboard with a wooden bottom, as AB, fig. 13, pl. 1: it ought to be put into a large iron mortar, and to be loaded with a quantity of powder proportioned to the weight of the globe.

This small mortar must be of light wood, or of paper pasted together, and rolled up in the form of a cylinder, or truncated cone, the bottom excepted; which, as already said, must be of wood. The chamber for the powder AC must be pierced obliquely, with a small gimblet, as seen at BC; so that the aperture B, corresponding to the aperture of the metal mortar, the fire applied to the latter may be communicated to the powder which is at the bottom of the chamber AC, immediately below the globe. By these means the globe will catch fire, and make an agreeable noise as it rises into the air; but it would not succeed so well, if any vacuity were left between the powder and the globe.

A profile or perpendicular section of such a globe is represented by the right-angled parallelogram ABCD, fig. 13 n°. 2; the breadth of which AB is nearly equal to the height AD. The thickness of the wood, towards the two sides, L, M, is equal, as above said, to the twelfth part of the diameter of the globe; and the thickness, EF, of the

cover, is double the preceding, or equal to a sixth part of the diameter. The height GK or HI of the chamber, GHIK, where the match is applied, and which is terminated by the semicircle LGHM, is equal to the fourth part of the breadth AB; and its breadth GH is equal to the sixth part of AB.

We must here observe that it is dangerous to put wooden covers, such as EF, on aerial balloons or globes; for these covers may be so heavy, as to wound those on whom they happen to fall. It will be sufficient to place turf or hay above the globe, in order that the powder may experience some resistance.

The globe must be filled with several pieces of cane or common reed, equal in length to the interior height of the globe, and charged with a slow composition, made of three ounces of pounded gunpowder, an ounce of sulphur moistened with a small quantity of petroleum oil, and two ounces of charcoal; and in order that these reeds or canes may catch fire sooner, and with more facility, they must be charged at the lower ends, which rest on the bottom of the globe, with pulverised gunpowder moistened in the same manner with petroleum oil, or well besprinkled with brandy, and then dried.

The bottom of the globe ought to be covered with a little gunpowder half pulverised and half grained; which, when set on fire, by means of a match applied to the end of the chamber GH, will set fire to the lower part of the reed. But care must have been taken to fill the chamber with a composition similar to that in the reeds, or with another slow composition, made of 8 ounces of gunpowder, 4 ounces of saltpetre, 2 ounces of sulphur, and one ounce of charcoal: the whole must be well pounded and mixed.

Instead of reeds, the globe may be charged with running rockets, or paper petards, and a quantity of fiery stars or sparks mixed with pulverised gunpowder, placed without any order above these petards, which must be choaked at

unequal heights, that they may perform their effect at different times.

These globes may be constructed in various other ways, which it would be tedious here to enumerate. We shall only observe that when loaded, they must be well covered at the top; they must be wrapped up in a piece of cloth dipped in glue, and a piece of woollen cloth must be tied round them, so as to cover the hole which contains the match.

ARTICLE IX.

Jets of Fire.

Jets of fire are a kind of fixed rockets, the effect of which is to throw up into the air jets of fire, similar to jets of water. They serve also to represent cascades; for, if a series of such rockets be placed horizontally on the same line, it may be easily seen that the fire they emit, will resemble a sheet of water. When arranged in a circular form, like the radii of a circle, they form what is called a *fixed sun*.

To form jets of this kind, the cartridge for brilliant fires must, in thickness, be equal to a fourth part of the diameter, and for Chinese fire, only to a sixth part.

The cartridge is loaded on a nipple, having a point equal in length to the same diameter, and in thickness to a fourth part of it; but as it generally happens that the mouth of the jet becomes larger than is necessary for the effect of the fire, you must begin to charge the cartridge, as the Chinese do, by filling it to a height equal to a fourth part of the diameter with clay, which must be rammed down as if it were gunpowder. By these means the jet will ascend much higher. When the charge is completed with the composition you have made choice of, the cartridge must be closed with a tompion of wood, above which it must be choaked.

The train or match must be of the same composition as that employed for loading; otherwise the dilatation of the air contained in the hole made by the piercer, would cause the jet to burst.

Clayed rockets may be pierced with two holes near the neck, in order to have three jets in the same plane.

If a kind of top, pierced with a number of holes, be added to them, they will imitate a bubbling fountain.

Jets intended for representing sheets of fire ought not to be choaked. They must be placed in a horizontal position, or inclined a little downwards.

It appears to us that they might be choaked so as to form a kind of slit, and be pierced in the same manner; which would contribute to extend the sheet of fire still farther. A kind of long narrow mouths might even be provided for this particular purpose.

Principal Compositions for Jets of Fire.

1st. *For jets of 5 lines or less, of interior diameter.*

Chinese fire. Saltpetre 1 pound, pulverised gunpowder 1 pound, sulphur 8 ounces, charcoal 2 ounces.

White fire. Saltpetre 1 pound, pulverised gunpowder 8 ounces, sulphur 3 ounces, charcoal 2 ounces, iron sand of the first order 8 ounces.

2d. *For Jets of from 10 to 12 lines in diameter.*

Brilliant fire. Pulverised gunpowder 1 pound, iron-filings of a mean size, 5 ounces.

White fire. Saltpetere 1 pound, pulverised gunpowder 1 pound, sulphur 8 ounces, charcoal 2 ounces.

Chinese fire. Saltpetre 1 pound 4 ounces, sulphur 5 ounces, charcoal 5 ounces, sand of the third order 12 ounces.

3d. *For Jets of 15 or 18 lines in diameter.*

Chinese fire. Saltpetre 1 pound 4 ounces, sulphur 7

ounces, charcoal 5 ounces, of the six different kinds of sand mixed 12 ounces.

Father d'Incarville, in his memoirs on this subject, gives various other proportions for the composition of these jets; but we must confine ourselves to what has been here said, and refer the reader to the author's memoirs, which will be found in the *Manuel de l'Artificer.*

The saltpetre, pulverised gunpowder, and charcoal, are three times sifted through a hair sieve. The iron sand is besprinkled with sulphur, after being moistened with a little brandy, that the sulphur may adhere to it; and they are then mixed together: the sulphured sand is then spread over the first mixture, and the whole is mixed with a ladle only; for if a sieve were employed, it would separate the sand from the other materials. When sand larger than that of the second order is used, the composition is moistened with brandy, so that it forms itself into balls, and the jets are then loaded: if there were too much moisture, the sand would not perform its effect.

ARTICLE X.

Of Fires of Different Colours.

It is much to be wished that, for the sake of variety, different colours could be given to these fire-works at pleasure; but though we are acquainted with several materials which communicate to flame various colours, it has hitherto been possible to introduce only a very few colours into that of inflamed gunpowder.

To make white fire, the gunpowder must be mixed with iron or rather steel-filings.

To make red fire, iron sand of the first order must be employed in the same manner.

As copper filings, when thrown into a flame, render it green, it might be concluded, that if mixed with gunpowder, it would produce a green flame: but this experiment does not succeed. It is supposed that the flame

is too ardent, and consumes the inflammable part of the copper too soon. But it is probable that a sufficient number of trials have not yet been made; for is it not possible to lessen the force of gunpowder in a considerable degree, by increasing the dose of the charcoal?

However, the following are a few of those materials which, in books on pyrotechny, are said to possess the property of communicating various colours to fire-works.

Camphor mixed with the composition, makes the flame to appear of a pale white colour.

Raspings of ivory give a clear flame of a silver colour, inclining a little to that of lead; or rather a white dazzling flame.

Greek pitch produces a reddish flame, of a bronze colour.

Black pitch, a dusky flame, like a thick smoke, which obscures the atmosphere.

Sulphur, mixed in a moderate quantity, makes the flame appear bluish.

Sal ammoniac and verdigrise give a greenish flame.

Raspings of yellow amber communicate to the flame a lemon colour.

Crude antimony gives a russet colour.

Borax ought to produce a blue flame; for spirit of wine, in which sedative salt, one of the component parts of borax, is dissolved by the means of heat, burns with a beautiful green flame.

Much, however, still remains to be done in regard to this subject; but it would add to the beauty of artificial fire-works, if they could be varied by giving them different colours: this would be creating for the eyes a new pleasure.

ARTICLE XI.

Composition of a Paste proper for representing animals and other devices in fire.

It is to the Chinese also that we are indebted for this

method of representing figures with fire. For this purpose, take sulphur reduced to an impalpable powder, and having formed it into a paste with starch, cover with it the figure you are desirous of representing on fire: it is here to be observed, that the figure must first be coated over with clay, to prevent it from being burnt.

When the figure has been covered with this paste, besprinkle it while still moist with pulverised gunpowder; and when the whole is perfectly dry, arrange some small matches on the principal parts of it, that the fire may be speedily communicated to it on all sides.

The same paste may be employed on figures of clay, to form devices and various designs. Thus, for example, festoons, garlands, and other ornaments, the flowers of which might be imitated by fire of different colours, could be formed on the frieze of a piece of architecture covered with plaster. The Chinese imitate grapes exceedingly well, by mixing pounded sulphur with the pulp of the jujube, instead of flour paste.

ARTICLE XII.

Of Suns, both Fixed and Moveable.

None of the pyrotechnic inventions can be employed with so much success, in artificial fire-works, as suns; of which there are two kinds, fixed and revolving: the method of constructing both is very simple.

For fixed suns, cause to be constructed a round piece of wood, into the circumference of which can be screwed twelve or fifteen pieces in the form of radii; and to these radii attach jets of fire, the composition of which has been already described; so that they may appear as radii tending to the same centre, the mouth of the jet being towards the circumference. Apply a match in such a manner, that the fire communicated at the centre may be conveyed, at the same time, to the mouth of each of the jets, by which means, each throwing out its fire, there will be produced the appearance of a radiating sun. We here sup-

pose that the wheel is placed in a position perpendicular to the horizon.

These rockets or jets may be so arranged as to cross each other in an angular manner; in which case, instead of a sun, you will have a star, or a sort of cross resembling that of Malta. Some of these suns are made also with several rows of jets; these are called *glories*.

Revolving suns may be constructed in this manner. Provide a wooden wheel, of any size at pleasure, and brought into perfect equilibrium around its centre, in order that the least effort may make it turn round. Attach to the circumference of it fire-jets placed in the direction of the circumference; they must not be choaked at the bottom, and ought to be arranged in such a manner, that the mouth of the one shall be near the bottom of the other, so that when the fire of the one is ended, it may immediately proceed to another. It may be easily perceived, that when fire is applied to one of these jets, the recoil of the rocket will make the wheel turn round, unless it be too large and ponderous: for this reason, when these suns are of a considerable size, that is when they consist for example of 20 rockets, fire must be communicated at the same time to the first, the sixth, the eleventh, and the sixteenth; from which it will proceed to the second, the seventh, the twelfth, the seventeenth, and so on. These four rockets will make the wheel turn round with rapidity.

If two similar suns be placed one behind the other, and made to turn in a contrary direction, they will produce a very pretty effect of cross-fire.

Three or four suns, with horizontal axes passing through them, might be implanted in a vertical axis, moveable in the middle of a table. These suns, revolving around the table, will seem to pursue each other. It may be easily perceived that, to make them turn around the table, they must be fixed on their axes, and these axes, at the place

where they rest on the table, ought to be furnished with a very moveable roller.

We shall say nothing farther on artificial fire-works; because it is not possible in this work to give a complete treatise of pyrotechny. We shall therefore content ourselves with pointing out, to those who are fond of this art, a few of the best authors on the subject. One is, *Traité des Feux d'artifice* de M. Frezier, a new edition of which was published in 1745. We shall mention also the work of M. Perrinet d'Orval, entitled *Traité des Feux d'artifice, pour le Spectacle et pour la Guerre.* To these we may add *Le Manuel de l'Artificier*, Paris 1757, 12mo. which contains, in a very small compass, the whole substance of the art of making artificial fire-works: it is an abridgment of the latter work, augmented with several new and curious compositions, in regard to the Chinese fire, by Father d'Incarville.

ARTICLE XIII.

Of Ointment for Burns.

It is proper that we should terminate a treatise on pyrotechny by some remedy for burns; as accidents must often take place in handling such a dangerous element as fire. We shall therefore not hesitate to follow the example of Ozanam, who in this respect is himself a follower of Siemienowitcz, and the greater part of those who have written on this subject: we shall even confine ourselves to the remedy he proposes.

Boil fresh hog's lard in common water, over a slow fire; skim it continually till no more scum is left, and let the melted lard remain in the open air for three or four nights Melt it again in an earthen vessel, over a slow and moderate fire, and strain it into cold water through a piece of linen cloth; then wash it well in pure river or spring water, to free it from its salt, and to make it become white; then press it into a glazed earthen vessel and preserve it for use.

It generally happens, in cases of burning, that the skin rises in blisters, which however must not be opened till the third or fourth day after the ointment has been applied.

ARTICLE XIV.

Pyrotechny without Fire, and merely Optical.

As the inventions which we have here described are necessarily attended with considerable expense, and are besides dangerous, attempts have been made in modern times, and with a considerable degree of success, to imitate the different kinds of fire-works by optical effects, and to give them the appearance of motion, though in reality fixed. By means of this invention, the spectacle of artificial fire-works may be exhibited at a very small expence; and if the pieces employed are constructed with ingenuity, if the rules of perspective are properly observed, and if, in viewing the spectacle, glasses which magnify the objects and render them somewhat less distinct be employed, a very agreeable illusion will be produced.

The artificial fire-works imitated with most success by this invention, are fixed suns, gerbes and jets of fire, cascades, globes, pyramids, and columns moveable around their axes. To represent a gerbe of fire, take paper blackened on both sides, and very opake, and having delineated on a piece of white paper the figure of a gerbe of fire, apply it to the black paper, and with the point of a very sharp penknife make several slashes (pl. 2, fig. 14) in it, as 3, 5, or 7, proceeding from the origin of the gerbe: these lines must not be continued but cut through at unequal intervals. Pierce these intervals with unequal holes made with a pinking iron, pl. 2 fig. 14, in order to represent the sparks of such a gerbe. In short you must endeavour to paint, by these lines and holes, the well-known effect of the fire of inflamed gunpowder, when it issues through a small aperture.

According to the same principles, you may delineate the

cascades (fig. 16) and jets of fire which you are desirous of introducing into this exhibition, which is purely optical; and those jets of fire which proceed from the radii of suns, either fixed or moveable. It may easily be conceived that in this operation taste must be the guide.

If you are desirous of representing globes, pyramids, or revolving columns, draw the outlines of them on paper, and then cut them out in a helical form; that is, cut out spirals with the point of a pen-knife, and of a size proportioned to that of the piece.

It is to be observed also, that as these different pieces have different colours, they may be easily imitated, by pasting on the back of the paper, cut as here described, very fine silk paper coloured in the proper manner. As jets, for example, when loaded with Chinese fire, give a reddish light, you must paste to the back of these jets transparent paper, slightly tinged with red; and proceed in the same manner in regard to the other colours by which the different fire-works are distinguished.

When these preparations have been made, the next thing is to give motion, or the appearance of motion, to this fire, which may be done two ways according to circumstances.

If a jet of fire, for example, is to be represented, prick unequal holes, and at unequal distances from each other, in a band of paper, pl. 2 fig. 17, and then move this band, making it ascend between a light and the above jet: the rays of light which escape through the holes of the moveable paper will exhibit the appearance of sparks rising into the air. It is to be observed that one part of the paper must be whole, that another must be pierced with holes thinly scattered; that in another place they must be very close, and then moderately so: by these means it will represent those sudden jets of fire observed in fire-works.

To represent a cascade, the paper pierced with holes, instead of moving upwards, must be made to descend.

This motion may be easily produced by means of two

rollers, on one of which the paper is rolled up while it is unrolled from the other.

Suns are attended with some more difficulty; because in these it is necessary to represent fire proceeding from the centre to the circumference. The artifice for this purpose is as follows.

On strong paper describe a circle, equal in diameter to the sun which you are desirous to exhibit, or even somewhat larger; then trace out on this circle two spirals, at the distance of a line or half a line from each other, and open the interval between them with a penknife, in such a manner, that the paper may be cut from the circumference, decreasing in breadth to a certain distance from the centre, pl. 2 fig. 18; cut the remainder of the circle into spirals of the same kind, open and close alternately, then cement the paper circle to a small iron hoop, supported by two pieces of iron, crossing each other in its centre, and adjust the whole to a small machine, which will suffer it to revolve round its centre. If this moveable paper circle, cut in this manner, be placed before the representation of your sun, with a light behind it, as soon as it is made to move towards that side to which the convexity of the spirals is turned, the luminous spirals, or those which afford a passage to the light, will give, on the image of the radii or jets of fire of your sun, the appearance of fire in continual motion, as if undulating from the centre to the circumference.

The appearance of motion may be given to columns, pyramids, and globes, cut through in the manner above described, by moving upwards, in a vertical direction, a band of paper cut through into apertures inclined at an angle somewhat different from that of the spirals. By these means the spectators will imagine that they see fire continually circulating and ascending along these spirals; and the result will be a sort of illusion, in consequence of which the columns or pyramids will seem to revolve with them.

But we shall not enlarge farther on this subject; it is sufficient to have explained the principle on which this cheap kind of pyrotechny can be exhibited; the taste of the artist may suggest to him many things to give more reality to this representation, and to render the deception stronger.

We shall however add a few words respecting illuminations which form a part of pyrotechny.

Take some prints representing a castle, or palace, &c; and having coloured them properly, cement paper to the back of them, in such a manner that they shall be only semi-transparent; then, with pinking irons of different sizes, prick small holes in the places and on the lines where lamps are generally placed, as along the sides of the windows, on the cornices, on balustrades, &c. But care must be taken to make these holes smaller and closer, according to the perspective diminution of the figure. With other irons of a larger size, cut out, [illegible] places, some stronger lights; so as to represent the [illegible] &c. Cut out also the panes in some of the windows, and cement to the back of them transparent paper of a green or red colour, to represent curtains drawn before them, and concealing an illuminated apartment.

When the print is cut in this manner, place it in the front of a sort of small theatre, strongly illuminated from the back part, and look at it through a convex glass of a pretty long focus, like that used in those small machines called optical boxes. If the rules of perspective have been properly observed in the prints, and if the lights and shades have been distributed with taste, this spectacle will be highly agreeable. It may be intermixed with some of the pyrotechnic artifices above described; as such illuminations are in general accompanied with fire-works.

END OF THE THIRD VOLUME.

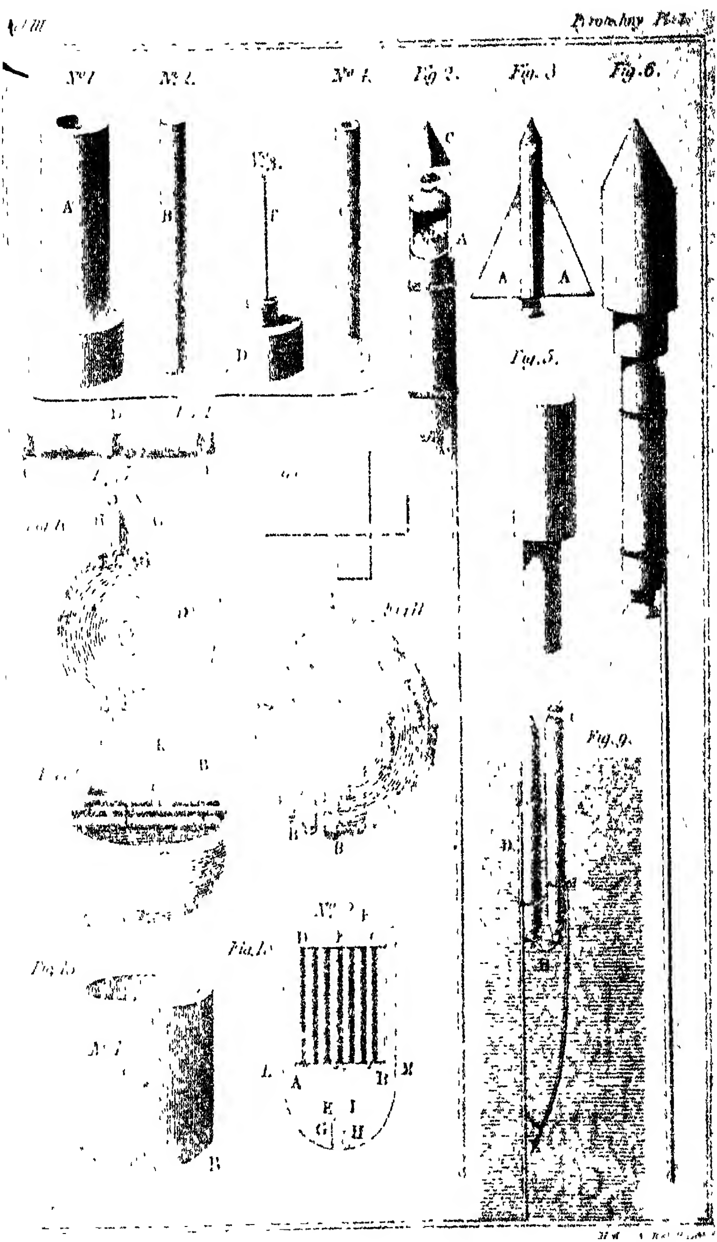


Fig. 2.
Fig. 3
Fig. 6.
Fig. 5.
Fig. 9.